School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

MA4822

Measurement and Sensing Systems

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Outline

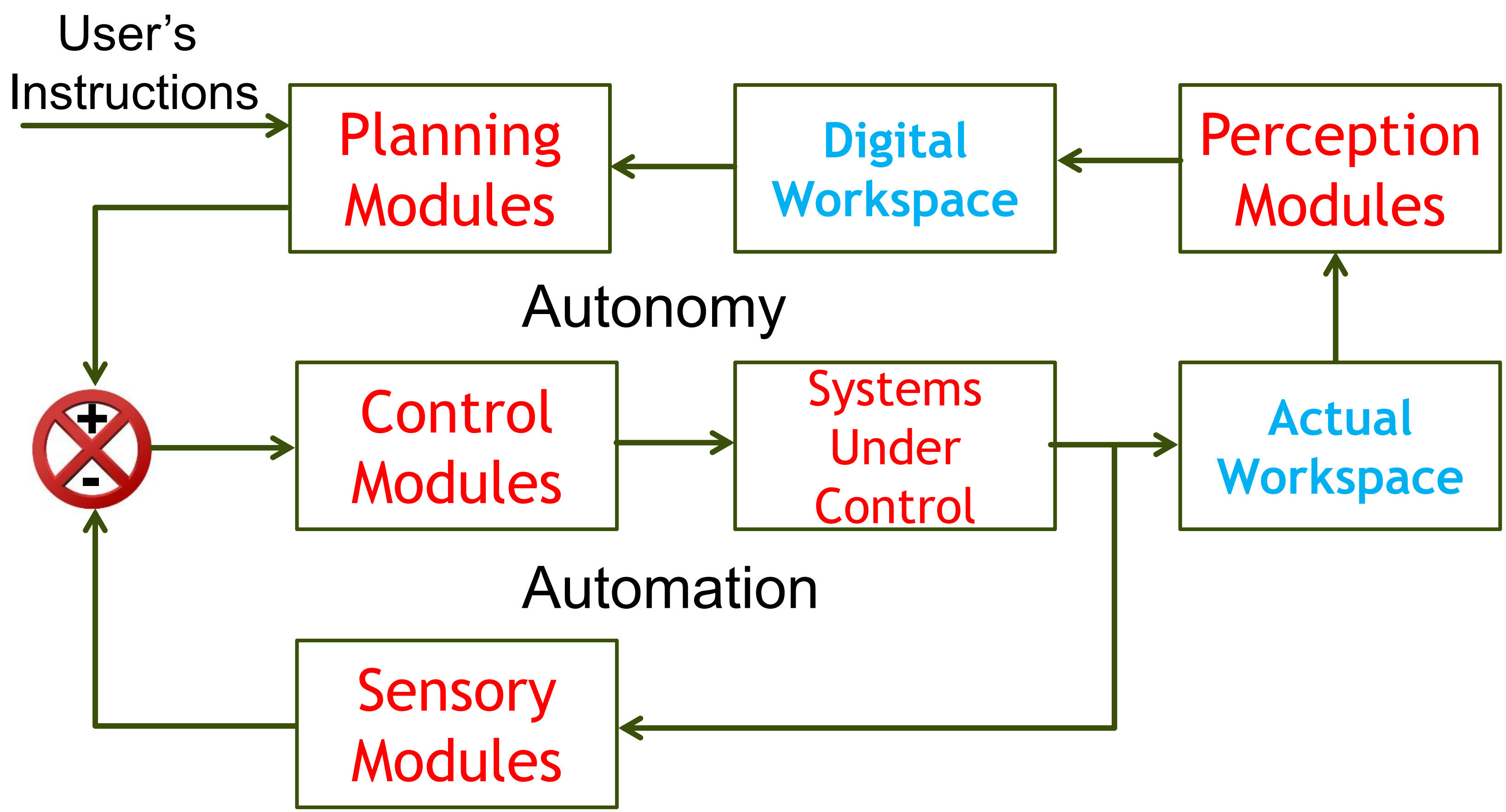
- Module 1: Foundation of AI Sensors
- Module 2: Sensors for Electrical Systems
- Module 3: Sensors for Mechanical Systems
- Module 4: Sensors for All Environments
- Module 5: Sensors for All Industries

Remember your mission as MAE undergraduates …

- You are here to grow your knowledge and skills so as to be able to design machines with controllable behaviors and hopefully in some intelligent ways.

How to fulfill your mission?

- To apply learnt knowledge and skills into the implementation of the following universal blueprint underlying all the intelligent machines or systems.



Why to study this course?

We are living inside an ocean of signals

How to study this course?

- To put yourselves into the mindset of designers of networked sensors as products:
  - Who are the users?
  - What are the needs of users?
  - What are your Internet of Sensors, which could meet the needs of your users or buyers?
  - What are the solutions behind the design of your Internet of Sensors?

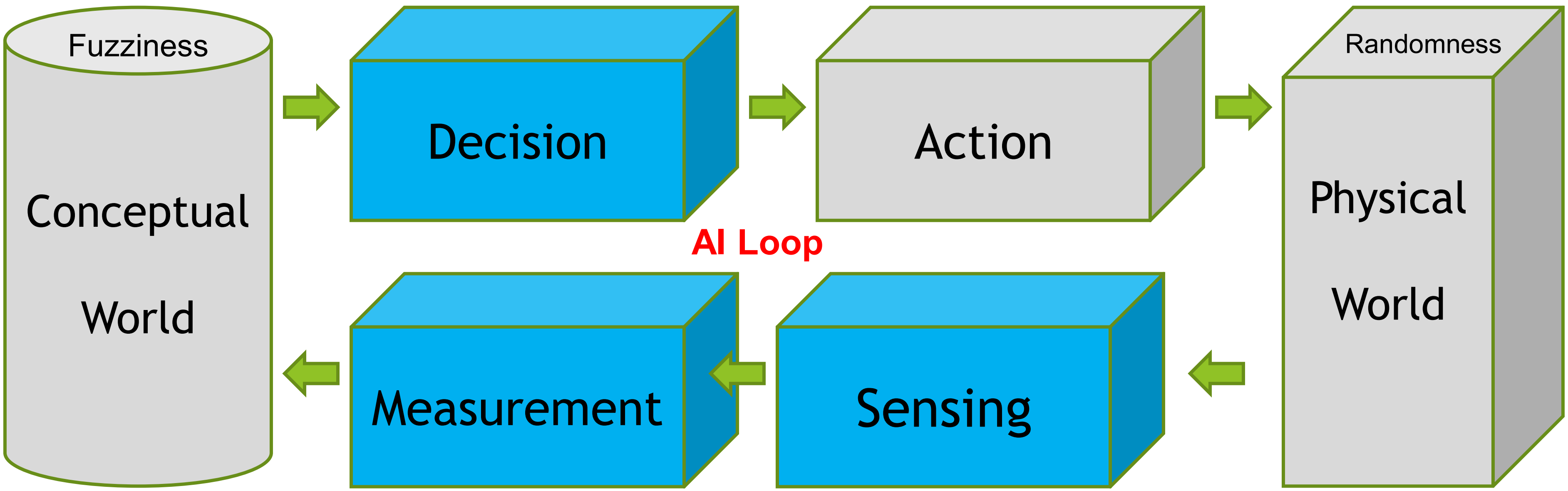
Practice with MATLAB

Market Demands or Needs
↓
Product Specifications
↓
Design Specifications
↓
Conceptual Design
↓
Selection of Materials/Components/Devices
↓
Embodiment Design
↓
Prototyping
↓
Optimizing
↓
Production
↓
Marketing

What are you going to study in this course?

Module 1: Foundation of AI Sensors

- Basics of Physical World
- Randomness of Physical World
- Basics of Conceptual Worlds
- Fuzziness of Conceptual Worlds



Module 2: For Electrical Systems

- Measurement of Voltage
- Measurement of Current
- Measurement of Resistance
- Measurement of Capacitance
- Measurement of Inductance

Module 3: For Mechanical Systems

- Measurement of Position
- Measurement of Velocity
- Measurement of Acceleration
- Measurement of Force
- Measurement of Torque

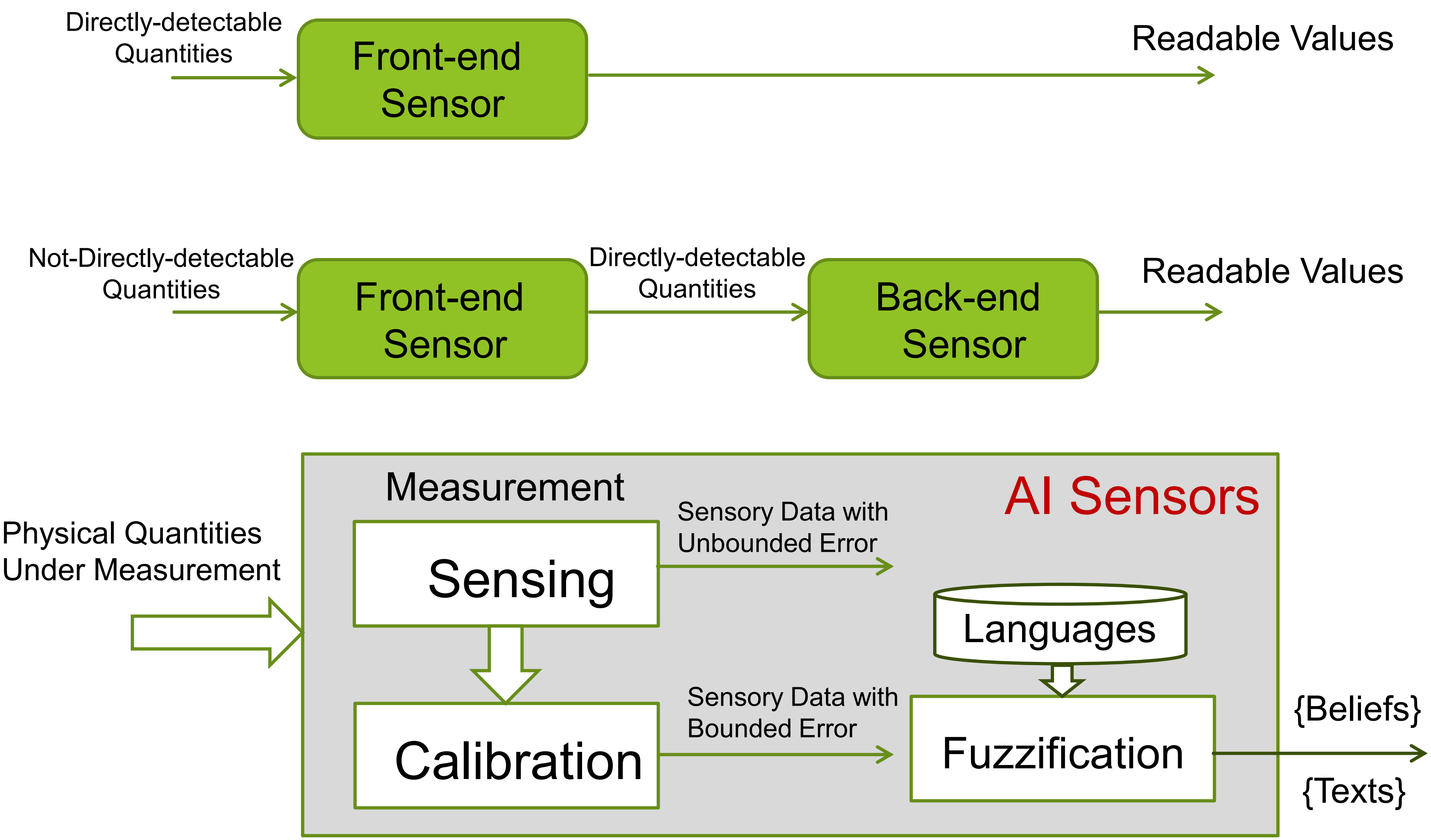
Module 4: For All Environments

- Measurement of Pressure
- Measurement of Temperature
- Measurement of Humidity
- Measurement of Vibration
- Measurement of Air Quality

Module 5: For All Industries

- Measurement of Fluid Level
- Measurement of Flow Rate
- Measurement of Sound/Voice
- Measurement of Photometry
- Measurement of Geometry

How to apply?


Directly-detectable Quantities
Front-end Sensor
Readable Values
Not-Directly-detectable Quantities
Front-end Sensor
Directly-detectable Quantities
Back-end Sensor
Readable Values
Physical Quantities Under Measurement
Measurement
Sensing
Calibration
Sensory Data with Unbounded Error
Sensory Data with Bounded Error
AI Sensors
Languages
Fuzzification
{Beliefs}
{Texts}

Today's Lectures …

- Module 1: Foundation of AI Sensors
- Module 2: Sensors for Electrical Systems
- Module 3: Sensors for Mechanical Systems
- Module 4: Sensors for All Environments
- Module 5: Sensors for All Industries

NANYANG TECHNOLOGICAL UNIVERSITY

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3

MA4822

Sensors for Mechanical Systems

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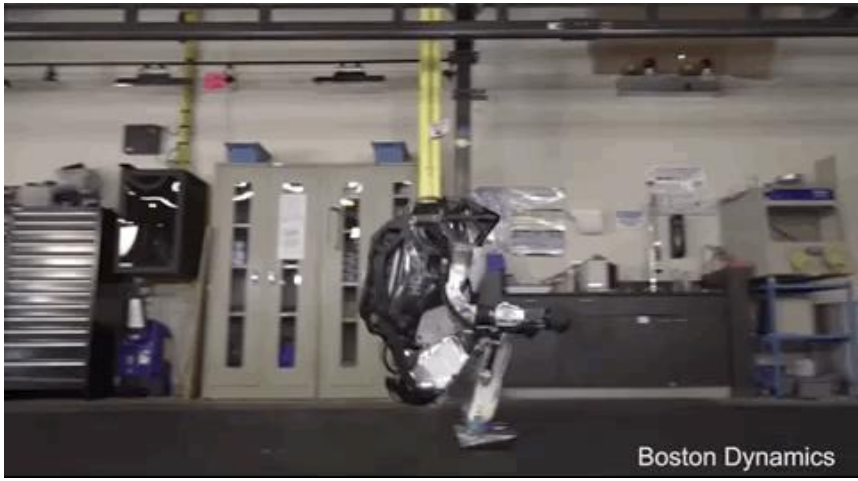
Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque

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School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3 Lecture 1

MA4822

Measurement of Position

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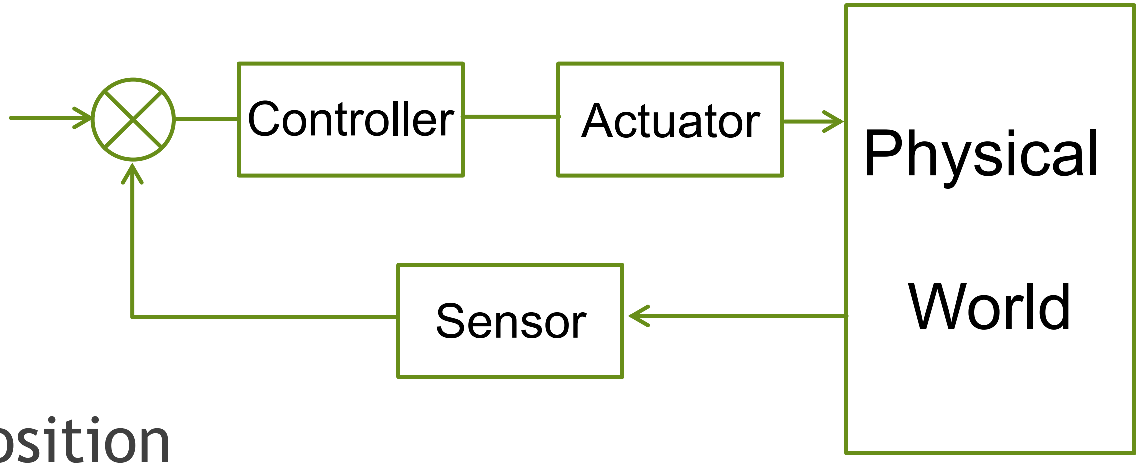
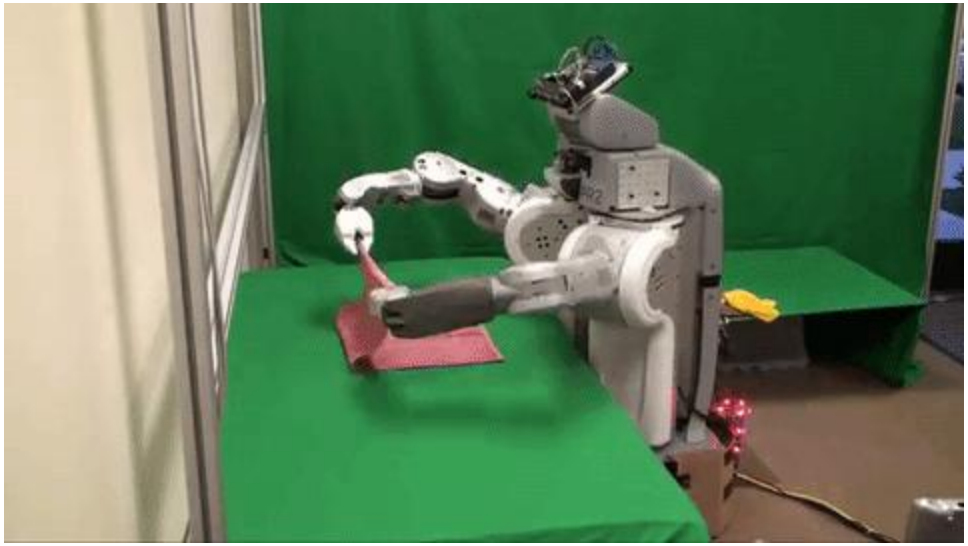
(Learning, Teaching) <o> (Research, Innovation) <o> (Leadership, Service)

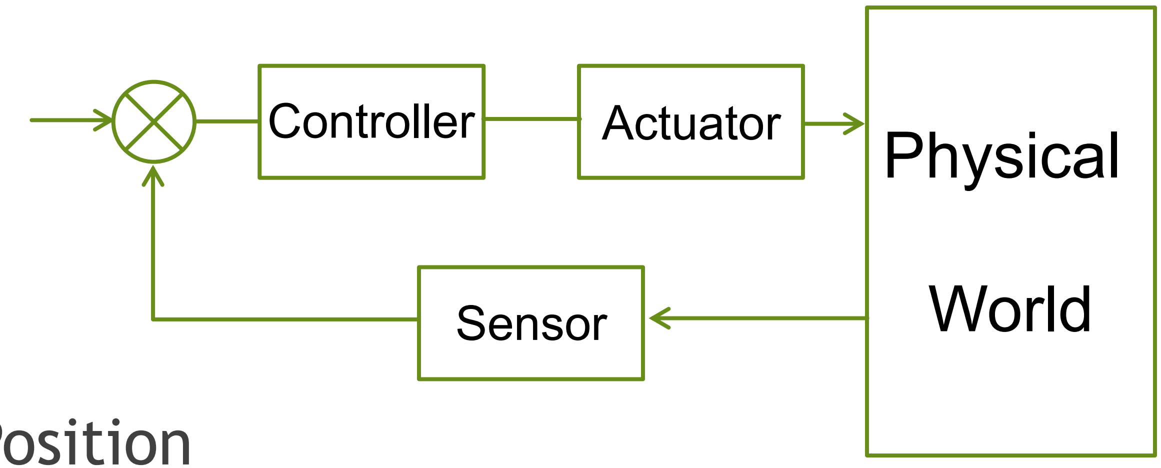
Outline



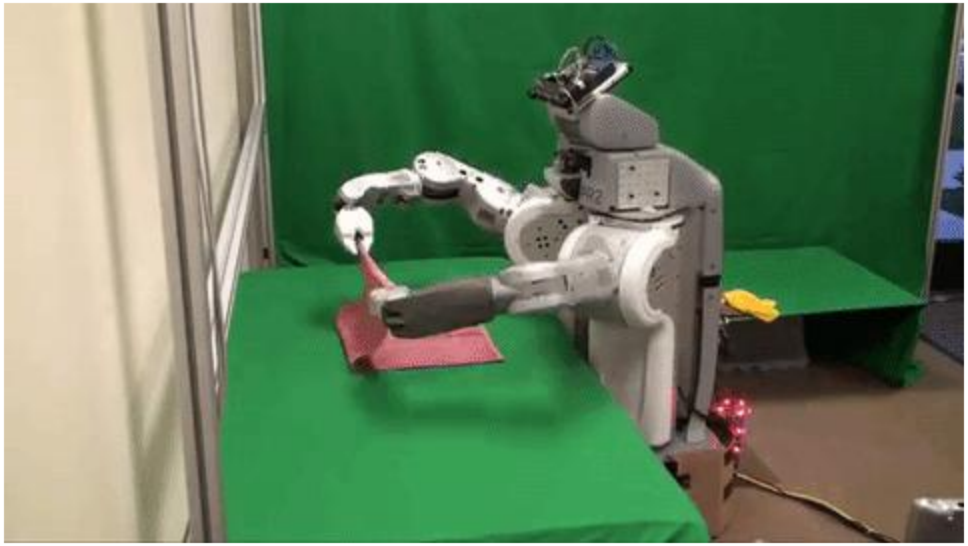
- Understanding of Position
- Computation of Position
- Measurement of Position



Outline



- Understanding of Position
- Computation of Position
- Measurement of Position



Physical World Consists of Entities

- Any existence in space and time is called an entity.

All entities have two common references

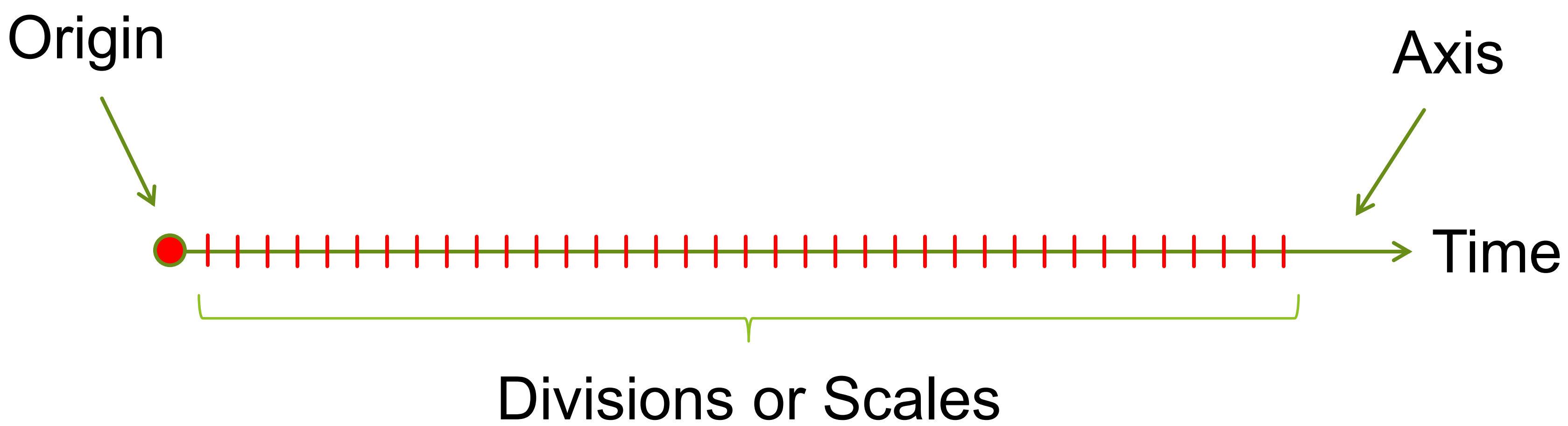
Reference for Space

- It provides reference to space
- It has origin
- It has multiple axes
- It has divisions or scales

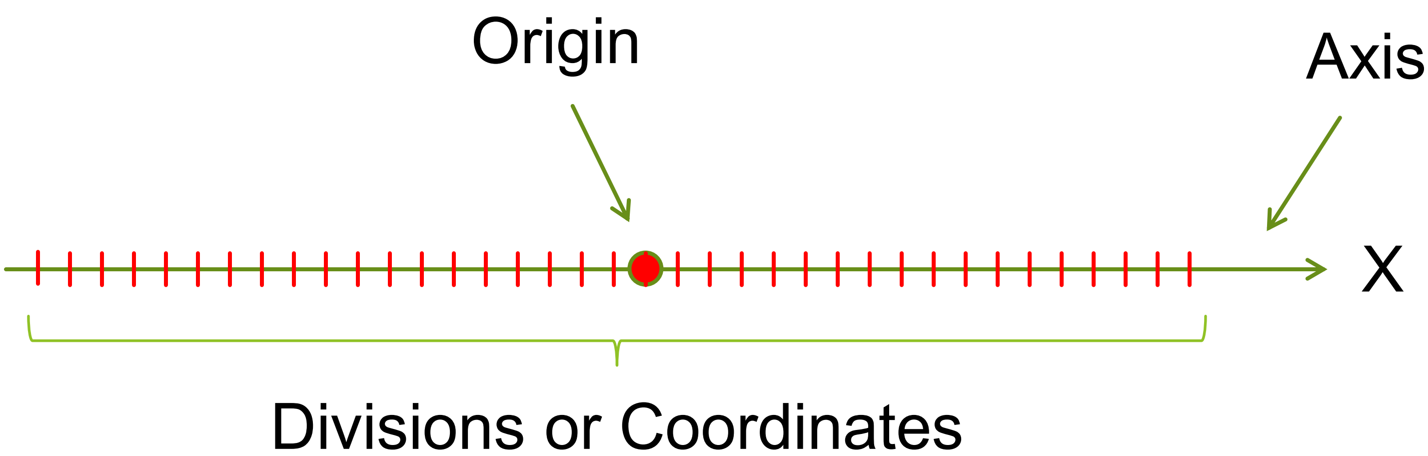
Reference for Time

- It provides reference to time
- It has origin
- It has one axis
- It has divisions or scales

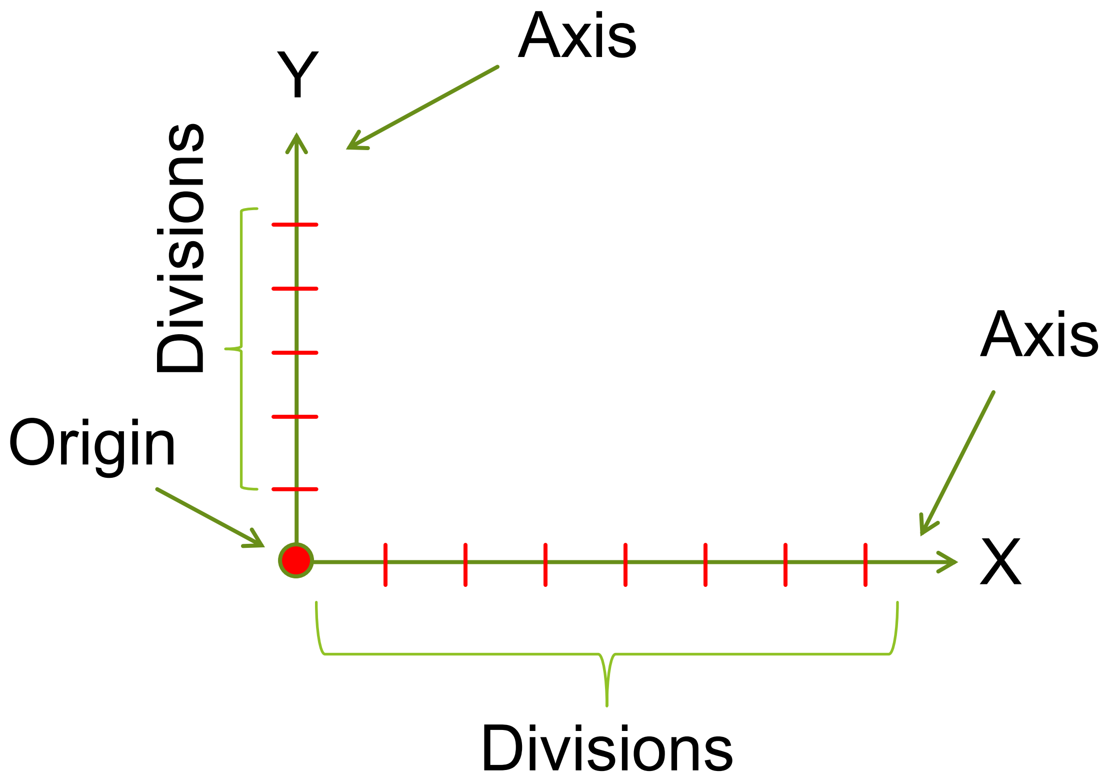
Reference for Time



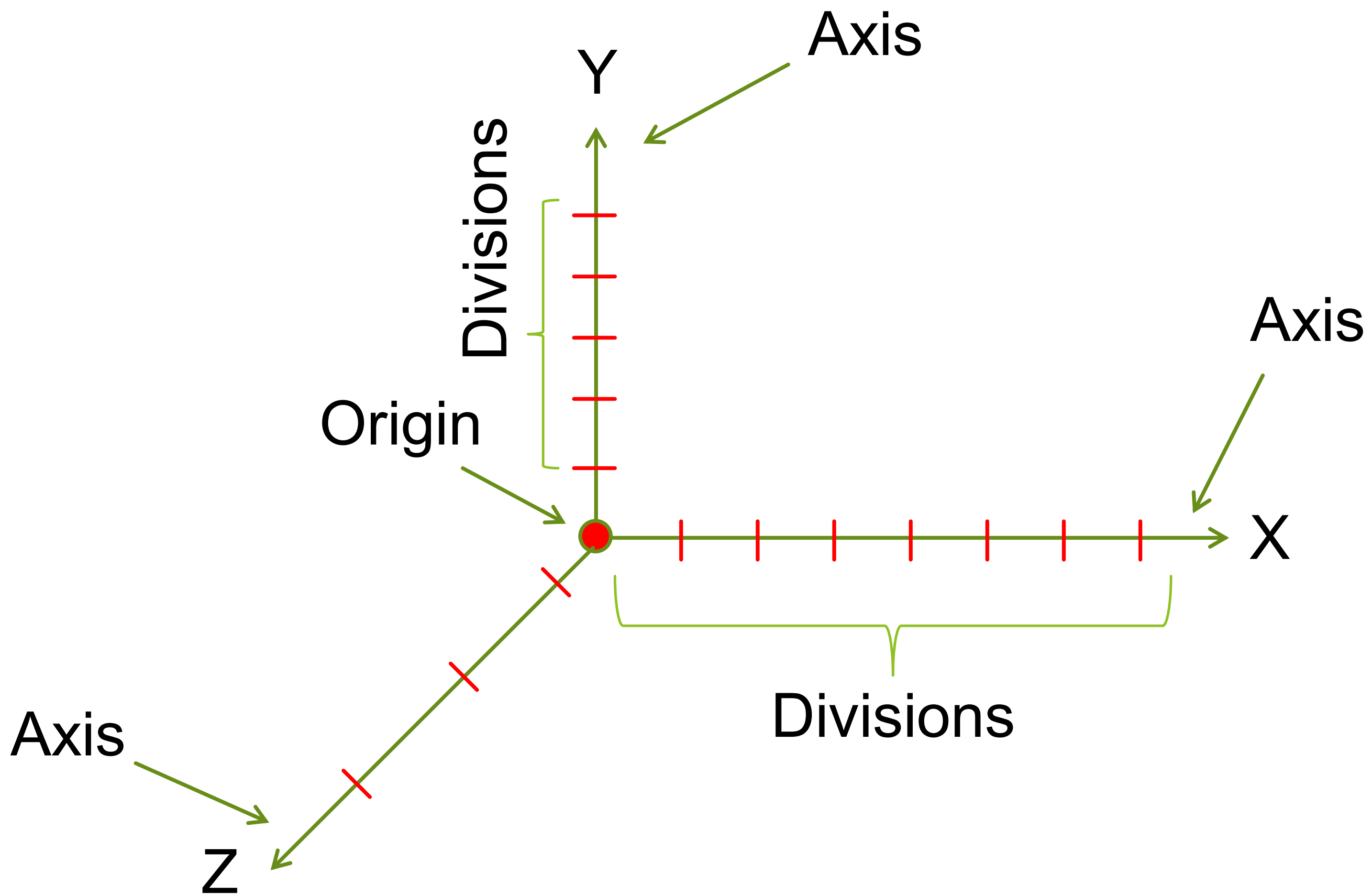
Reference for One-Dimensional Space



Reference for Two-Dimensional Space



Reference for Three-Dimensional Space

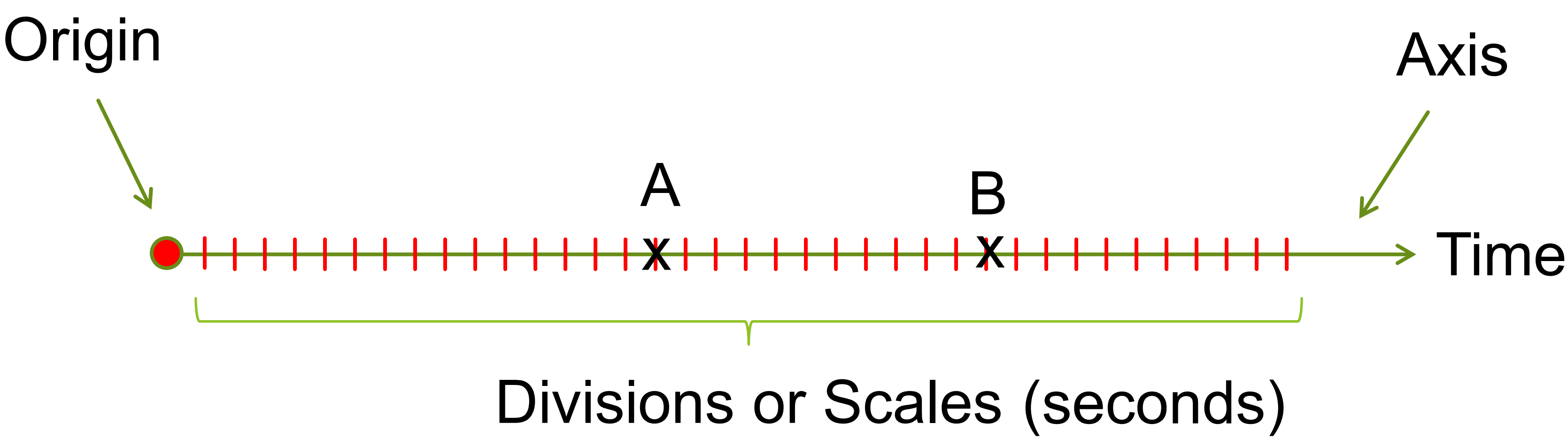


Definition of Coordinates

- A coordinate is the distance between the origin and a point along an axis.
- The distance is measured by the divisions or scales along an axis.

Example of Coordinates in Time

- What are the times at points A and B?
- Answer: $t_A = 16.0s$ $t_B = 27.0s$



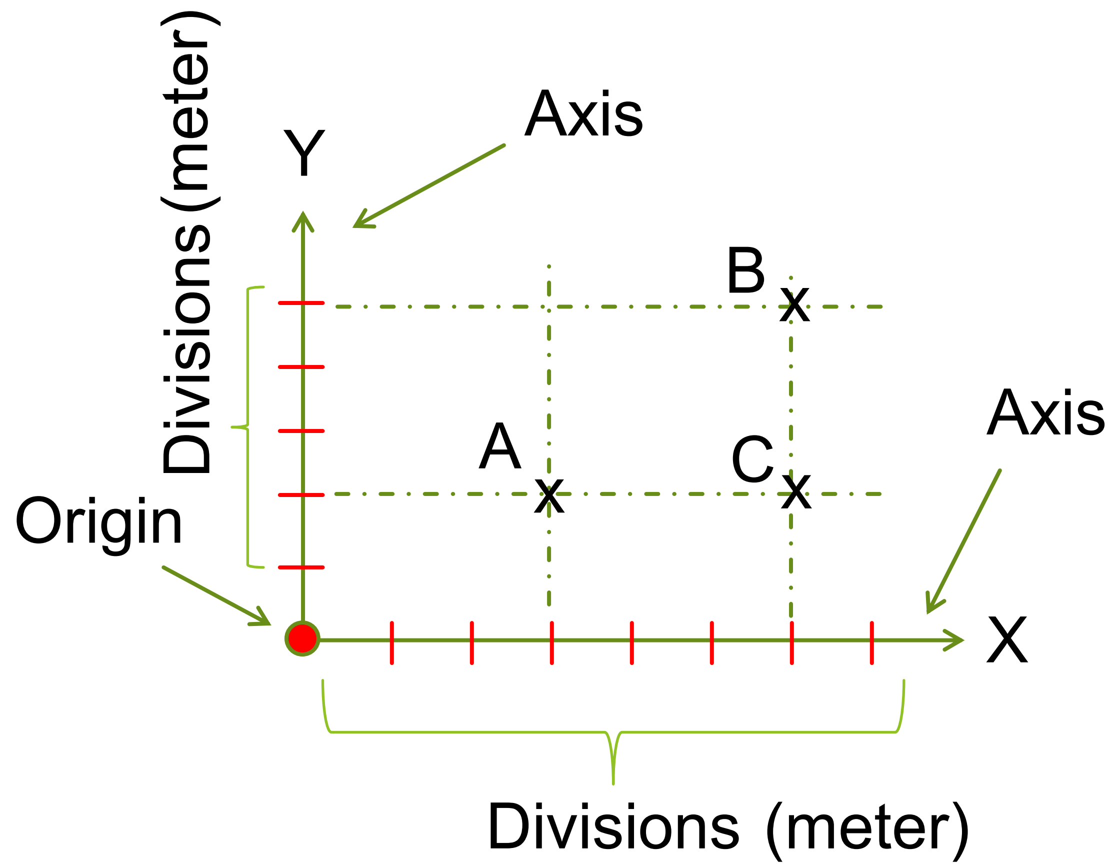
Example of Coordinates in Space

- What are the coordinates of points A and B?

- Answer:

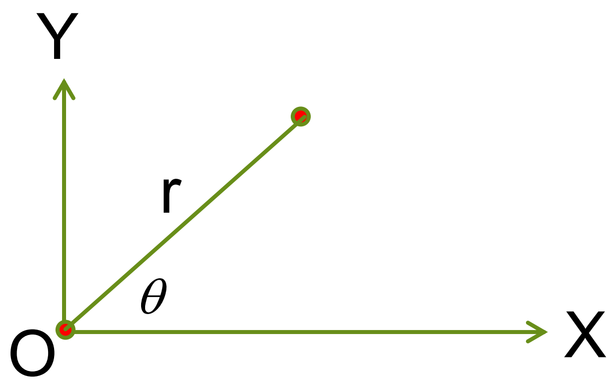
A=(3.0, 2.0) (m)

B=(6.0, 5.0) (m)



Polar Coordinates in Two-dimensional Space

- A polar coordinate of a point in space consists of two values:
  - One is called linear coordinate which is the distance between the origin and the point.
  - Another is called angular coordinate which is the angle between one axis of the reference and the line that connects the origin to the point.



Example of Polar Coordinates

- What are the polar coordinates of points A and B?
- Answer:

For point A:

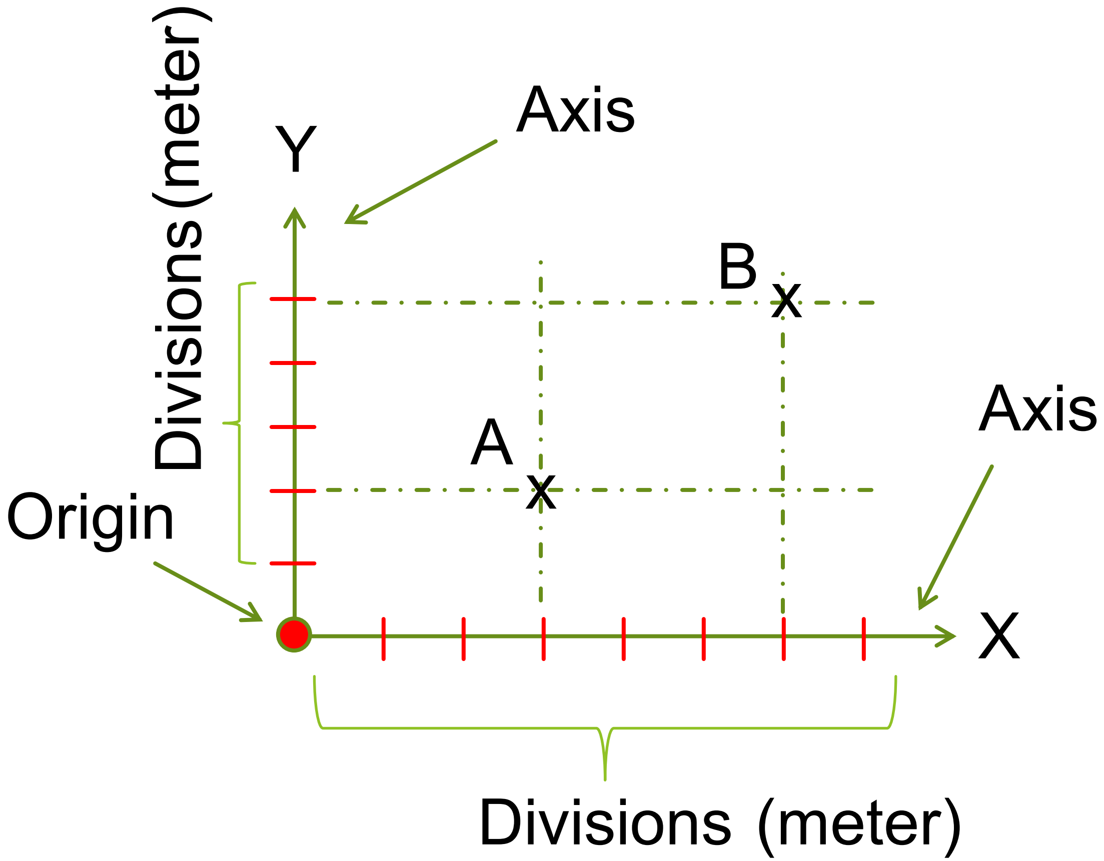
$$r_A = \sqrt{3^2 + 2^2}$$

$$\theta_A = \arctan(\frac{2}{3})$$

For point B:

$$r_B = \sqrt{6^2 + 5^2}$$

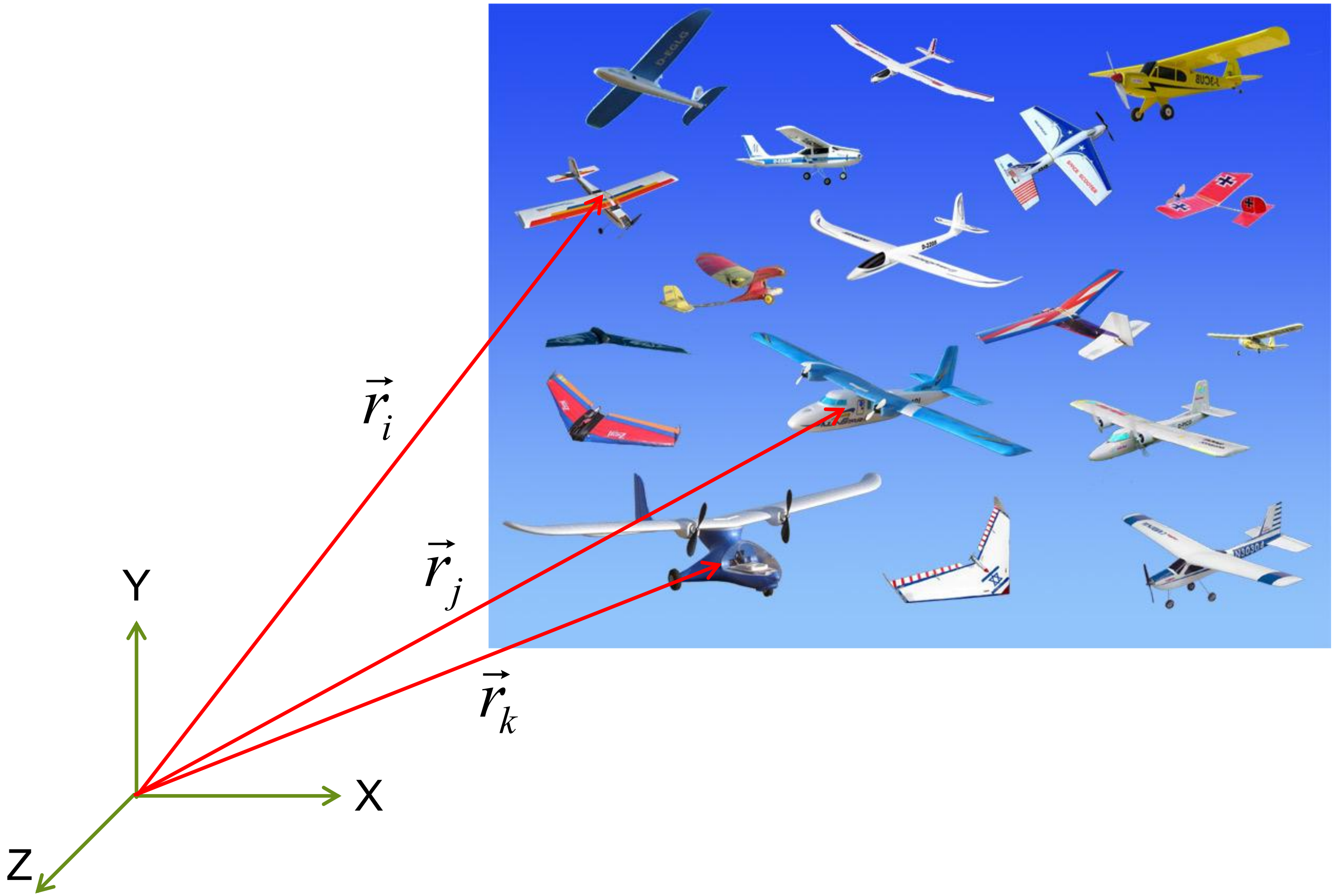
$$\theta_B = \arctan(\frac{5}{6})$$



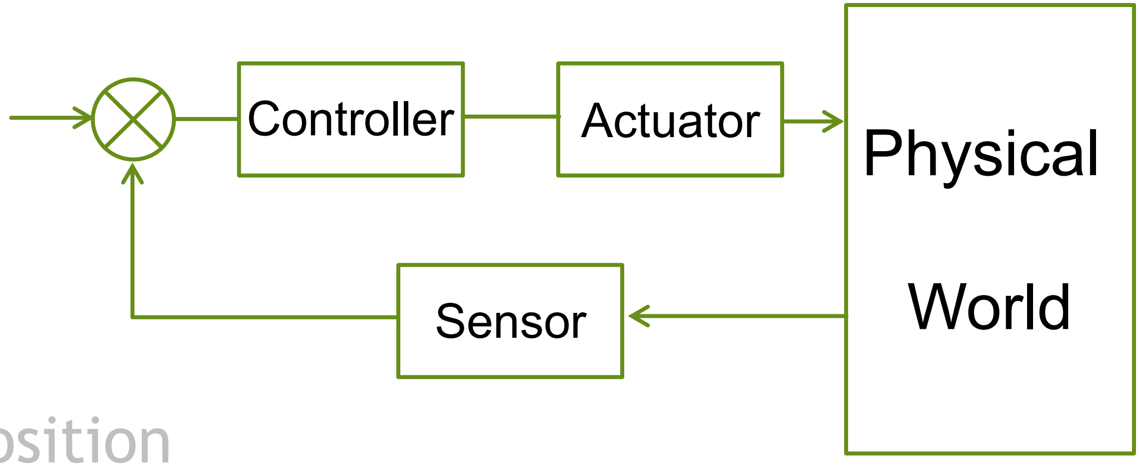
Definition of Positions

- The coordinates of a point with respect to a reference are called the position of the point.
- A reference of coordinates is also called a coordinate system.
- A position is normally represented by a vector.

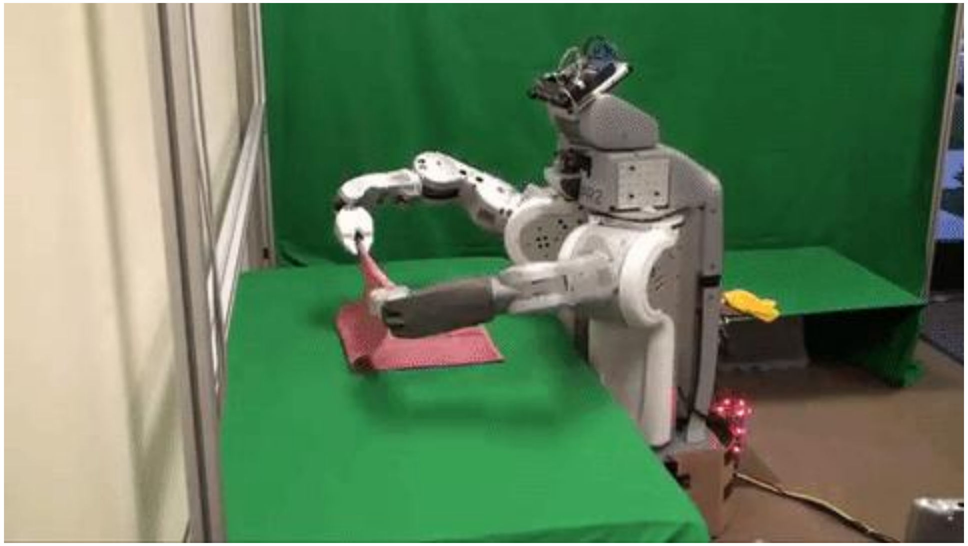
Example of Position Vectors



Outline



- Understanding of Position
- Computation of Position
- Measurement of Position



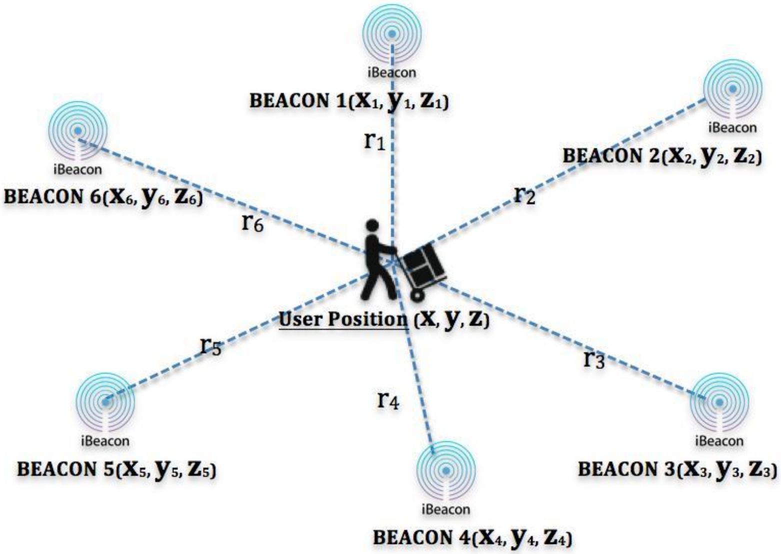
Observations

- Positions could be computed from the knowledge about velocities.
- Positions could be computed from the knowledge about accelerations.
- Two-dimensional positions could be computed from the knowledge about one-dimensional positions (i.e. distances).
- Three dimensional positions could also be computed from the knowledge about one-dimensional positions (i.e. distances).

Example of Computing Positions from Constant Velocities

- An object is moving along X axis with a constant velocity v. What is the time function of its positions?

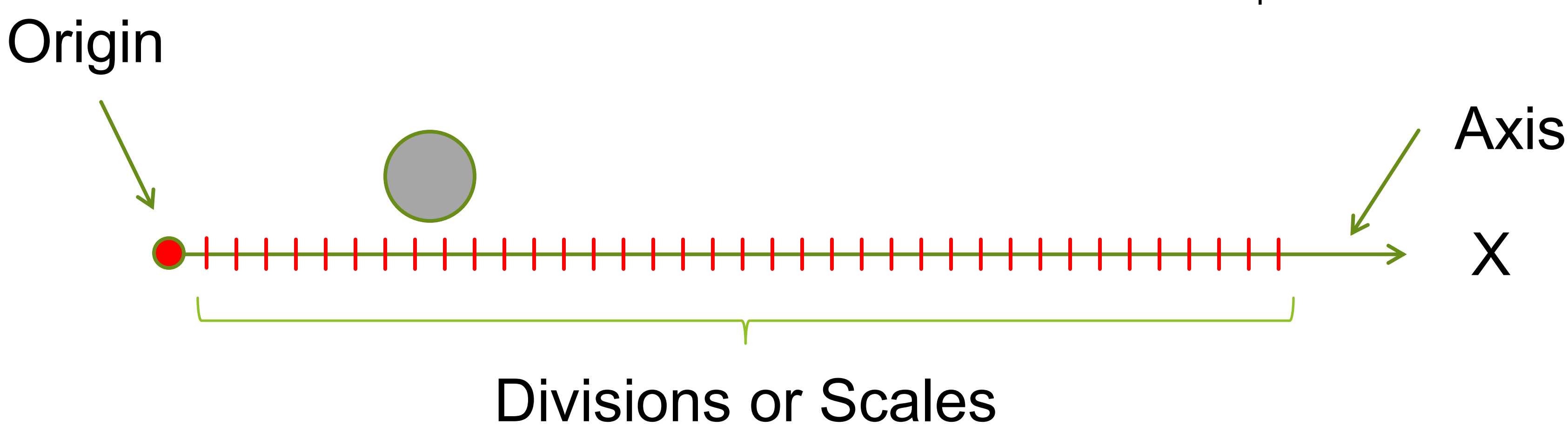
- Answer: $x(t) = x(0) + v \times t$



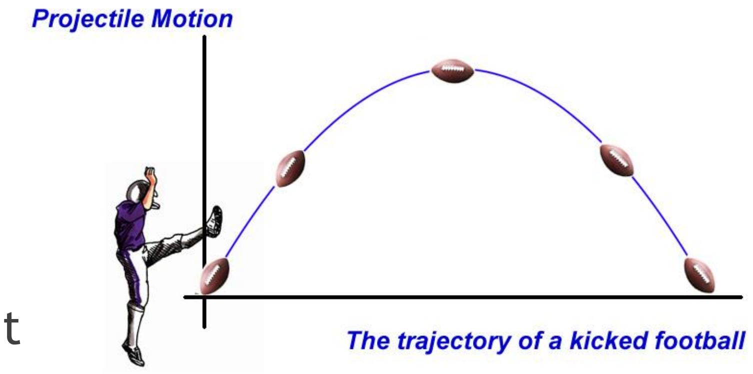
Signal could travel at a constant speed

⇩

Solution to compute distances

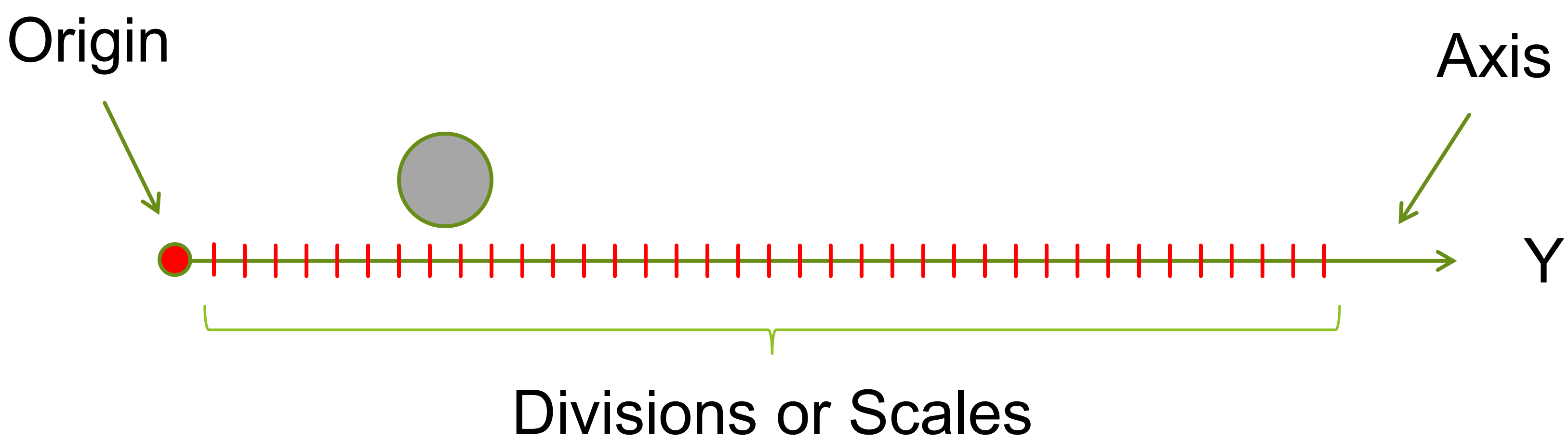


Example of Computing Positions from Constant Accelerations



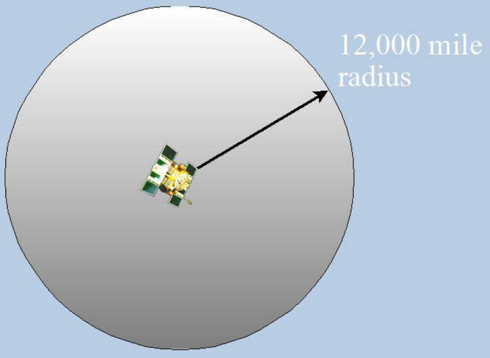
- An object is moving along Y axis with a constant acceleration a. If the object's initial velocity is v0, what is the time function of its positions?

- Answer: $$y(t) = y(0) + v_0 \times t + \frac{1}{2} a \times t^2$$

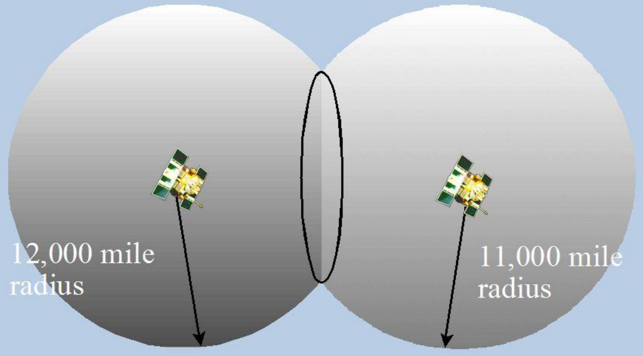


How to determine positions from distances?

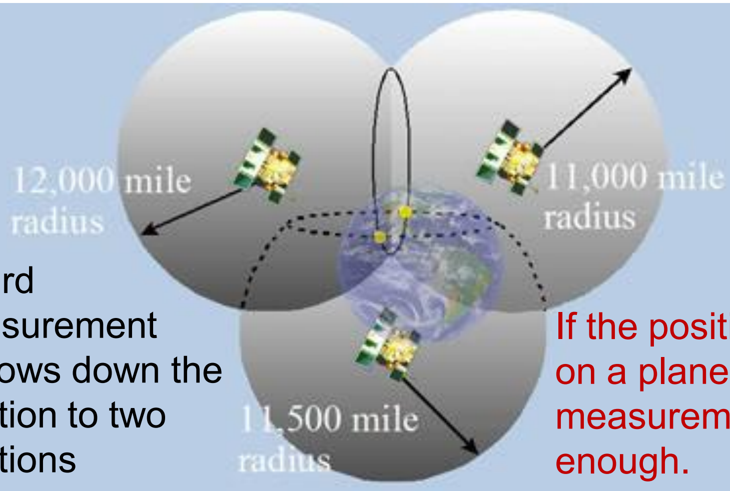
One measurement narrows down our position to the surface of a sphere



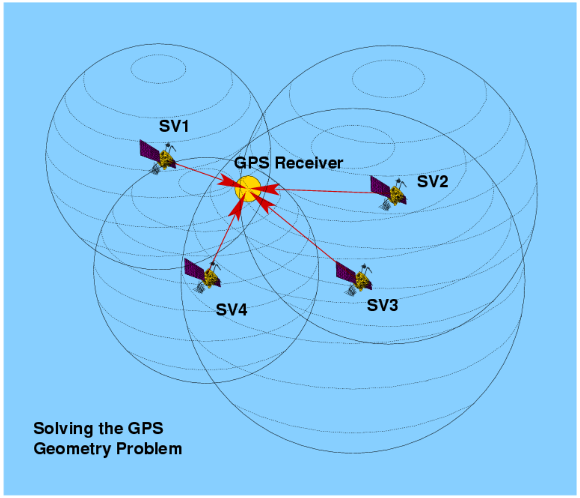
A second measurement narrows down our position to the intersection of two spheres



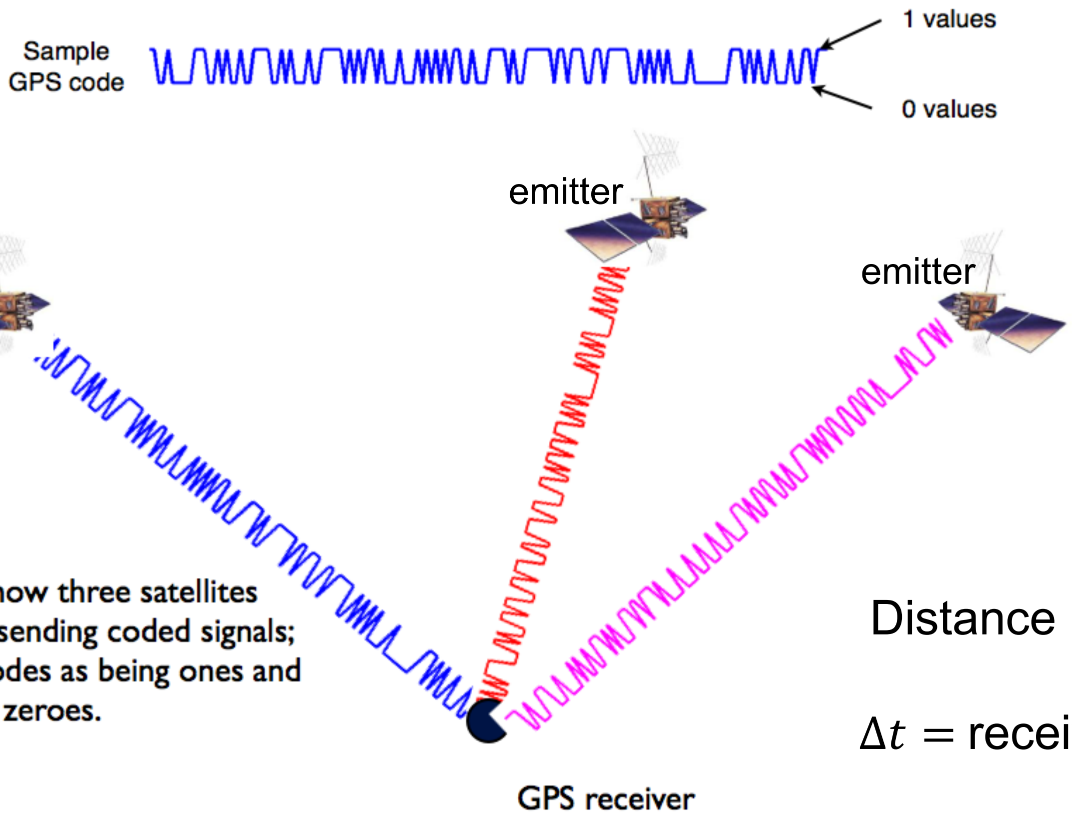
A third measurement narrows down the solution to two locations



If the positions are on a plane, three measurements are enough.



How to determine distances between two locations? Answer: To use coded-signal emitters and receivers



Here we show three satellites continuously sending coded signals; think of the codes as being ones and zeroes.

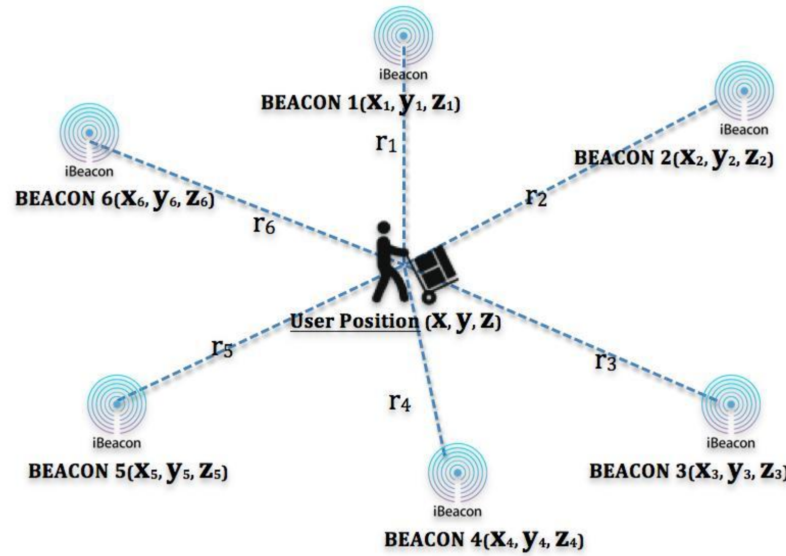
Distance = speed of light x Δt

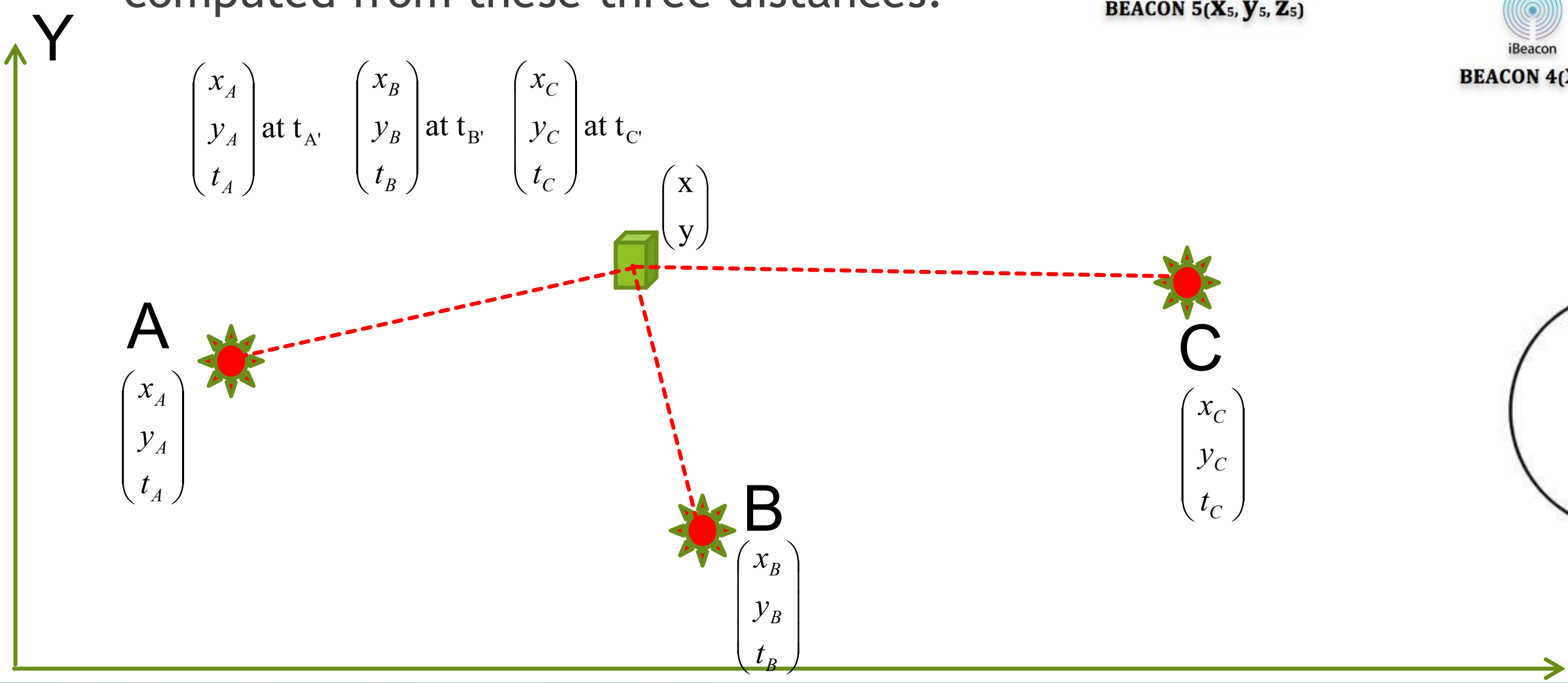
Δt = receiving time – sending time

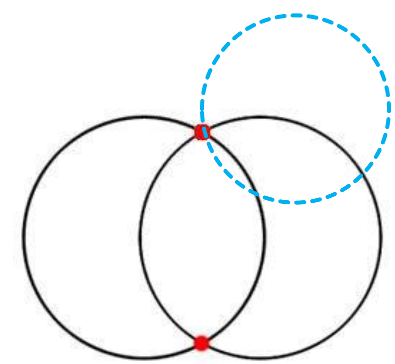
The receiver is going to try to decrypt each of the GPS signals separately.

Example of Computing Two-Dimensional Positions

- Three points define a two-dimensional space. Then, we put three time-signal emitters at three known points. When a target carries a time-signal receiver, it can measure three distances to these three time-signal emitters. Then, the coordinates of the target's position can be computed from these three distances.





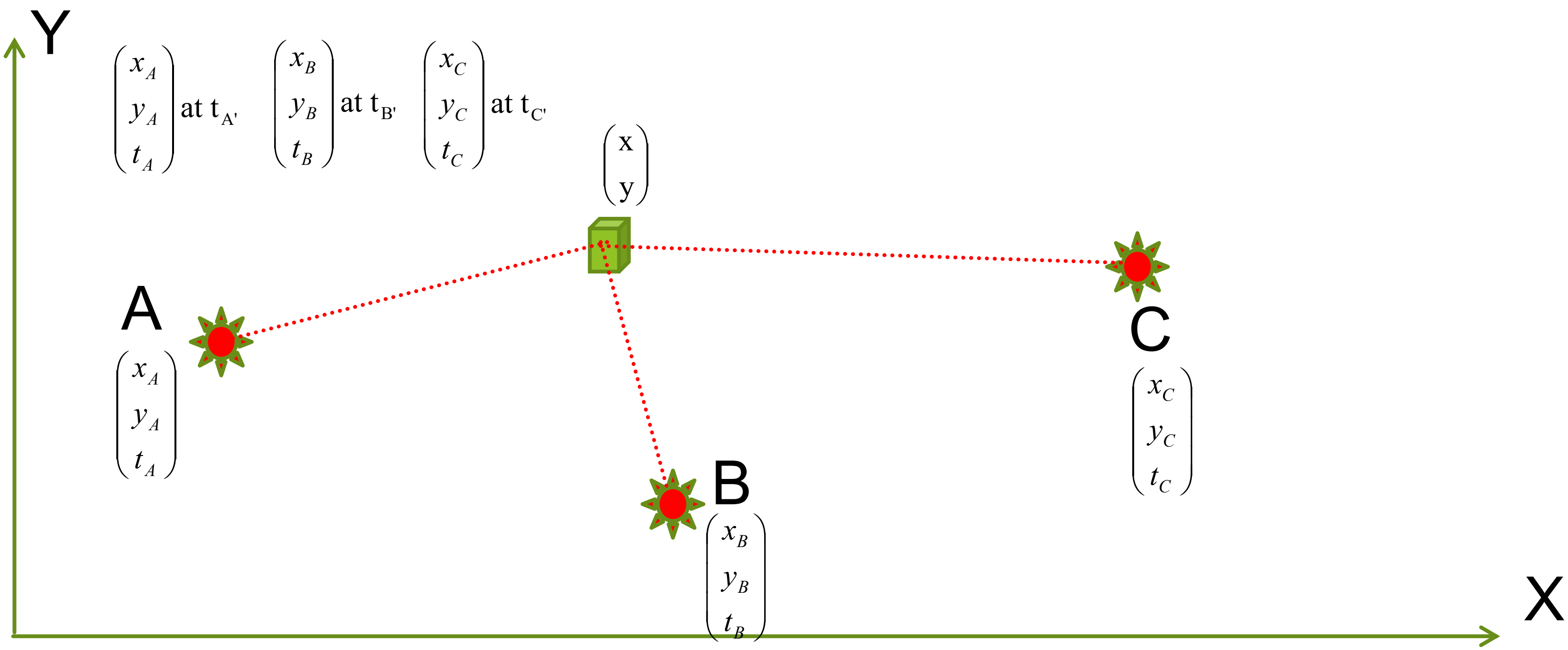


Details of Solution

- Equations of Three Circles:

$$\begin{cases}(x-x_A)^2+(y-y_A)^2=d_A^2=(v\bullet(t_{A'}-t_A))^2\\(x-x_B)^2+(y-y_B)^2=d_B^2=(v\bullet(t_{B'}-t_B))^2\\(x-x_C)^2+(y-y_C)^2=d_C^2=(v\bullet(t_{C'}-t_C))^2\end{cases}$$

v: Speed of signal



Details of Solution (continued)

$$\begin{cases} x^2 - 2x_A x + x_A^2 + y^2 - 2y_A y + y_A^2 = d_A^2 \\ x^2 - 2x_B x + x_B^2 + y^2 - 2y_B y + y_B^2 = d_B^2 \\ x^2 - 2x_C x + x_C^2 + y^2 - 2y_C y + y_C^2 = d_C^2 \end{cases}$$

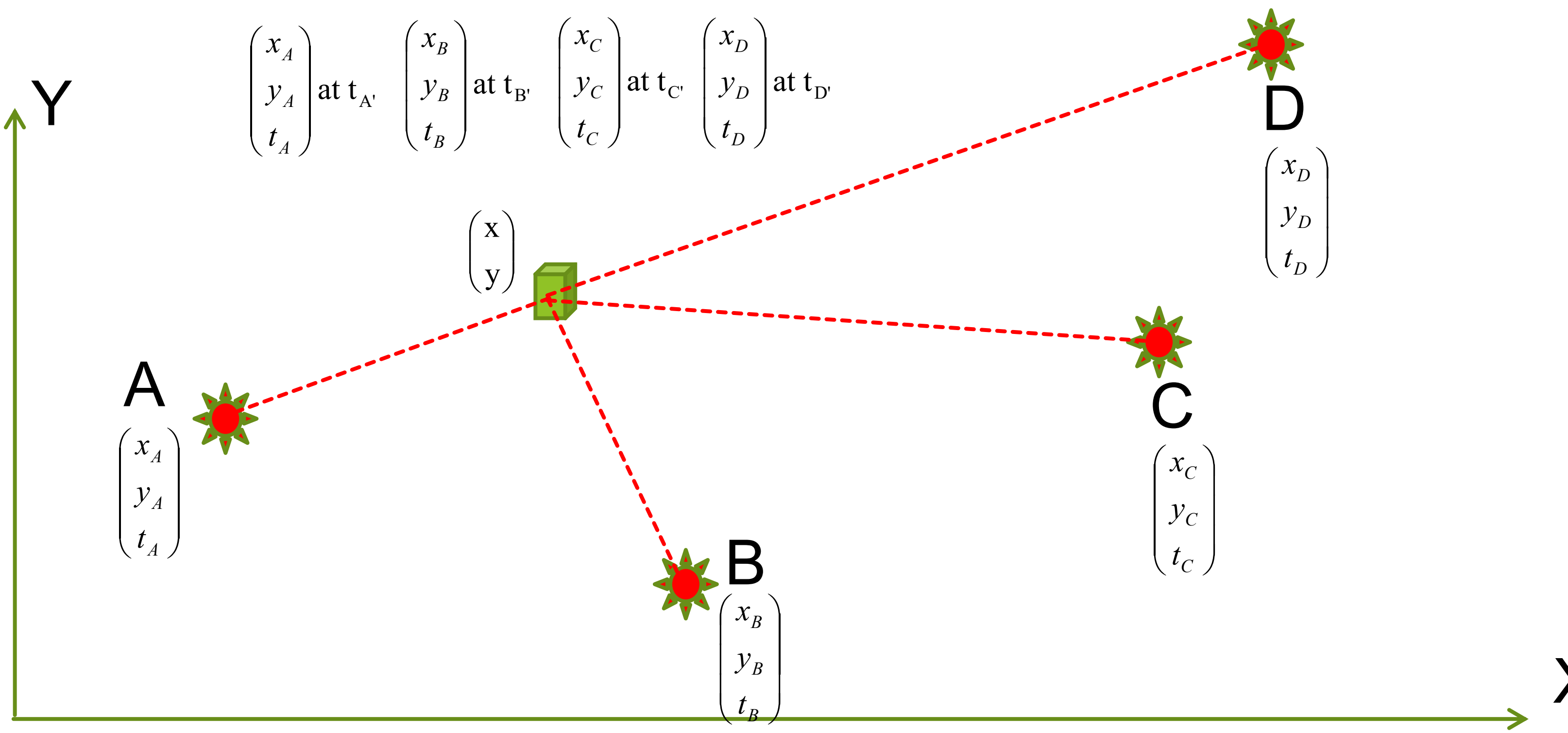
$$\begin{cases} 2(x_B - x_A)x + 2(y_B - y_A)y = d_A^2 - d_B^2 + x_B^2 - x_A^2 + y_B^2 - y_A^2 \\ 2(x_C - x_A)x + 2(y_C - y_A)y = d_A^2 - d_C^2 + x_C^2 - x_A^2 + y_C^2 - y_A^2 \end{cases}$$

$$A = \begin{bmatrix} 2(x_B - x_A) & 2(y_B - y_A) \\ 2(x_C - x_A) & 2(y_C - y_A) \end{bmatrix} \qquad B = \begin{bmatrix} d_A^2 - d_B^2 + x_B^2 - x_A^2 + y_B^2 - y_A^2 \\ d_A^2 - d_C^2 + x_C^2 - x_A^2 + y_C^2 - y_A^2 \end{bmatrix}$$

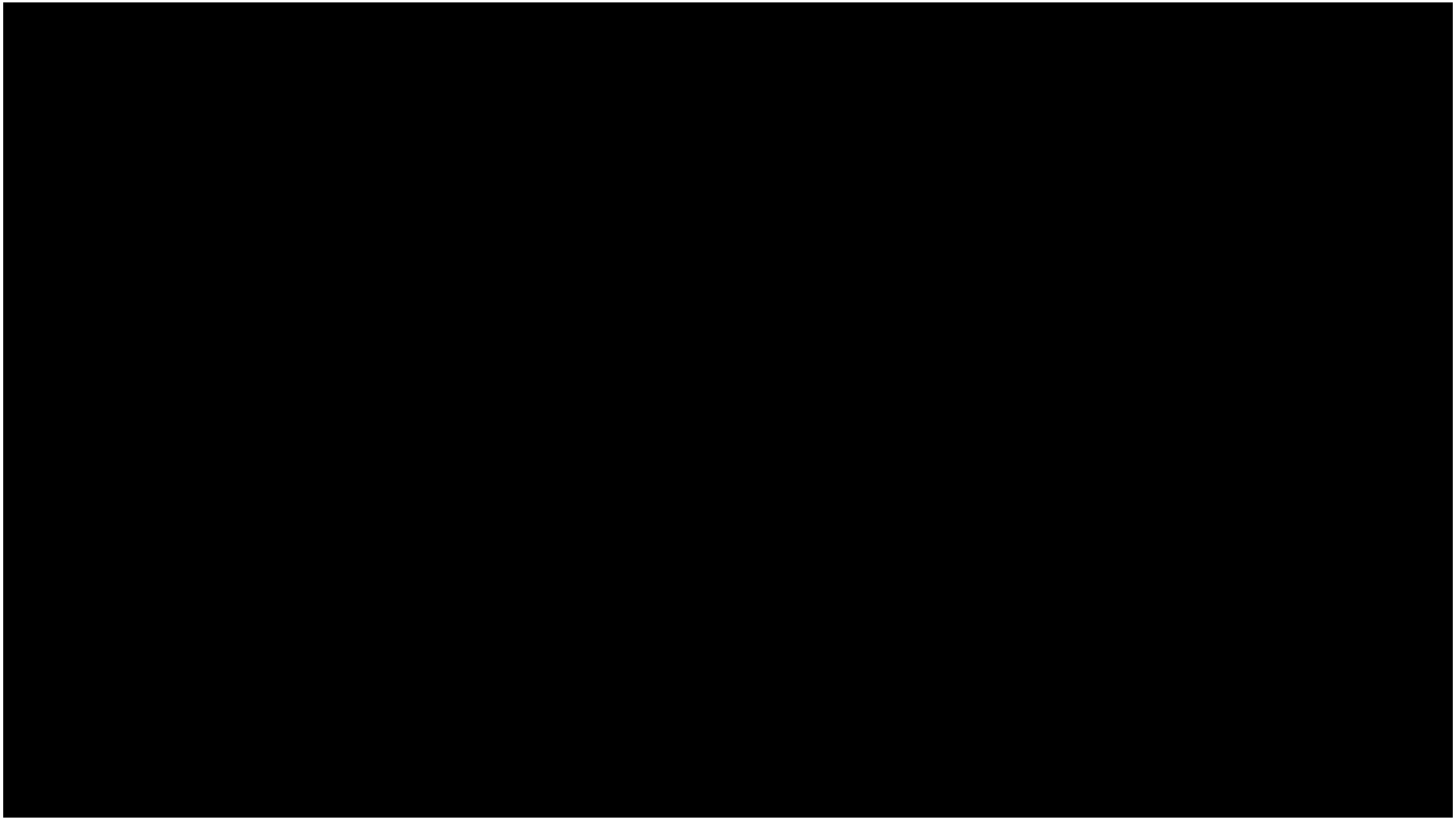
$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

Exercise

- In a two-dimensional space, we have four time-signal emitters. If a moving target has a receiver which can receive the time-signal from these four emitters, what is the solution of determining the position of the target?

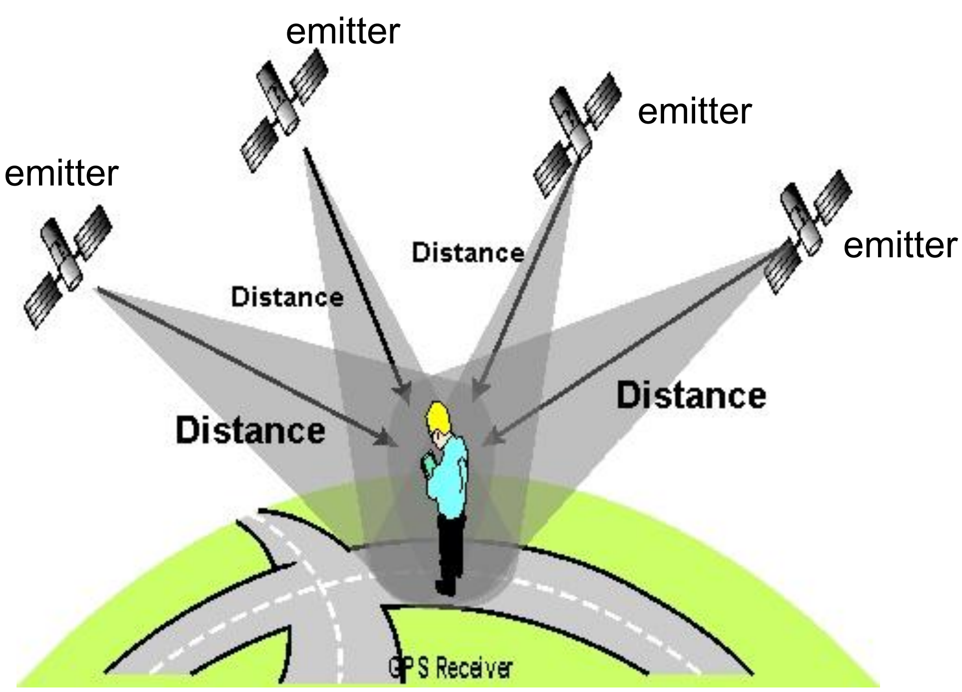


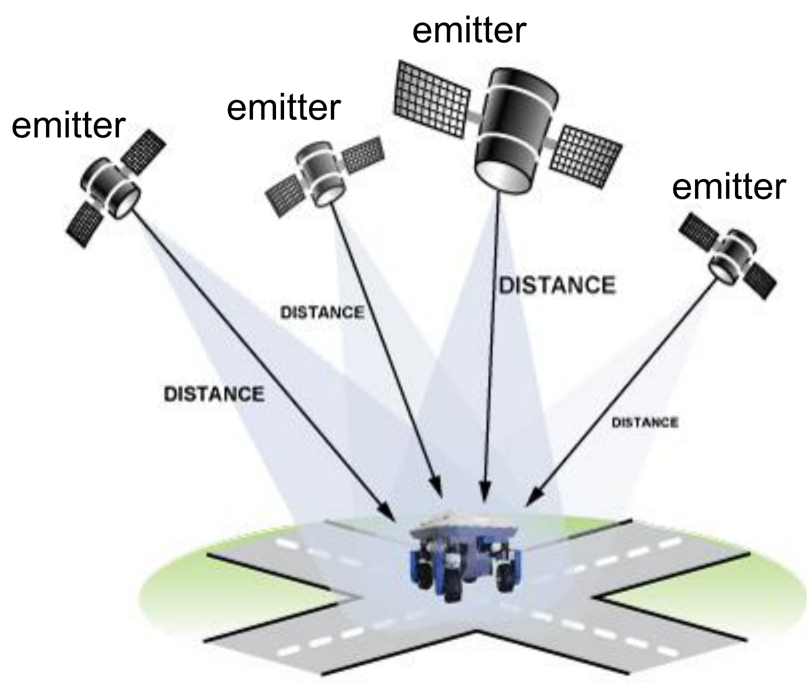
Example of Implementation



Example of Computing Three-Dimensional Positions

- Four points define a three-dimensional space. Then, we put four time-signal emitters at four known points. When a target carries a time-signal receiver, it can measure four distances to these four time-signal emitters. Then, the coordinates of the target's position can be computed from these four distances.

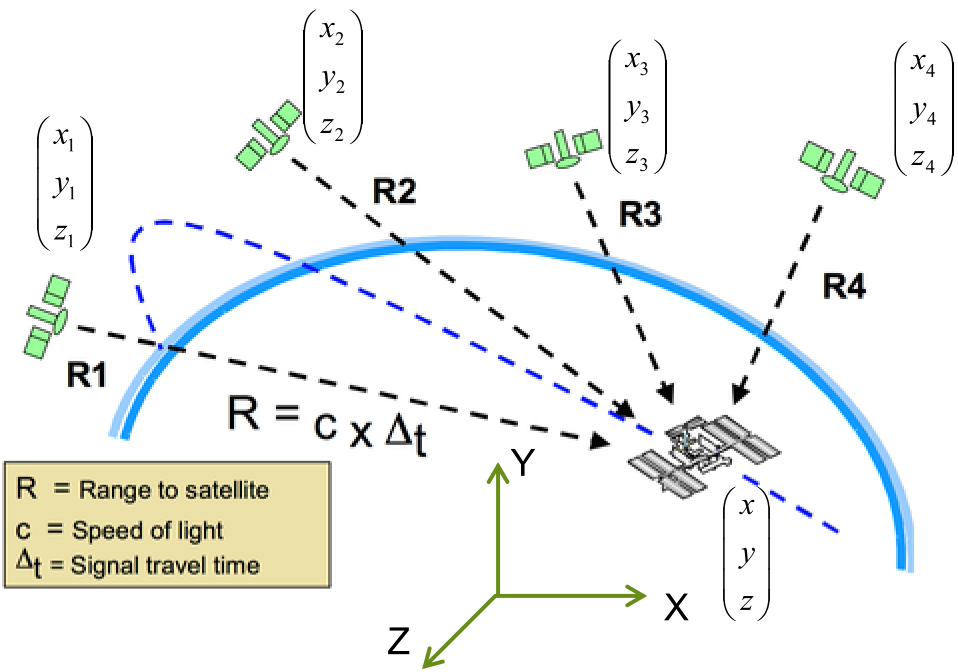




Details of Solution

- Equations of Four Spheres:

$$
\begin{cases}
(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=R_1^2 \\
(x-x_2)^2+(y-y_2)^2+(z-z_2)^2=R_2^2 \\
(x-x_3)^2+(y-y_3)^2+(z-z_3)^2=R_3^2 \\
(x-x_4)^2+(y-y_4)^2+(z-z_4)^2=R_4^2
\end{cases}
$$



Details of Solution

$$\begin{cases} (x-x_1)^2+(y-y_1)^2+(z-z_1)^2=R_1^2 \\ (x-x_2)^2+(y-y_2)^2+(z-z_2)^2=R_2^2 \\ (x-x_3)^2+(y-y_3)^2+(z-z_3)^2=R_3^2 \\ (x-x_4)^2+(y-y_4)^2+(z-z_4)^2=R_4^2 \end{cases}$$

$$\begin{cases} 2(x_2-x_1)x+2(y_2-y_1)y+2(z_2-z_1)z=R_1^2-R_2^2+x_2^2-x_1^2+y_2^2-y_1^2+z_2^2-z_1^2 \\ 2(x_3-x_1)x+2(y_3-y_1)y+2(z_3-z_1)z=R_1^2-R_3^2+x_3^2-x_1^2+y_3^2-y_1^2+z_3^2-z_1^2 \\ 2(x_4-x_1)x+2(y_4-y_1)y+2(z_4-z_1)z=R_1^2-R_4^2+x_4^2-x_1^2+y_4^2-y_1^2+z_4^2-z_1^2 \end{cases}$$

Details of Solution

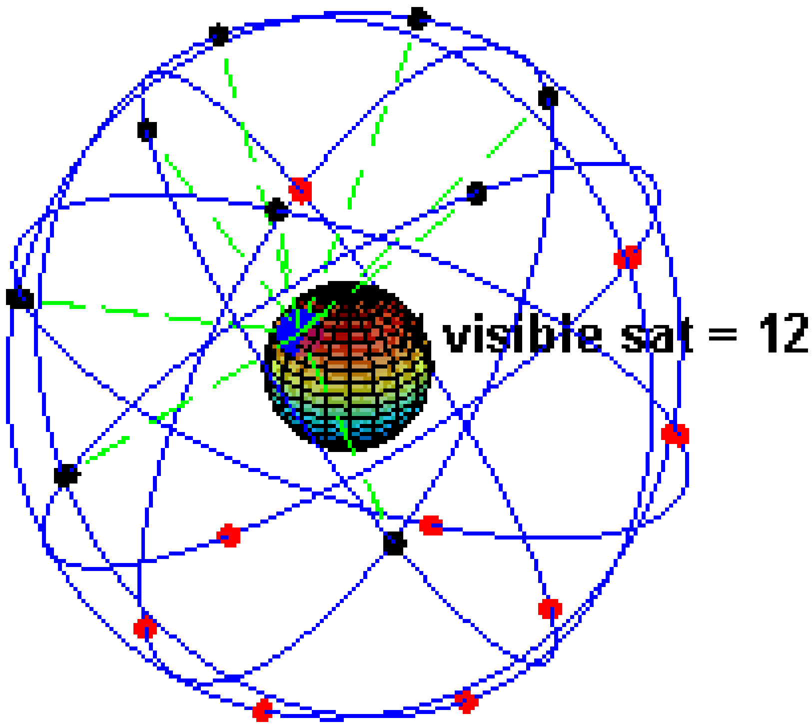
$$\begin{cases} 2(x_2 - x_1)x + 2(y_2 - y_1)y + 2(z_2 - z_1)z = R_1^2 - R_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 + z_2^2 - z_1^2 \\ 2(x_3 - x_1)x + 2(y_3 - y_1)y + 2(z_3 - z_1)z = R_1^2 - R_3^2 + x_3^2 - x_1^2 + y_3^2 - y_1^2 + z_3^2 - z_1^2 \\ 2(x_4 - x_1)x + 2(y_4 - y_1)y + 2(z_4 - z_1)z = R_1^2 - R_4^2 + x_4^2 - x_1^2 + y_4^2 - y_1^2 + z_4^2 - z_1^2 \end{cases}$$

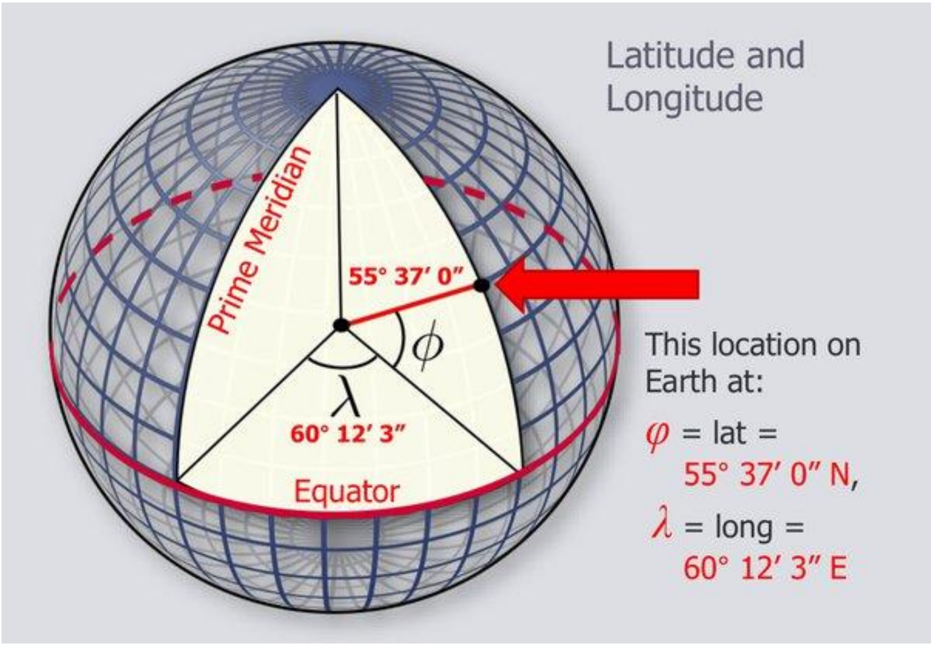
$$\begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_3 - z_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} R_1^2 - R_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 + z_2^2 - z_1^2 \\ R_1^2 - R_3^2 + x_3^2 - x_1^2 + y_3^2 - y_1^2 + z_3^2 - z_1^2 \\ R_1^2 - R_4^2 + x_4^2 - x_1^2 + y_4^2 - y_1^2 + z_4^2 - z_1^2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_3 - z_1 \end{bmatrix}^{-1} \begin{bmatrix} R_1^2 - R_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 + z_2^2 - z_1^2 \\ R_1^2 - R_3^2 + x_3^2 - x_1^2 + y_3^2 - y_1^2 + z_3^2 - z_1^2 \\ R_1^2 - R_4^2 + x_4^2 - x_1^2 + y_4^2 - y_1^2 + z_4^2 - z_1^2 \end{bmatrix}$$

Pay Attention to Coordinate Conversion

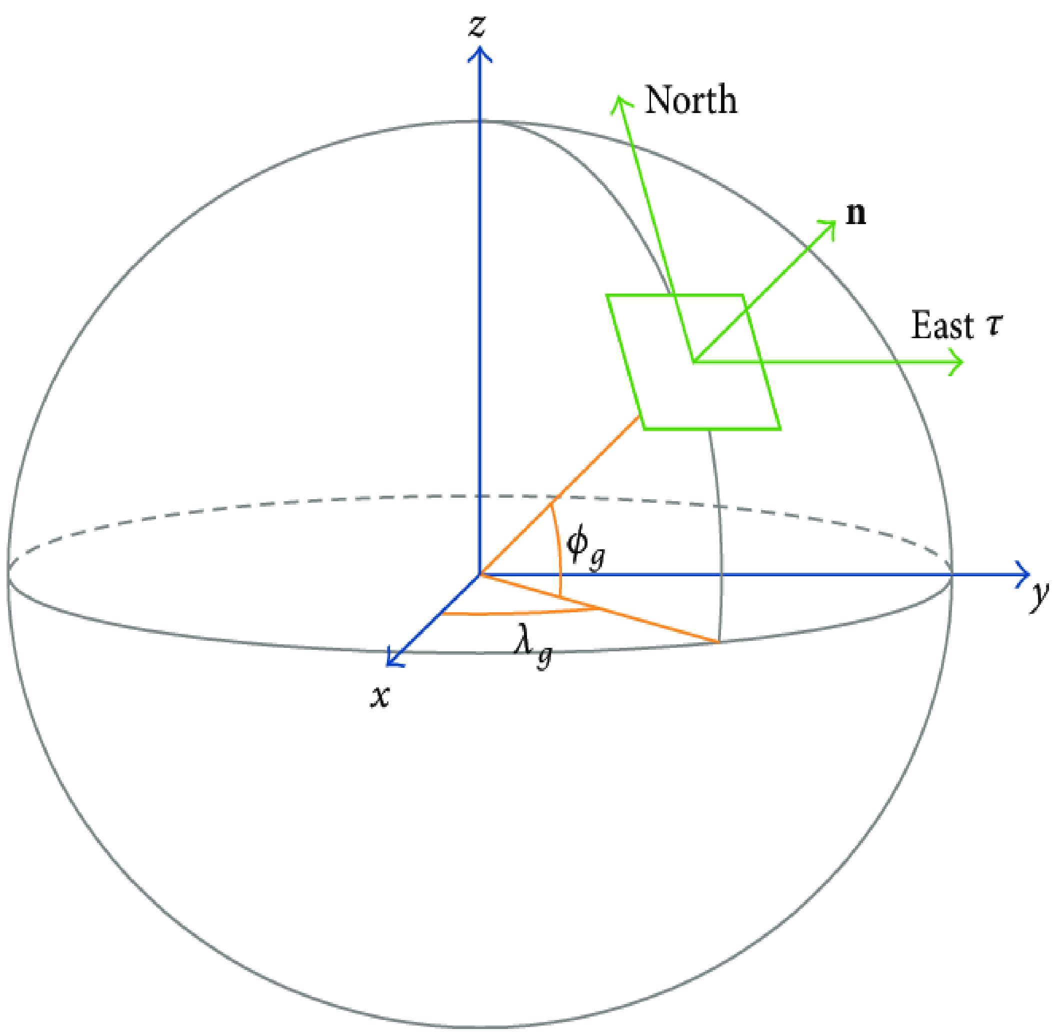
From Global Polar Coordinates to Local Cartesian Coordinates

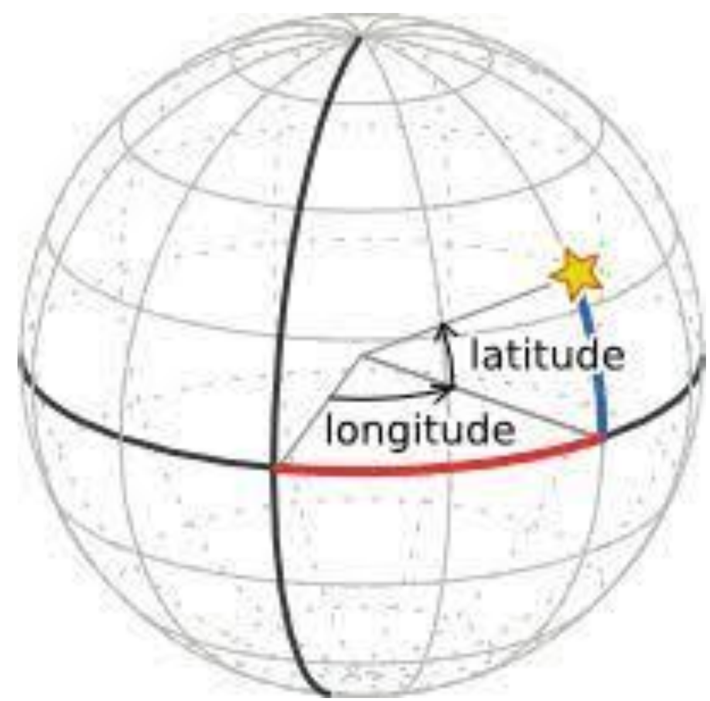




GPS Coordinates = 3D Polar Coordinates

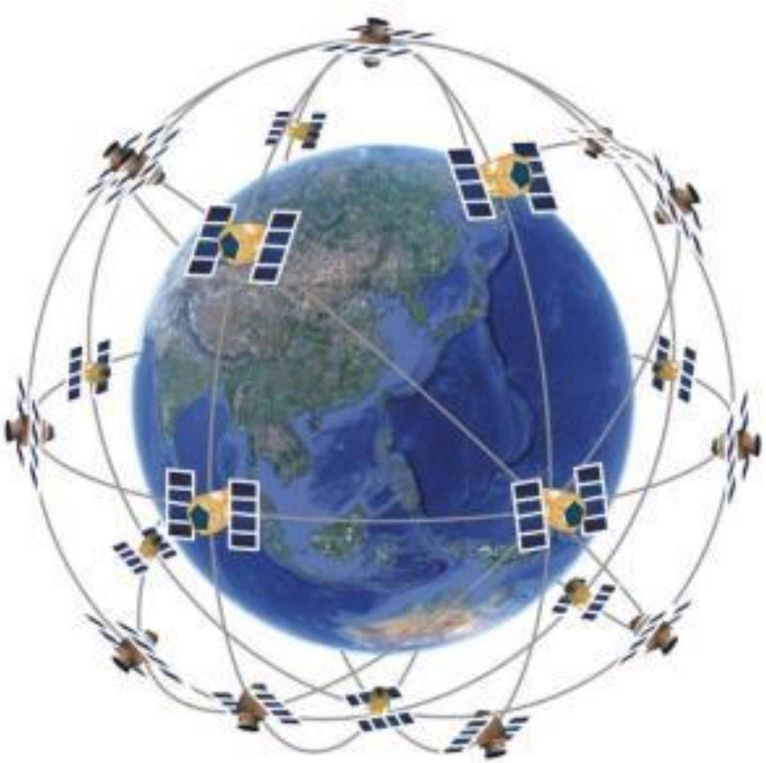
From Global Polar Coordinates to Local Cartesian Coordinates



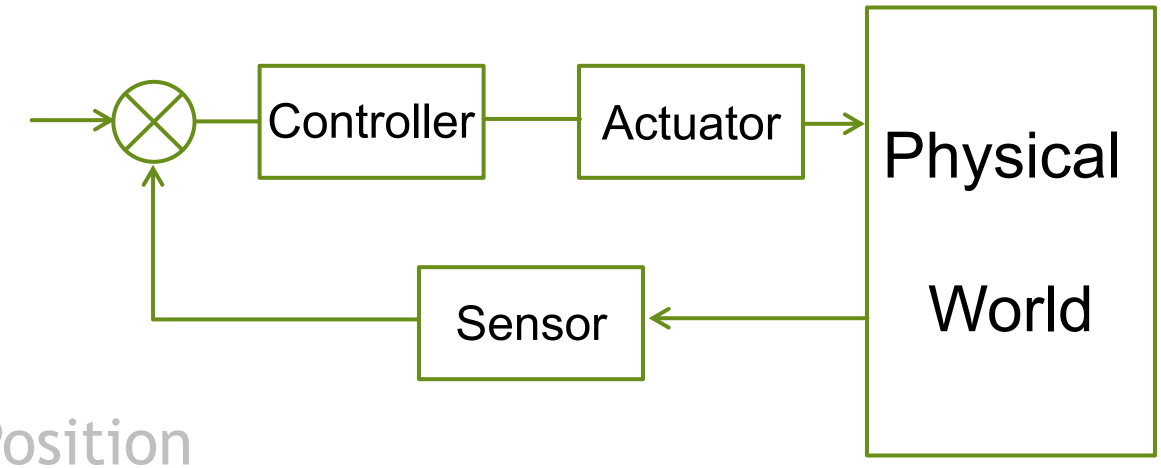


Exercise

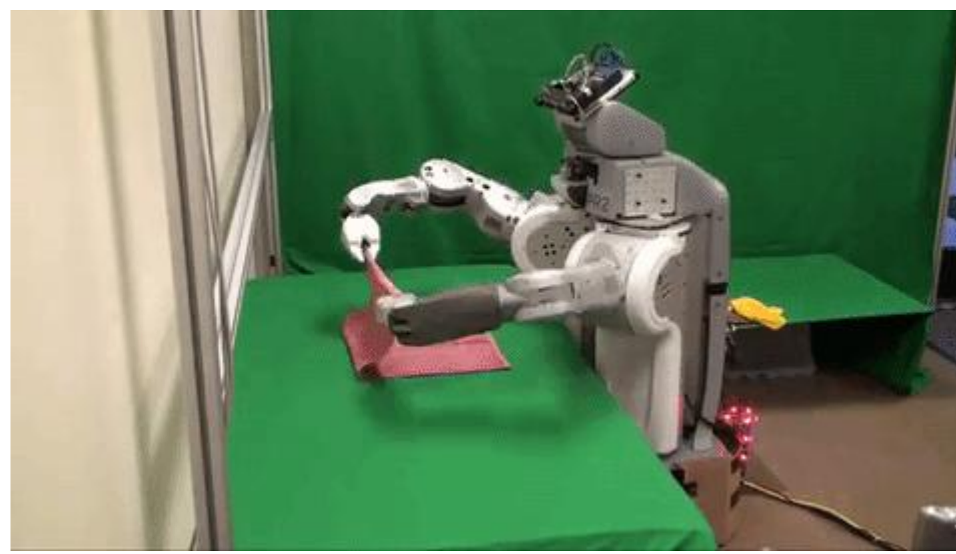
- A target carries a time-signal receiver. It can compute eight distances to eight time-signal emitters. What is the solution which determines the coordinates of the target's position from these eight distances?



Outline



- Understanding of Position
- Computation of Position
- Measurement of Position



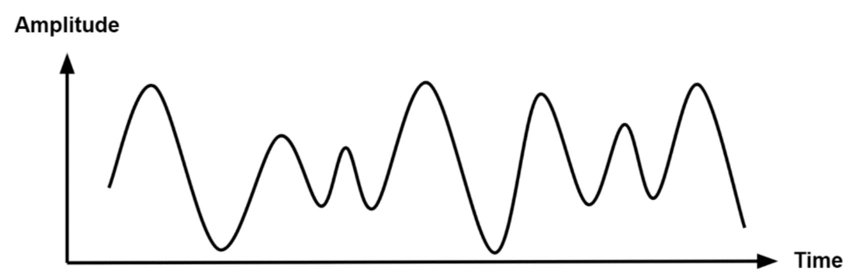
Applications: Positions in Navigation

Applications: Positions in Manipulation

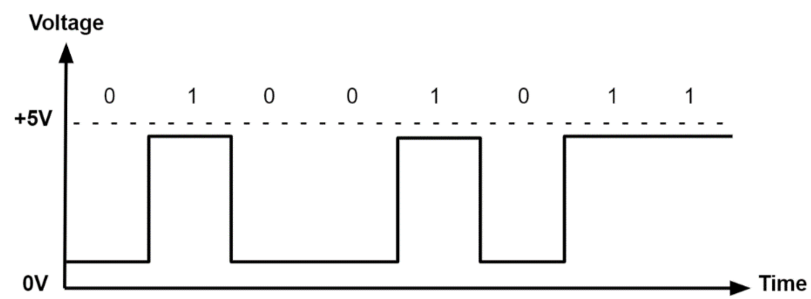
Applications: Positions in Grasping

Two Observations

- Observation 1:
  - All microcontrollers are programmable sensors of DC voltages ⇒ Design Principle 1

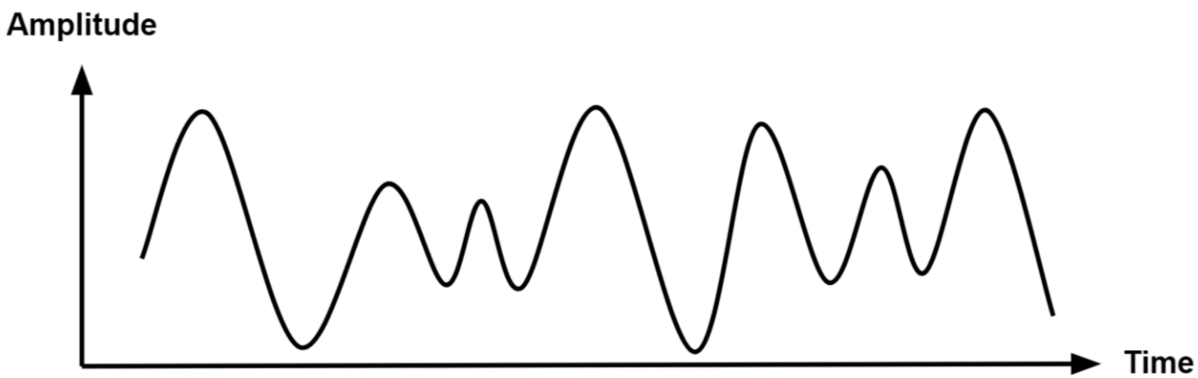


- Observation 2:
  - All microcontrollers are programmable sensors of digital signals ⇒ Design Principle 2



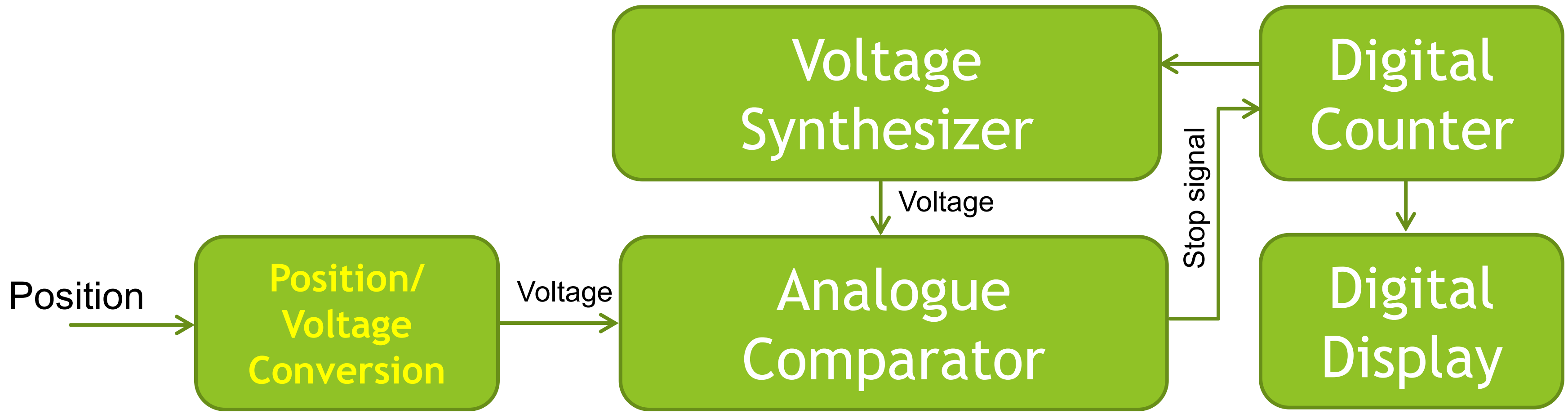
Principle 1: Using Microcontrollers as Sensors of DC Voltages

- DC voltages could be digitally measured.
- Positions could be converted into voltages.
- Hence, positions could be digitally measured.



How to apply principle 1 to design digital measurement and sensing systems for positions?

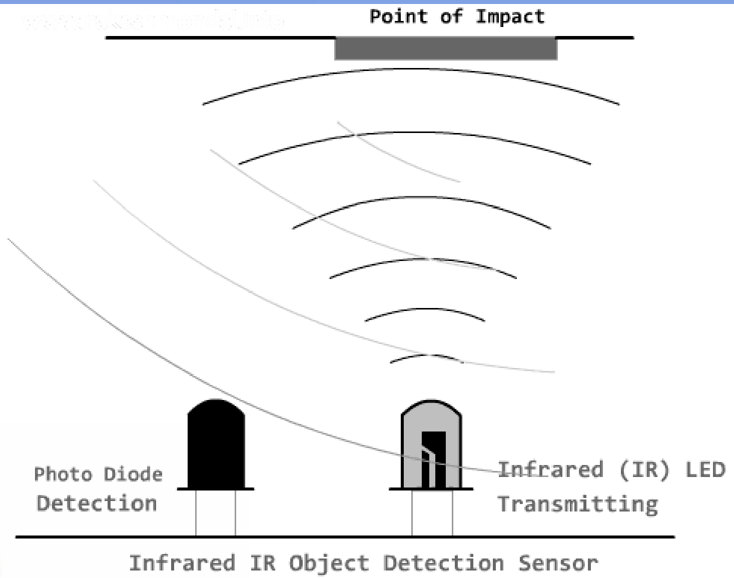
- Position is converted to voltage which is measured by digital voltmeter (e.g. microcontrollers).

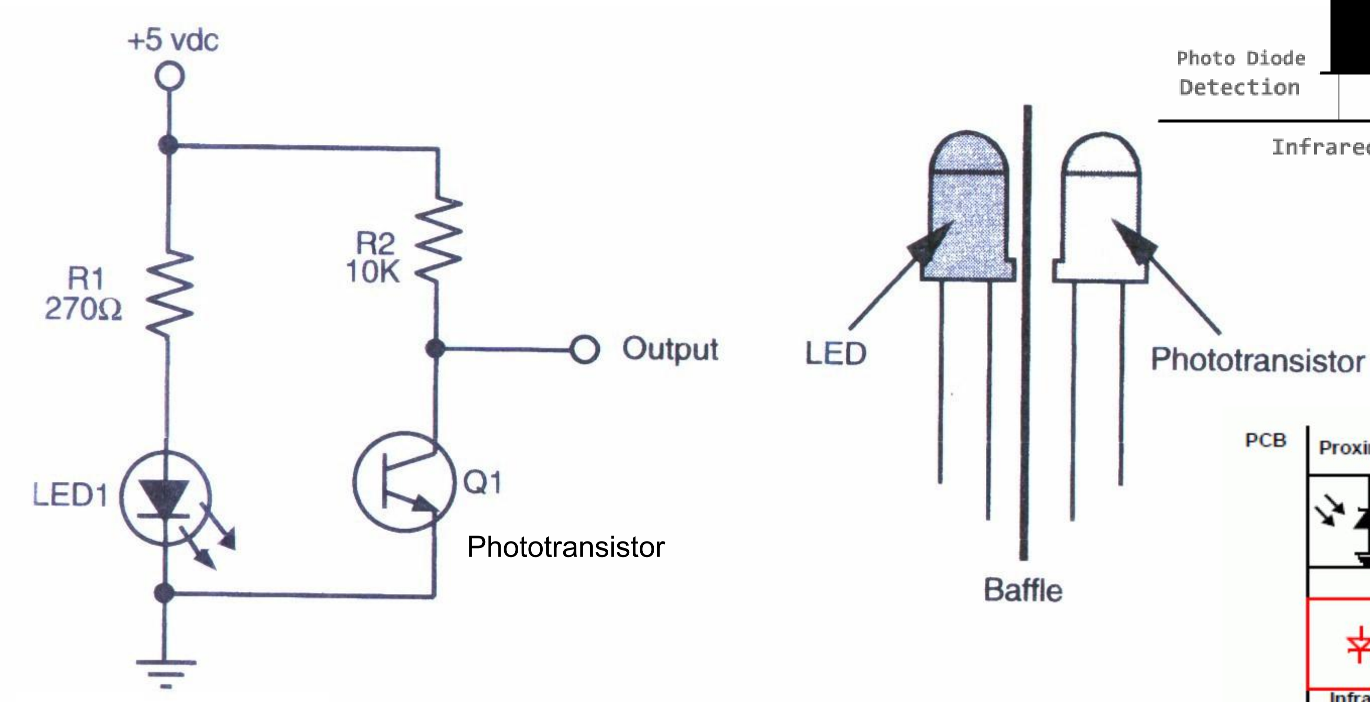


All microcontrollers are programmable digital sensors of voltage!

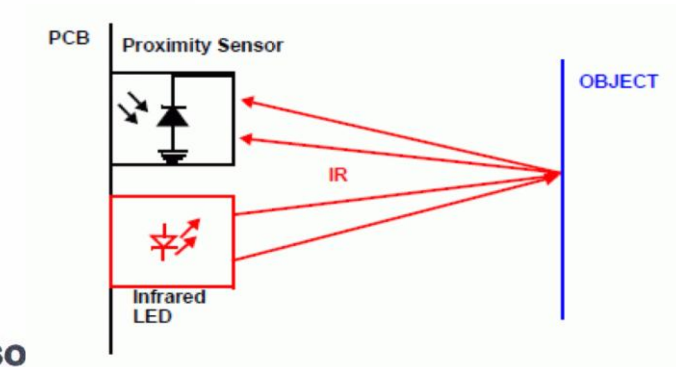
Using Infrared Signals to Convert Linear Positions into DC Voltages

- Infrared Distance Sensor: basic design



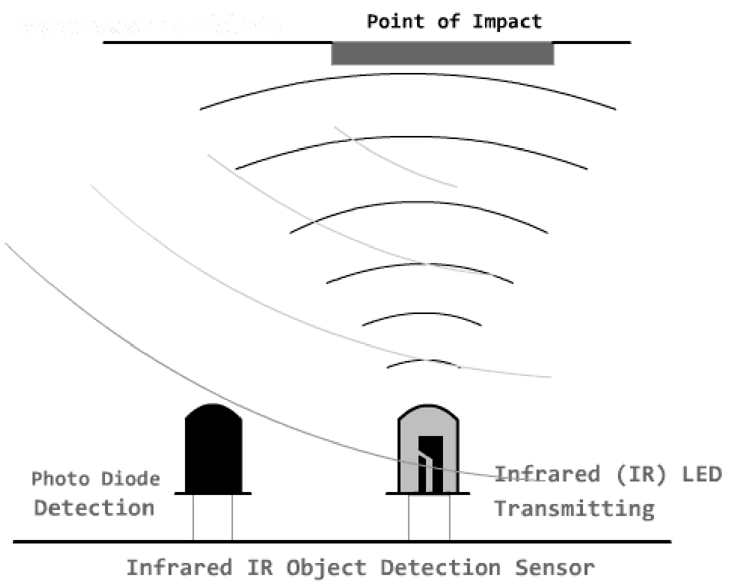


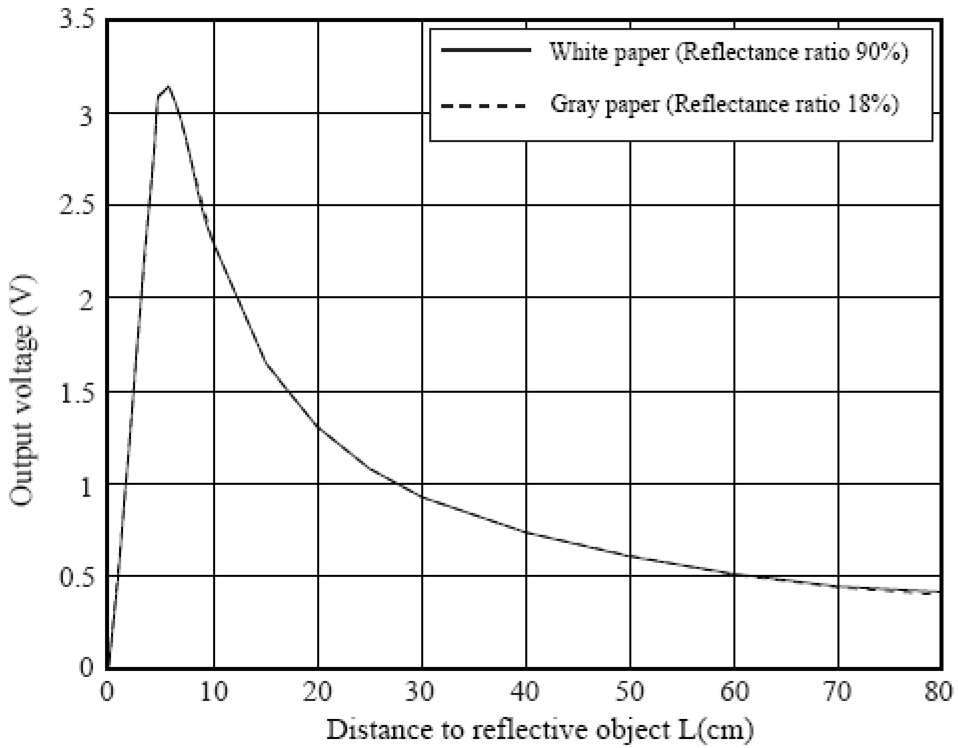
The basic design of the infrared proximity senso



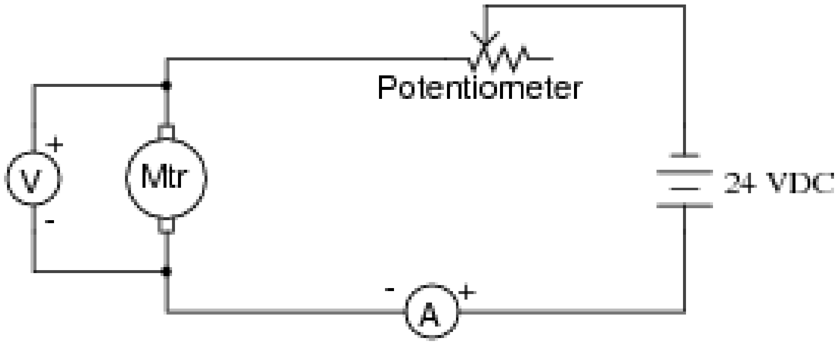
Using Infrared Signals to Convert Linear Positions into DC Voltages (continued)

- Infrared Distance Sensor: input-output relationship

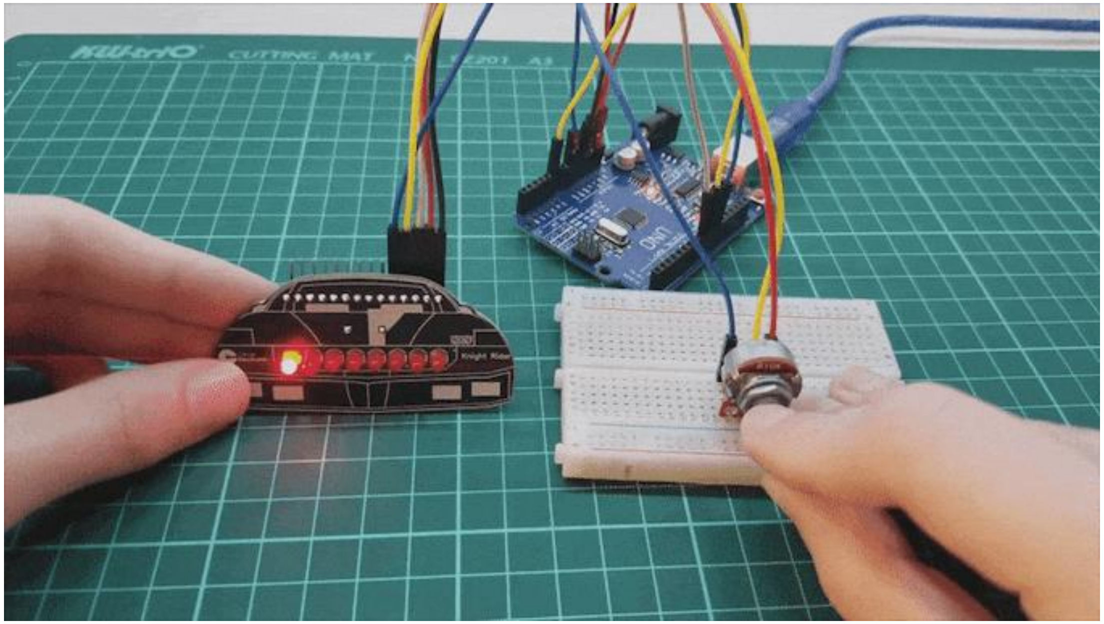


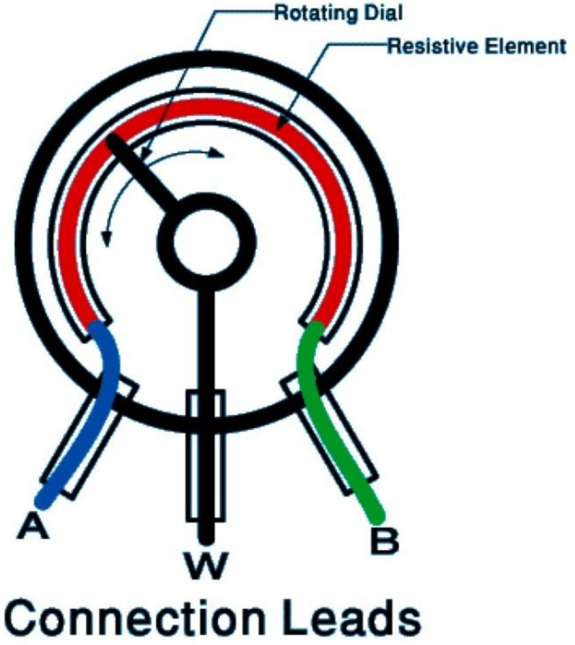


Using Potentiometer to Convert Linear/Angular Positions into DC Voltages



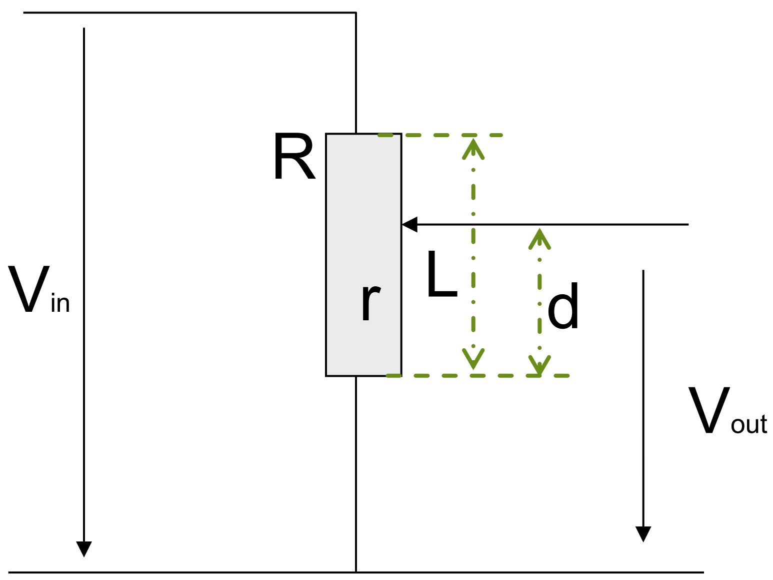
- Linear Potentiometer: Setup





Using Potentiometer to Convert Linear/Angular Positions into DC Voltages (continued)

- Linear Potentiometer: Equations

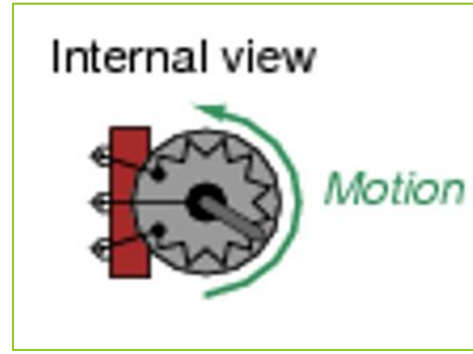


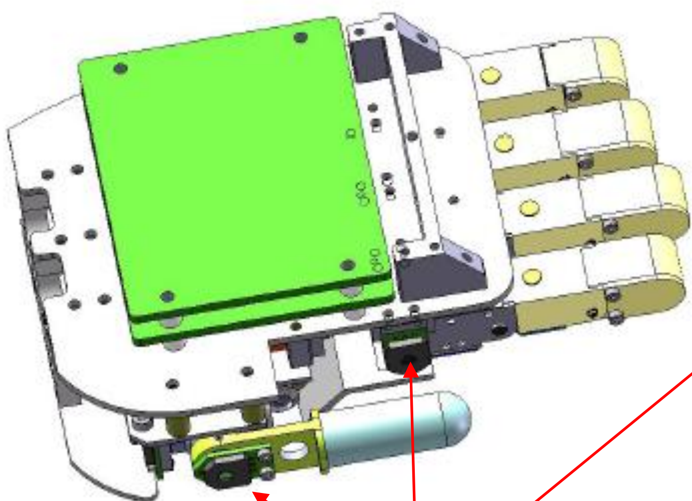
$$V_{out} = \frac{V_{in}}{R} \bullet r = \frac{V_{in}}{L} \bullet d$$

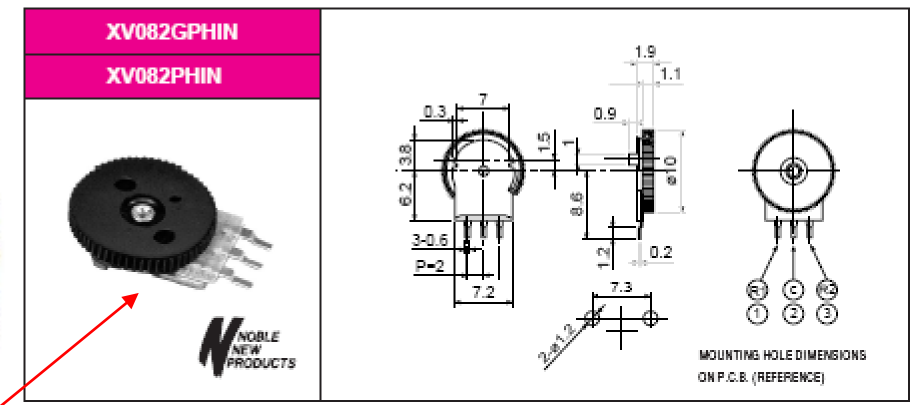
$$d = L\frac{V_{out}}{V_{in}}$$

Using Potentiometer to Convert Linear/Angular Positions into DC Voltages (continued)

- Rotational Potentiometer: Setup



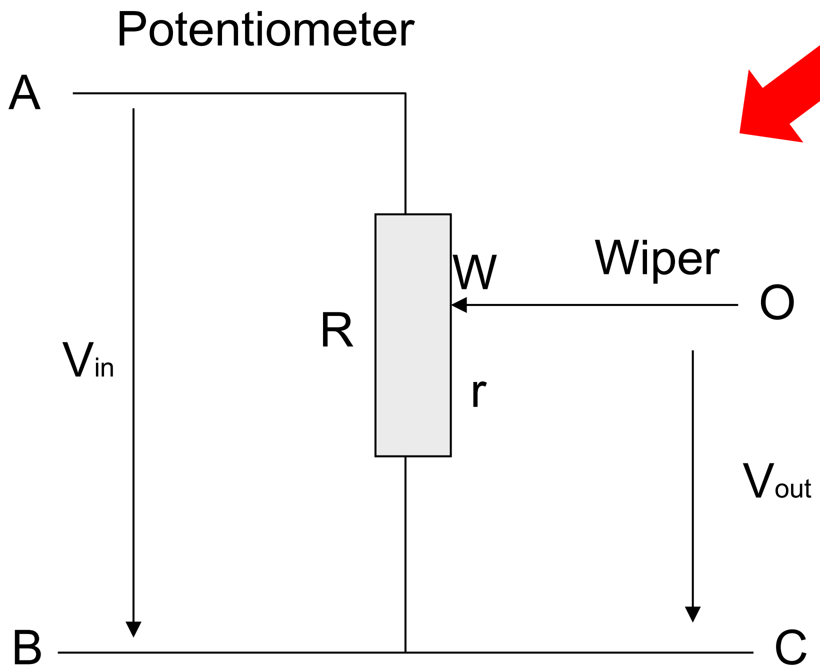


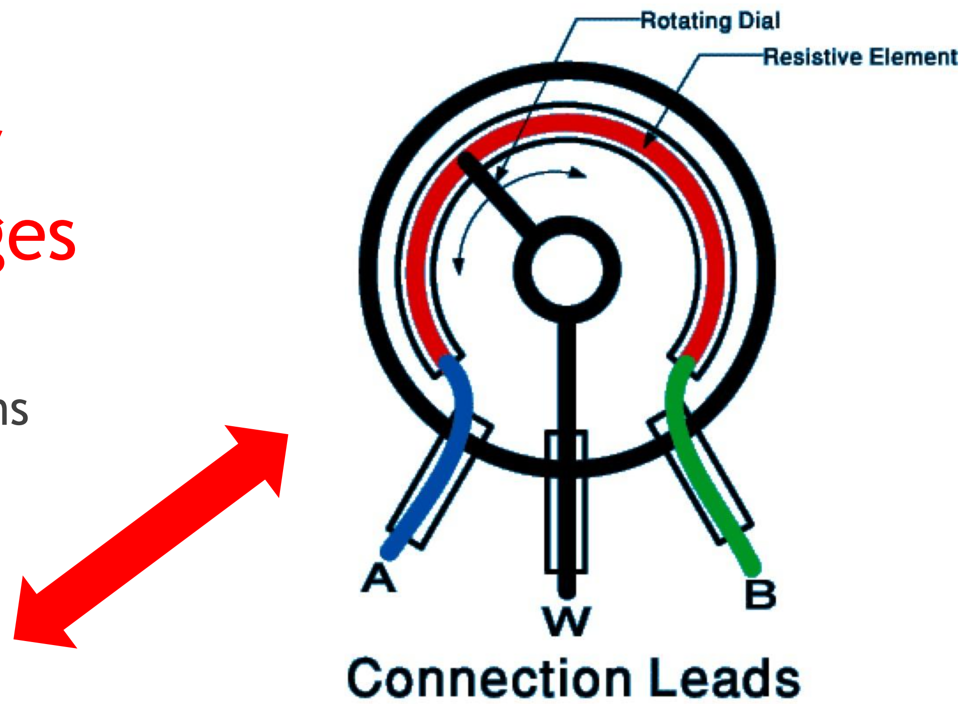


Potentiometer for measuring angular positions

Using Potentiometer to Convert Linear/Angular Positions into DC Voltages (continued)

- Angular Potentiometer: Equations



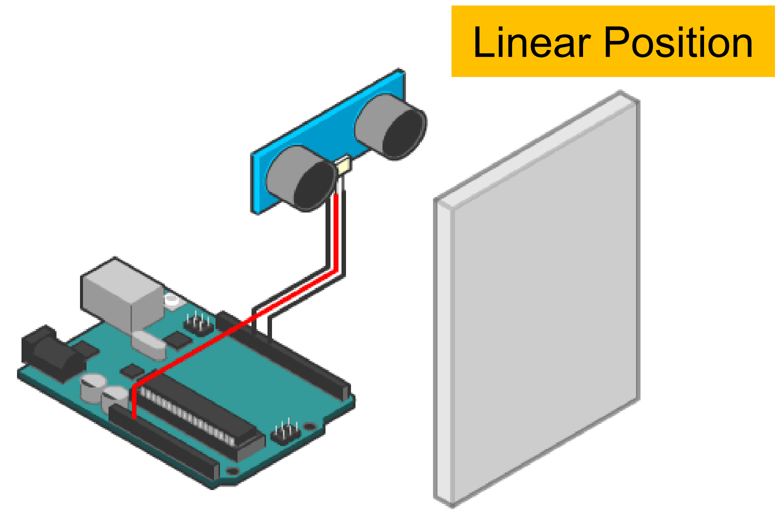


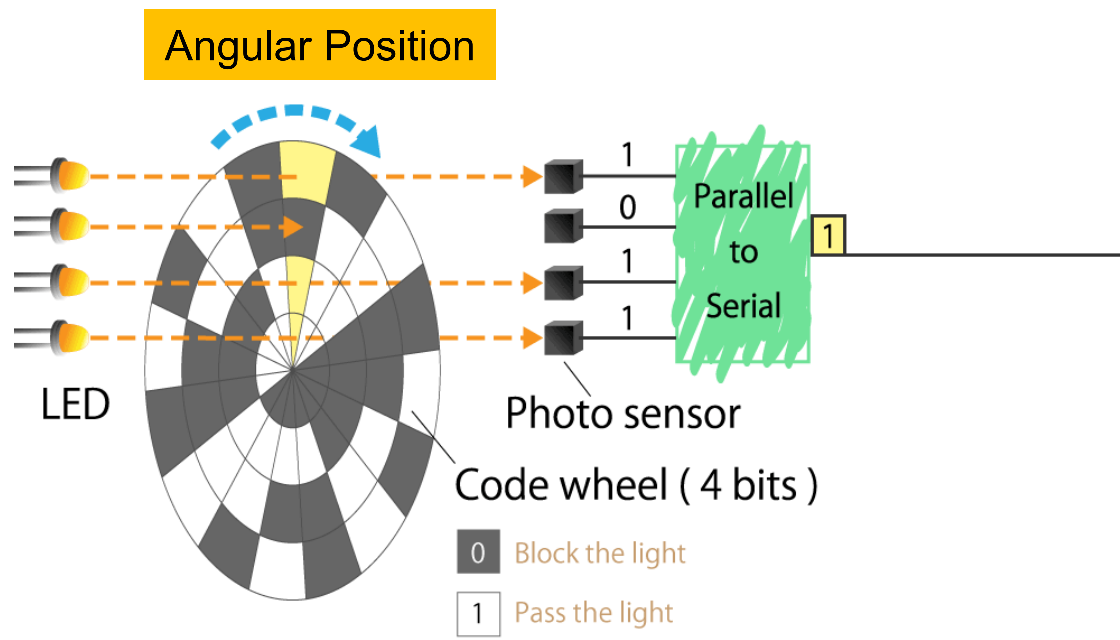
$$V_{out} = \frac{V_{in}}{R} \bullet r$$

$$= \frac{V_{in}}{2\pi L} \bullet (\theta L) = \frac{V_{in}}{2\pi} \bullet \theta$$

Principle 2: Using Microcontrollers as Sensors of Digital Signals

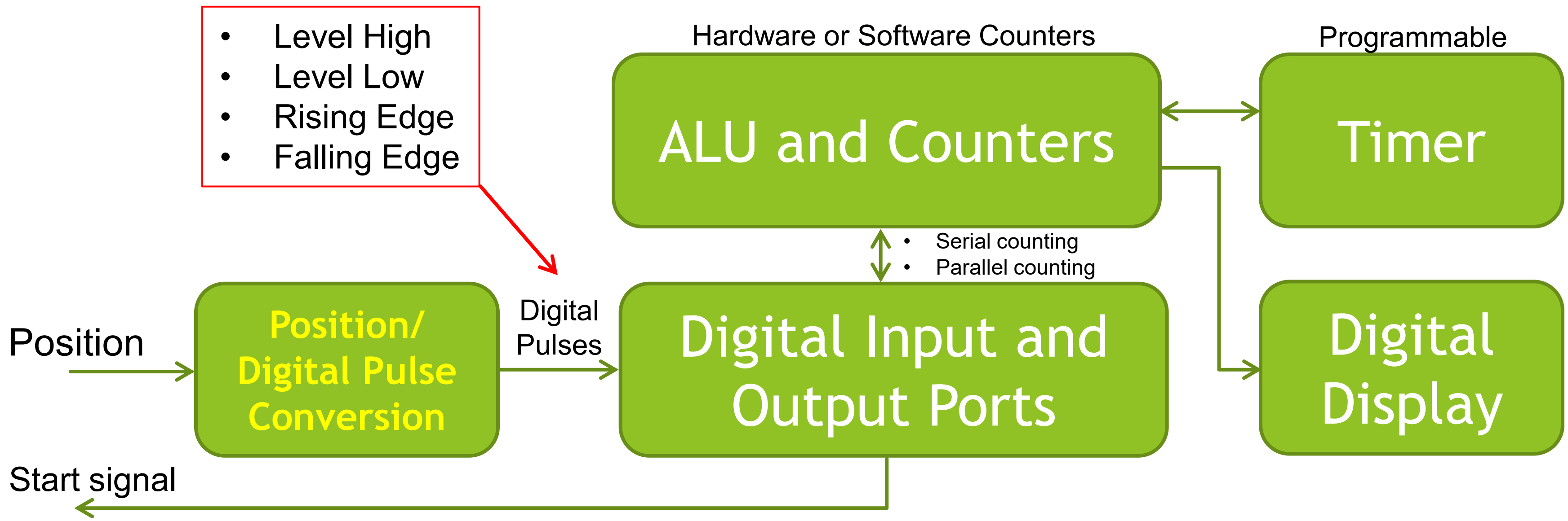
- Digital signals could be directly measured.
- Positions could be converted into digital signals of either time series or parallel bits.
- Hence, positions could be digitally measured.





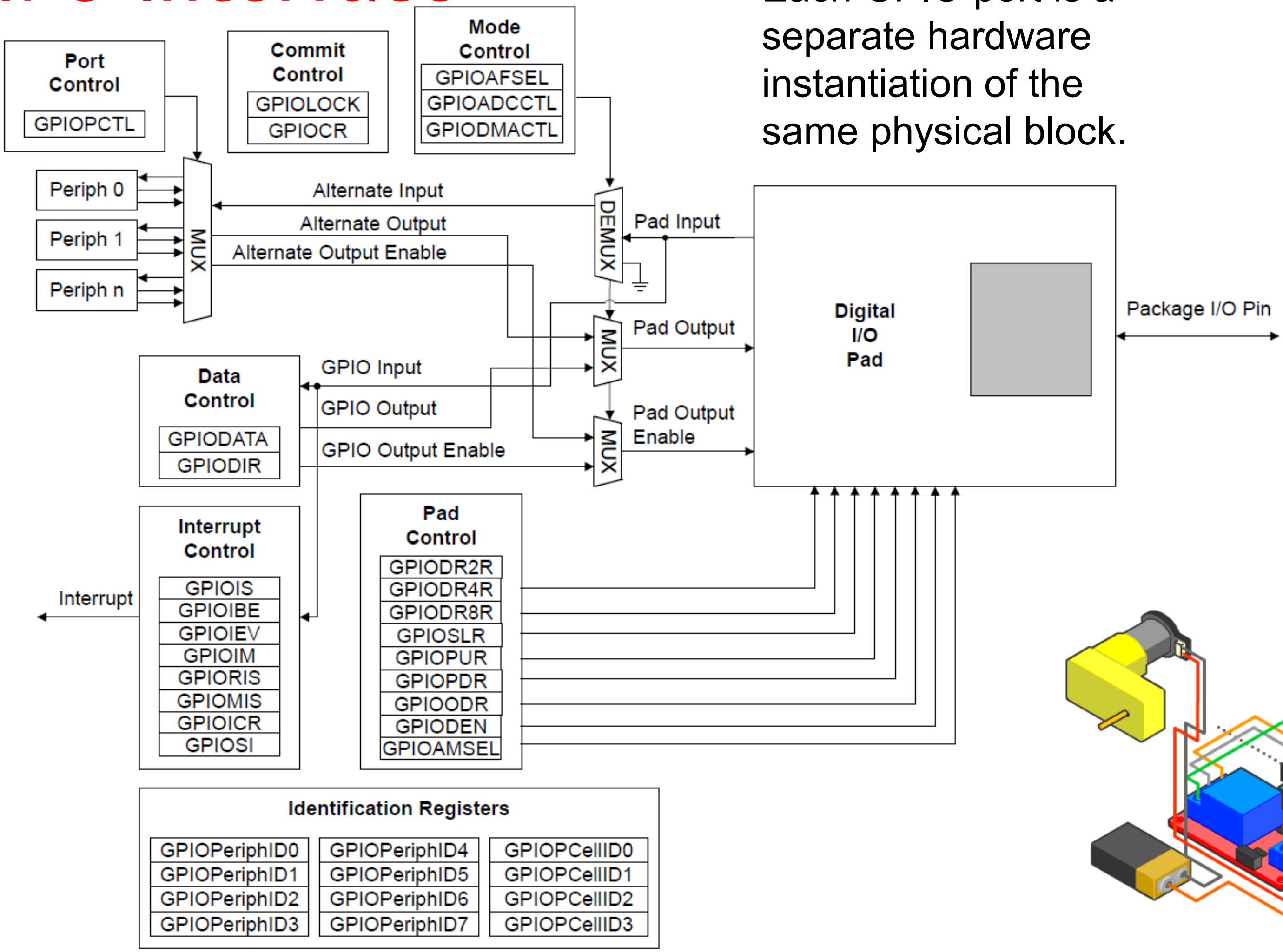
How to apply principle 2 to design digital measurement and sensing systems for positions?

- Position is converted to digital signals (i.e. pulses) which are to be measured by microcontrollers.

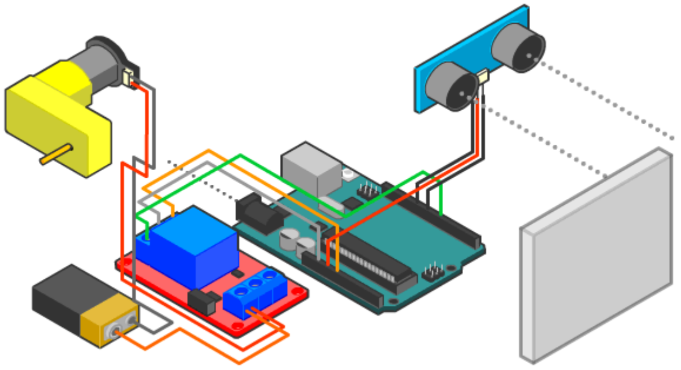


All microcontrollers are programmable sensors of digital signals!

Example of ARM Cortex Microcontroller's Digital I/O Interface

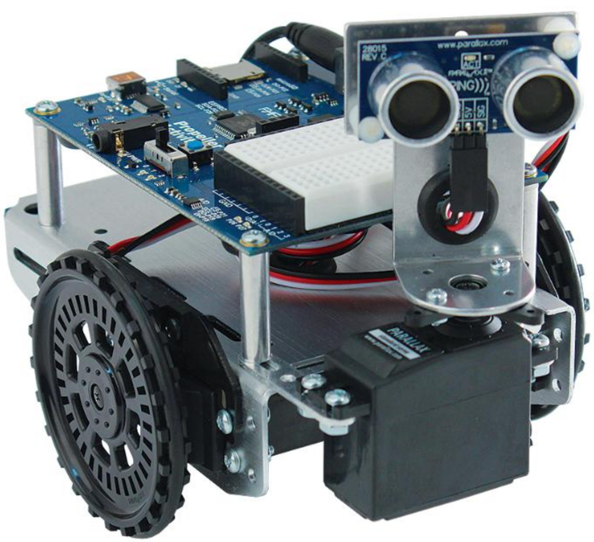


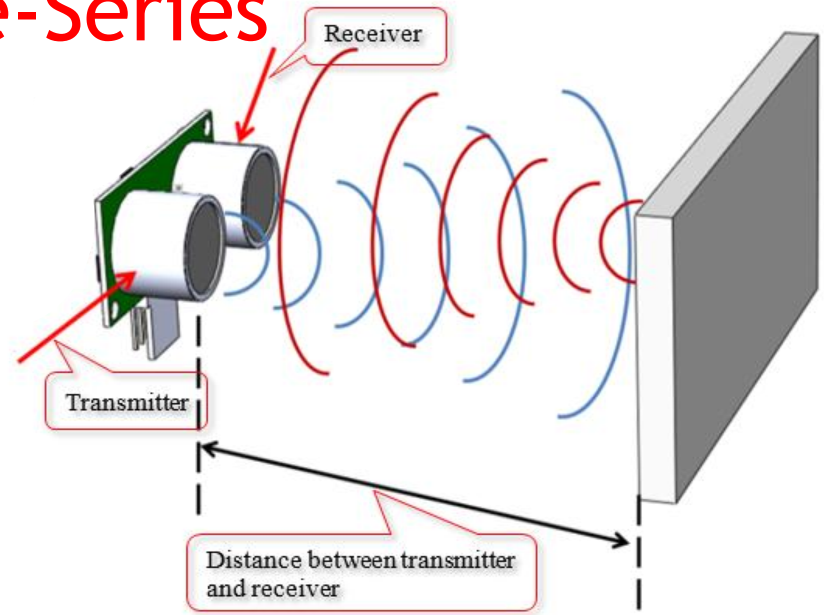
Each GPIO port is a separate hardware instantiation of the same physical block.

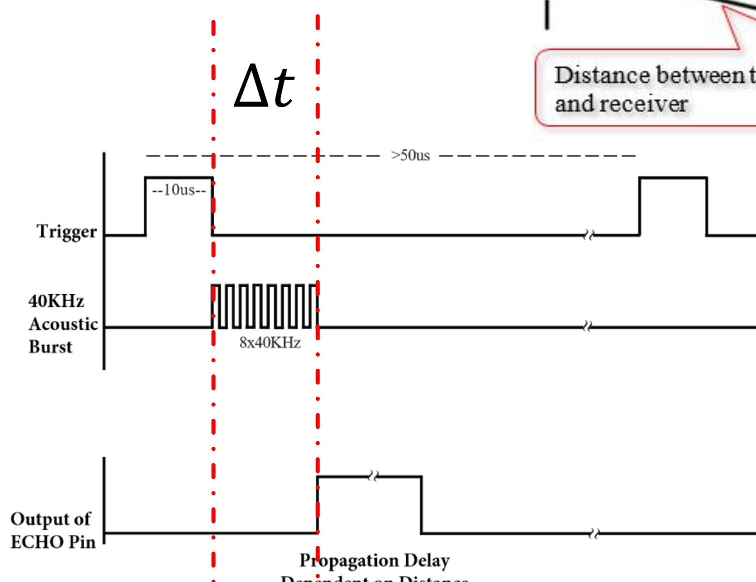


Example of Converting Positions into Digital Signals in the Form of Time-Series

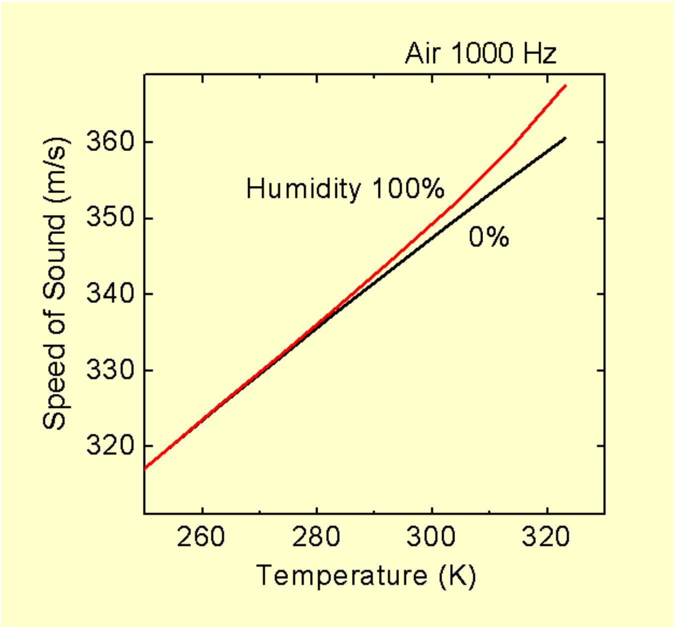
- Ultrasonic Distance Sensor: Setup





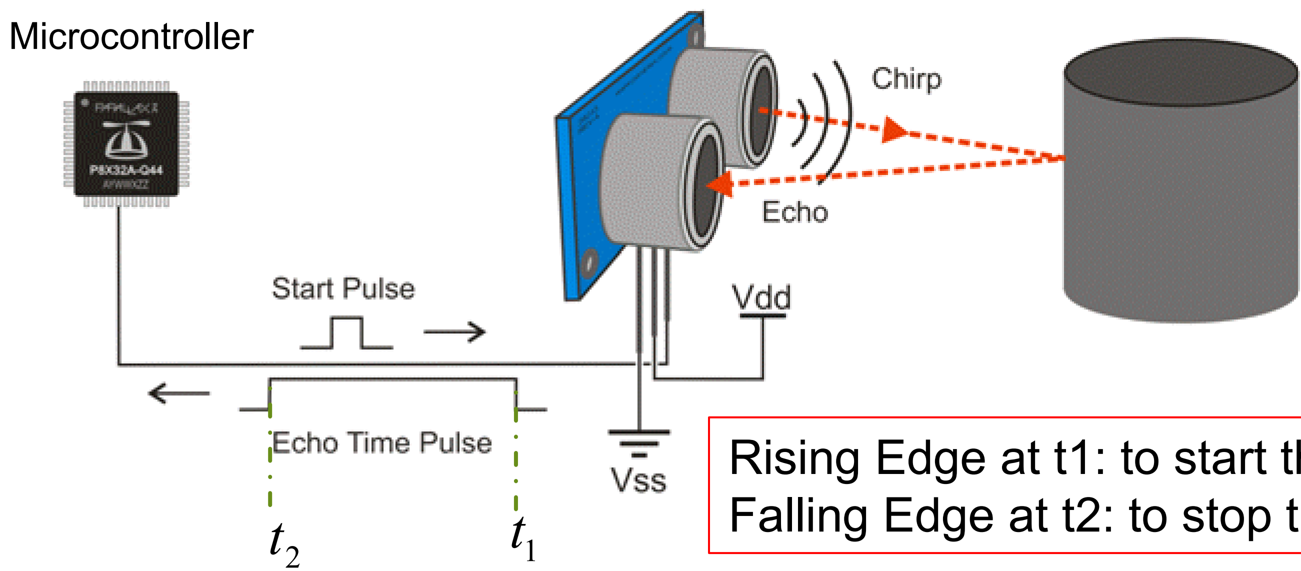


Example of Converting Positions into Digital Signals in the Form of Time-Series



- Ultrasonic Distance Sensor: Equations

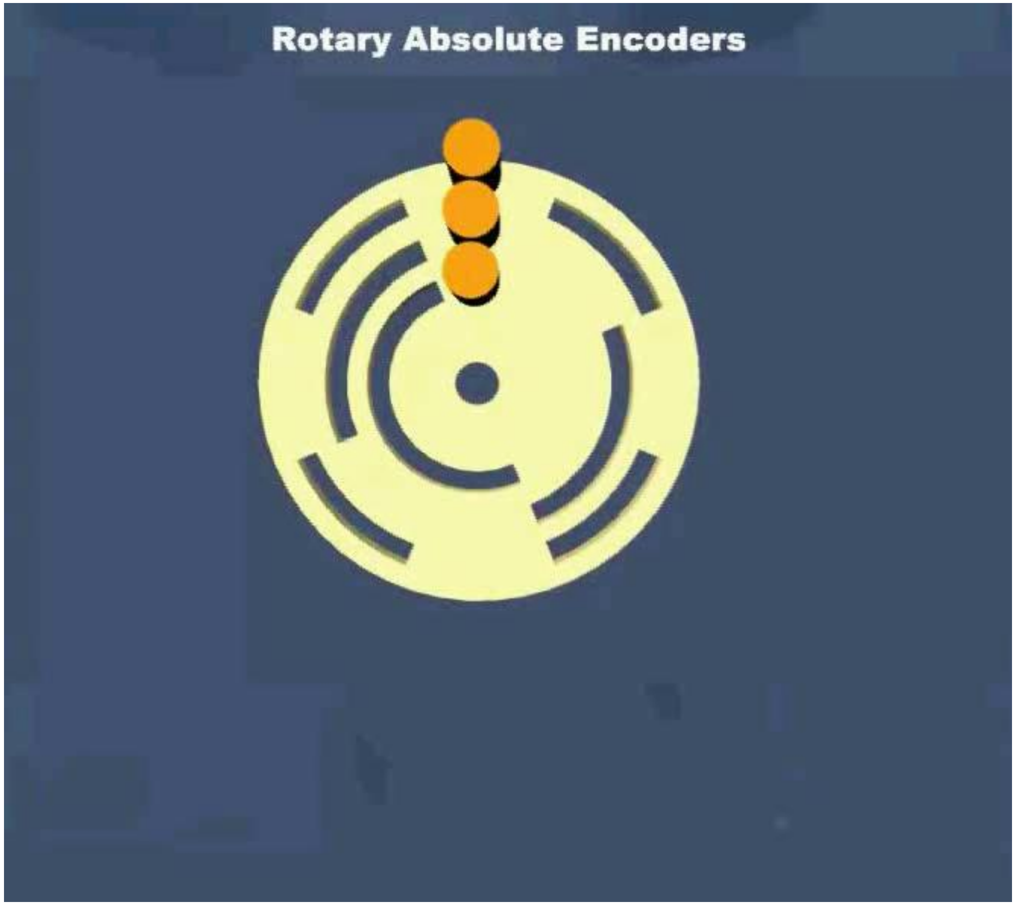
$$d = \frac{v_{sound}(t_2 - t_1)}{2}$$



Rising Edge at t1: to start the counting
Falling Edge at t2: to stop the counting

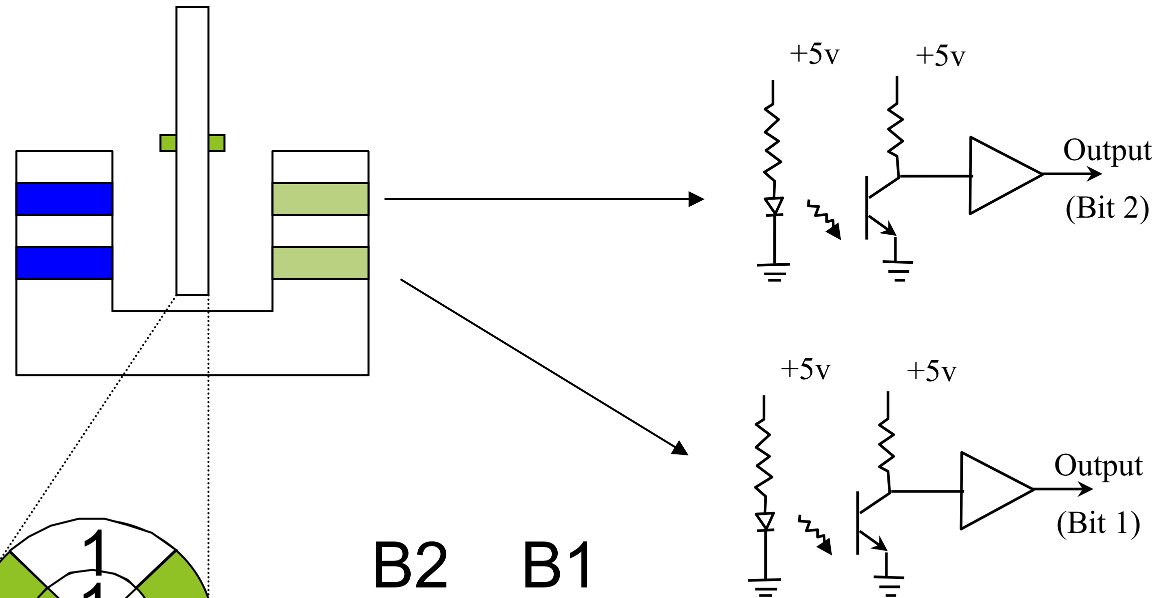
Example of Converting Angular Positions into Digital Signals in the Form of Parallel Bits

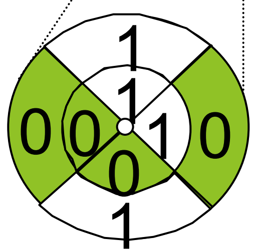
- Optical Absolute Encoder: Illustration



Example of Converting Angular Positions into Digital Signals in the Form of Parallel Bits

- Optical Absolute Encoder: Setup of Two-Bit Output





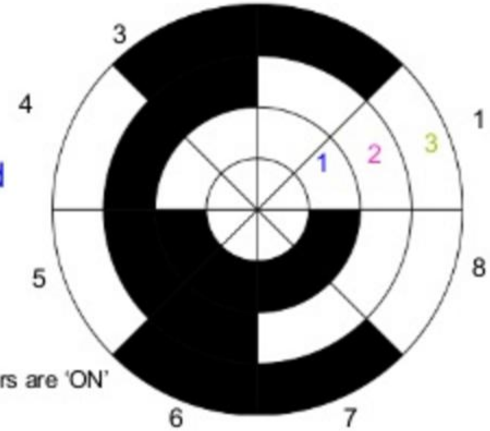
B2	B1
0	0
0	1
1	0
1	1

Example of Converting Angular Positions into Digital Signals in the Form of Parallel Bits

- Optical Absolute Encoder: Setup of Three-Bit Output

Gray Coding

3-bit Binary-Reflected Gray code (BRGC)



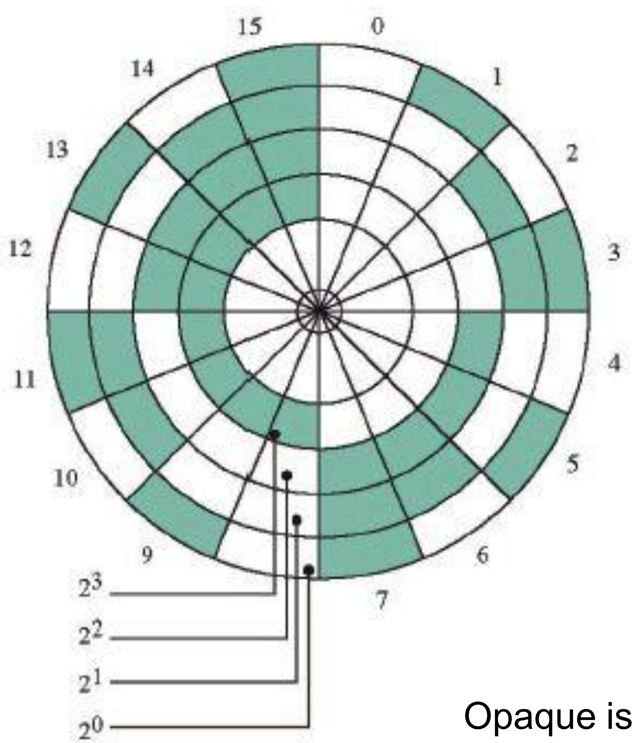
Black sectors are 'ON'

1 bit change from 1 sector to another

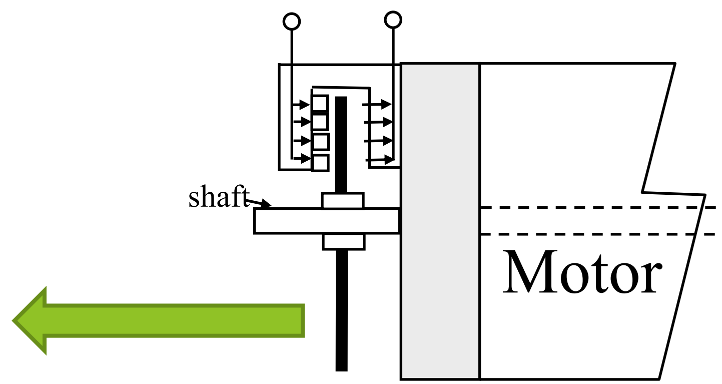
Sector	Contact 1	Contact 2	Contact 3	Angle
1	off	off	off	0° to 45°
2	off	off	on	45° to 90°
3	off	on	on	90° to 135°
4	off	on	off	135° to 180°
5	on	on	off	180° to 225°
6	on	on	on	225° to 270°
7	on	off	on	270° to 315°
8	on	off	off	315° to 360°

Example of Converting Angular Positions into Digital Signals in the Form of Parallel Bits

- Optical Absolute Encoder: Setup of Four-Bit Output



Opaque is “on”

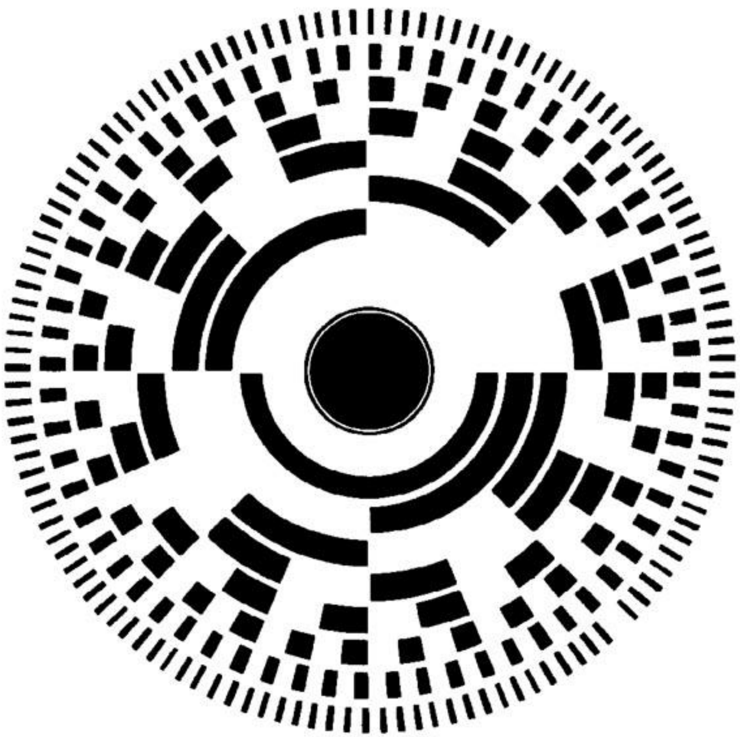


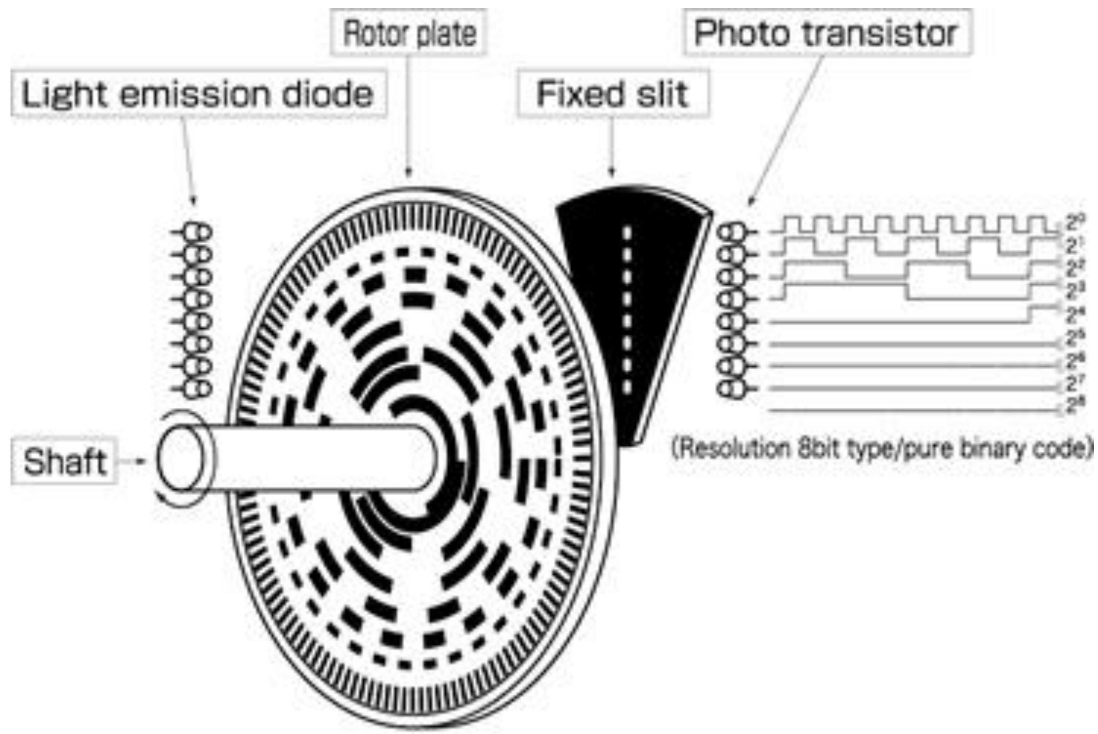
B3	B2	B1	B0
0	0	0	0
0	0	0	1
0	0	1	0
…			
1	1	1	1

Example of Converting Angular Positions into Digital Signals in the Form of Parallel Bits

- Optical Absolute Encoder: Setup of Eight-Bit Output

Black is “on”





Absolute Encoder Simplified Structure

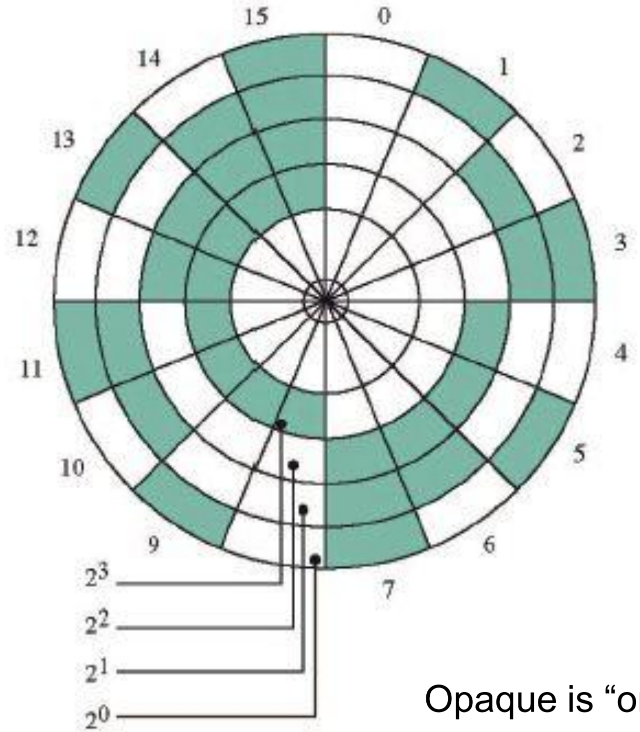
Example of Converting Angular Positions into Digital Signals

- Optical Absolute Encoder: Equations
- Measured Angular Position:

$$\theta = \frac{360^0}{2^N} \times (\text{Counter's Digital Value})$$

- Accuracy of Measurement:

$$\Delta\theta = \pm \frac{360^0}{2^N} \times \frac{1}{2}$$



Opaque is “on”

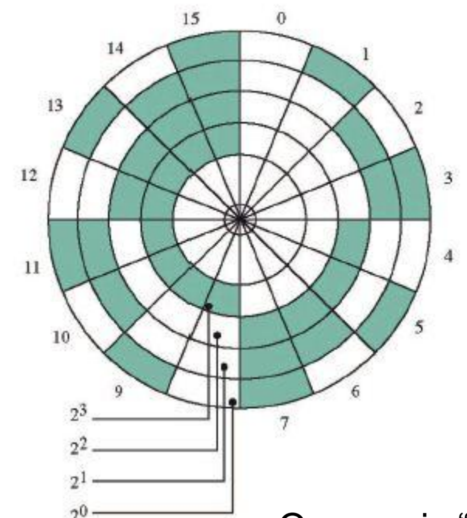
Exercise

- An optical absolute encoder has four-bit output. If the counter's digital value is 0x0100, what is the measured angular position and its accuracy?

- Answer:

$$\text{Angular Position} = \frac{360^0}{2^4} \times 4 = 90^0$$

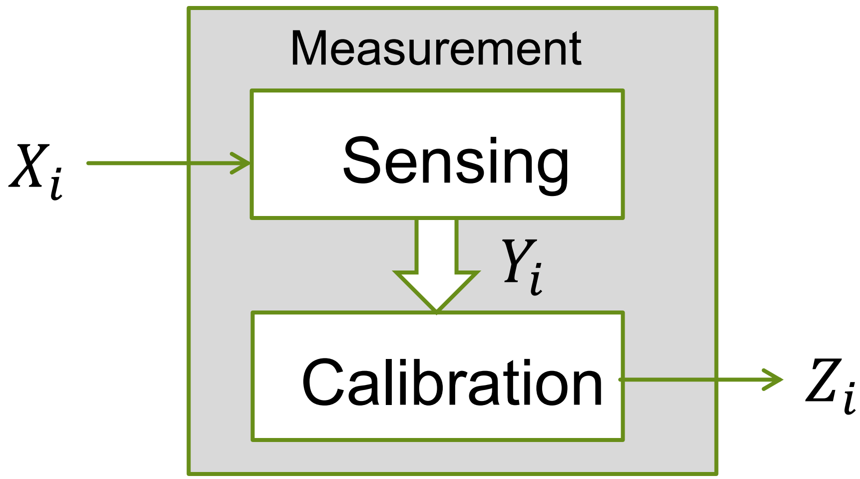
$$\text{Accuracy} = \frac{360^0}{2^4} \times \frac{1}{2} = 11.25^0$$

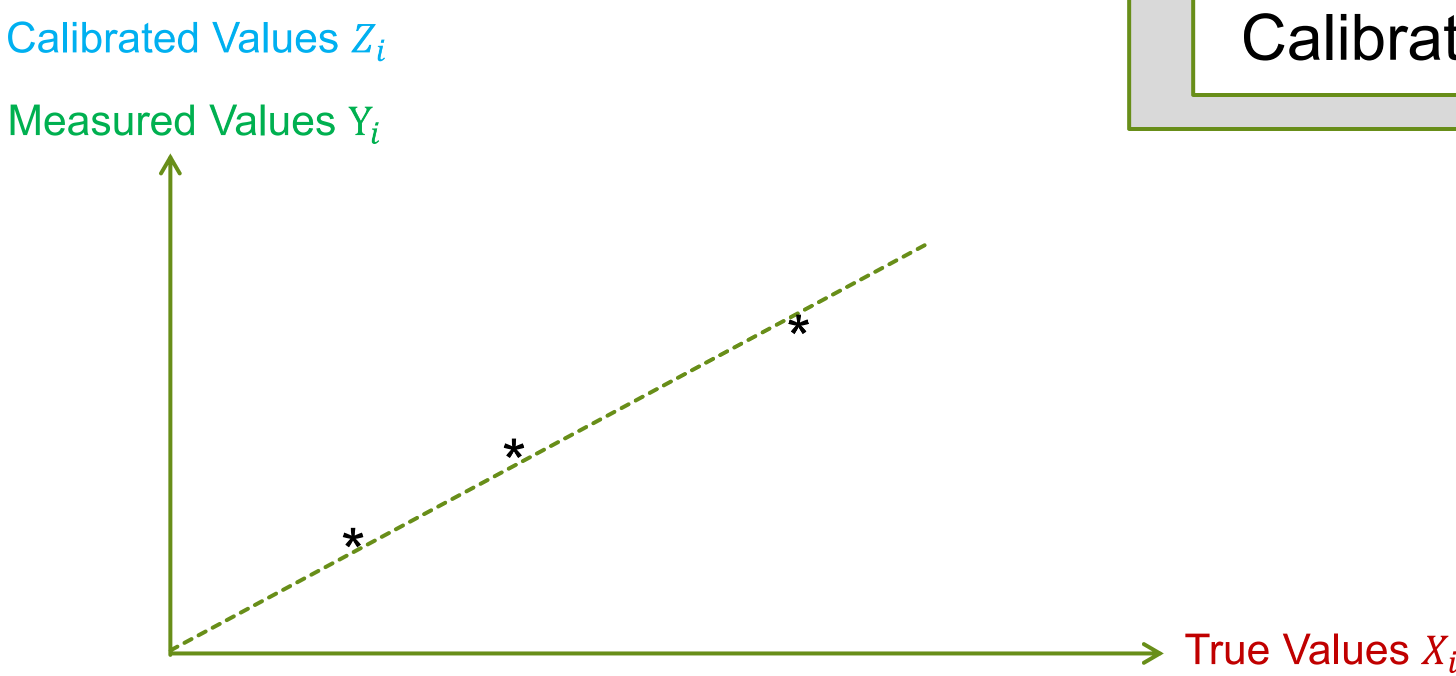


Opaque is "on"

Remember to Do Calibration

- Curve fitting for calibration:
  - Yi is produced by Xi
  - Zi is computed from Yi
  - Zi must be equal to Xi

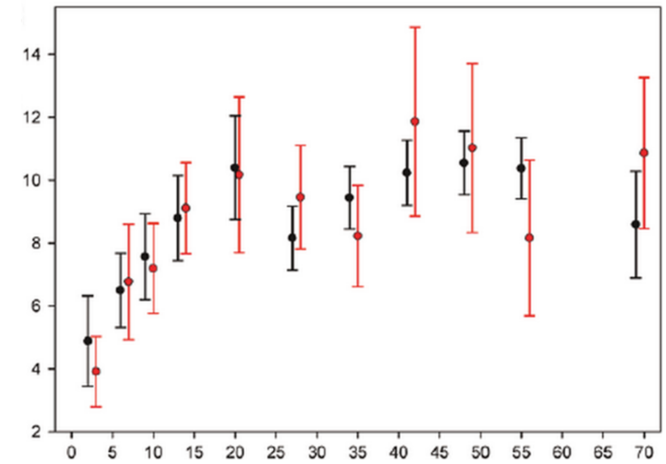


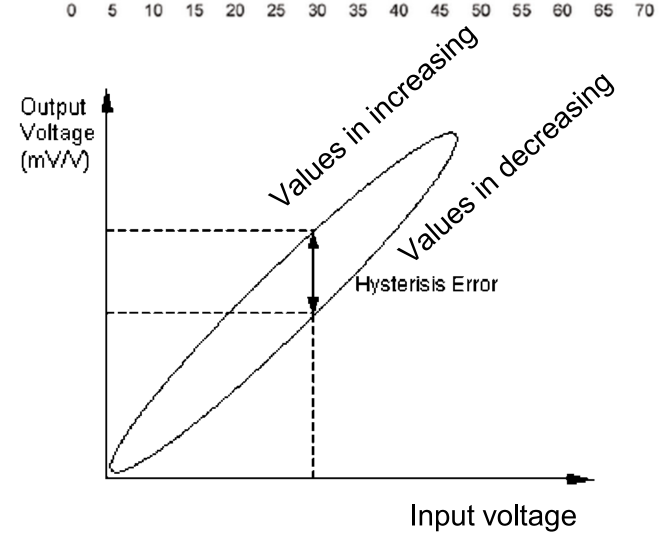


Remember to Do Error Analysis

- Systematic error = mean value – true value
- Repeatability error = value with maximum error – mean value
- Accuracy = value with minimum error – mean value
- Hysteresis error = |measured value in increasing – measured value in decreasing|

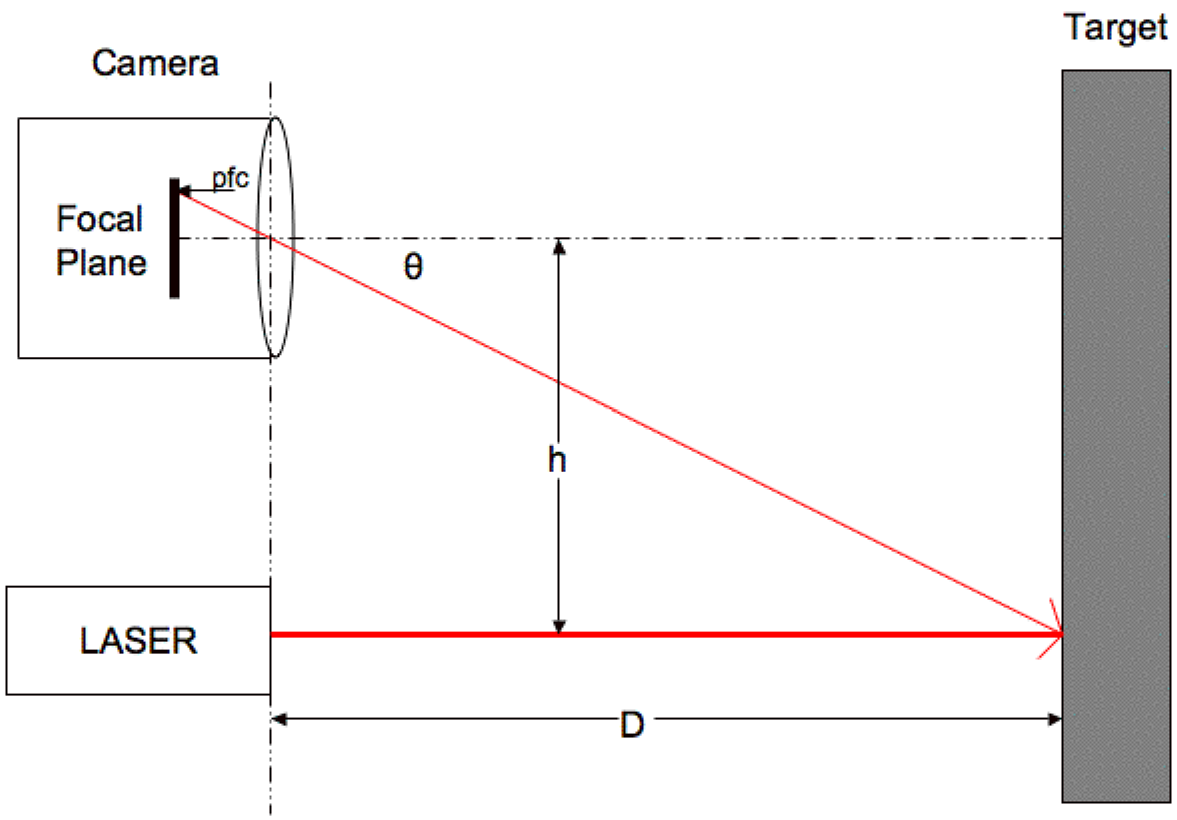
For each true value, we can do error analysis



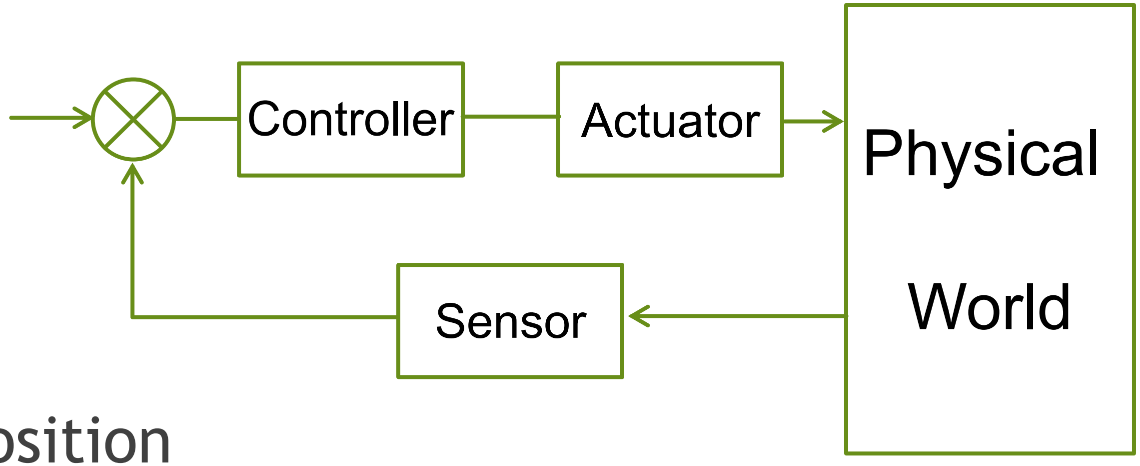


There Are Vision-based Methods

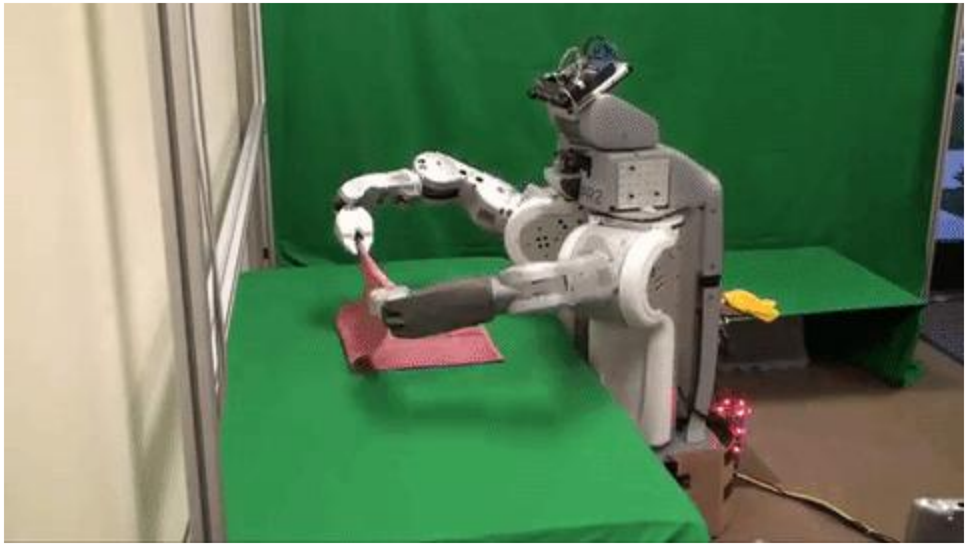
- We will study vision-based methods in separate lectures …



Summary

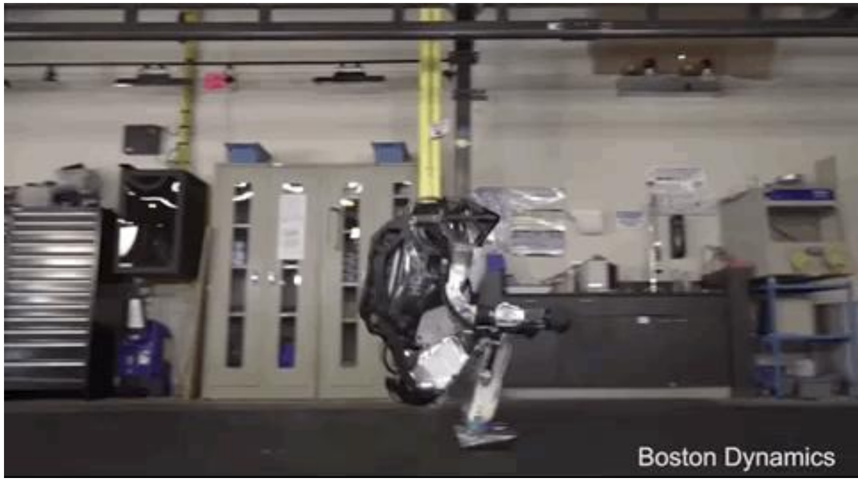


- Understanding of Position
- Computation of Position
- Measurement of Position



Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



Module 3 Lecture 2

MA4822

Measurement of Velocity

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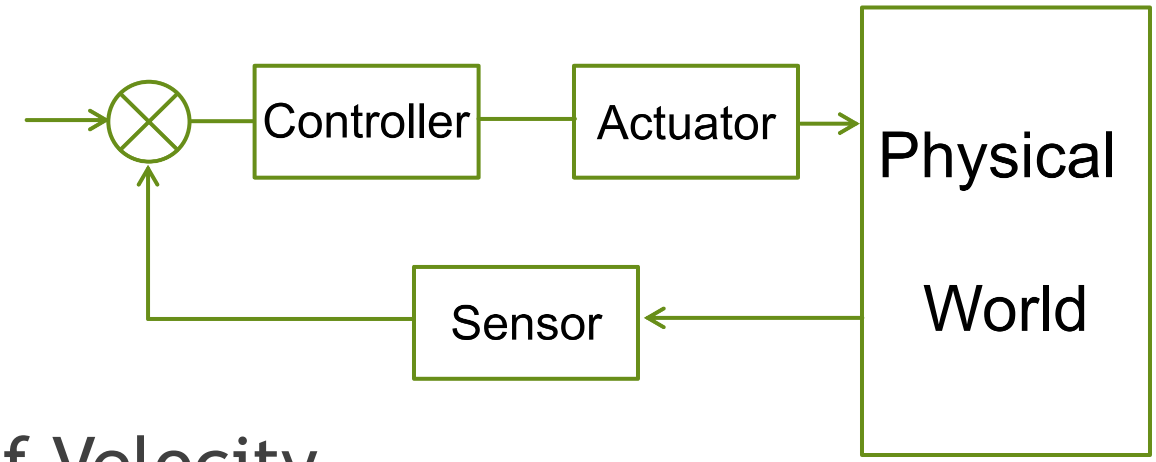
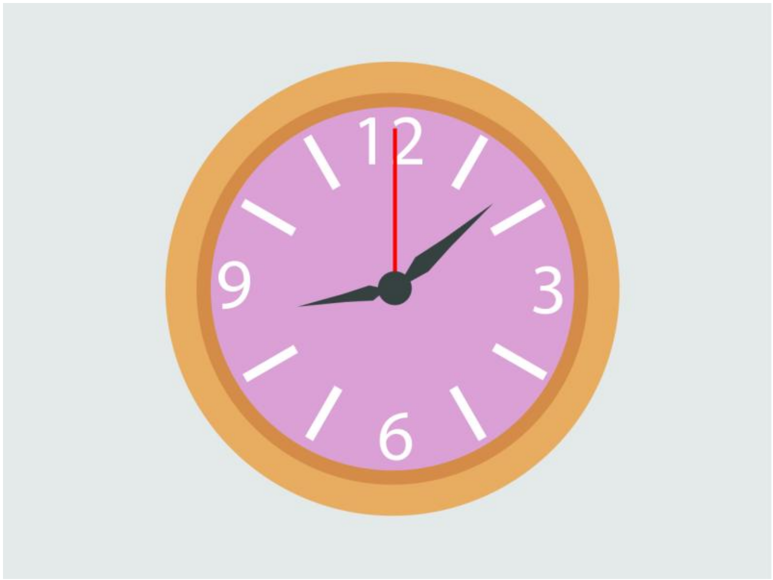
http://personal.ntu.edu.sg/mmxie

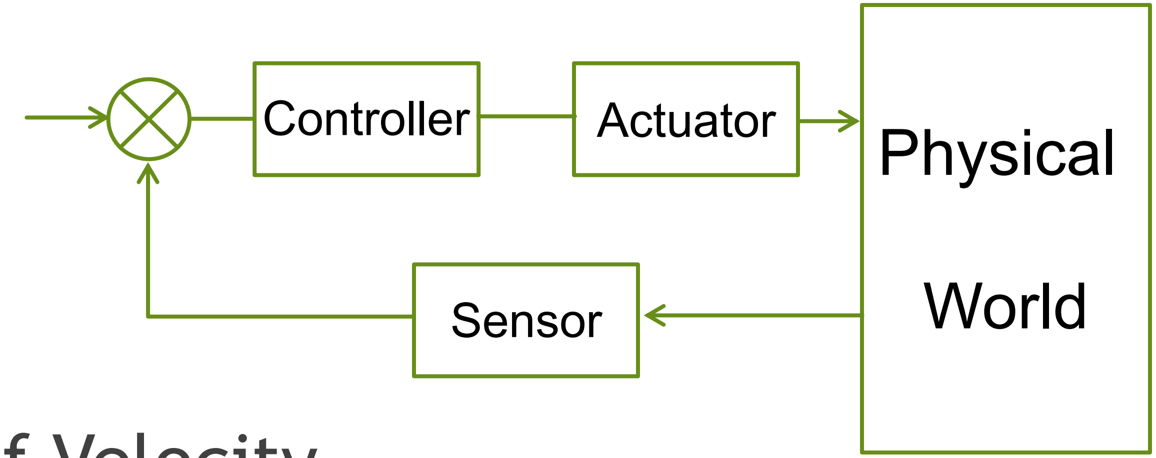
Outline



- Understanding of Velocity
- Computation of Velocity
- Measurement of Velocity

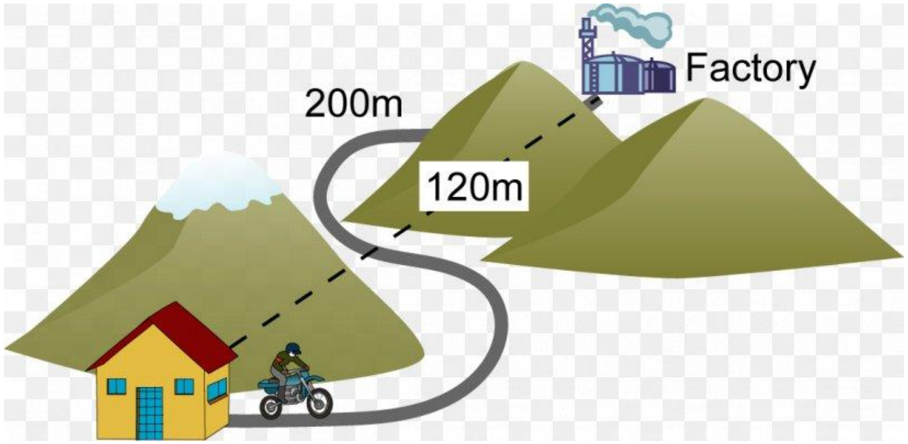


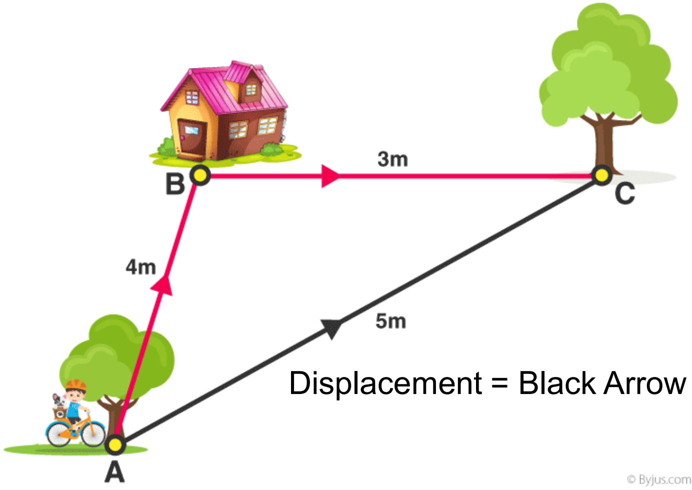
Outline

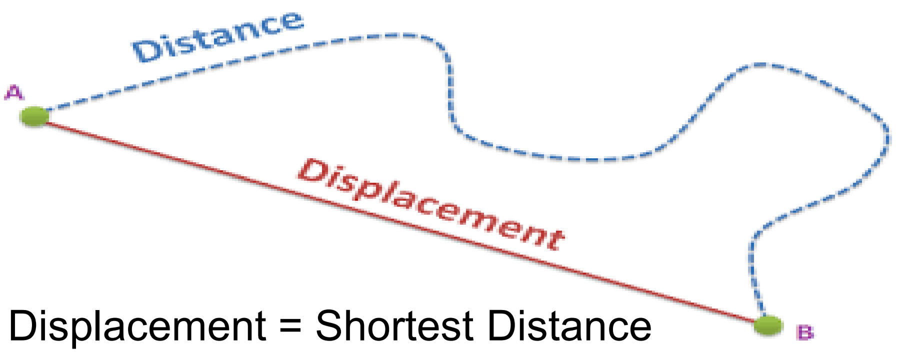


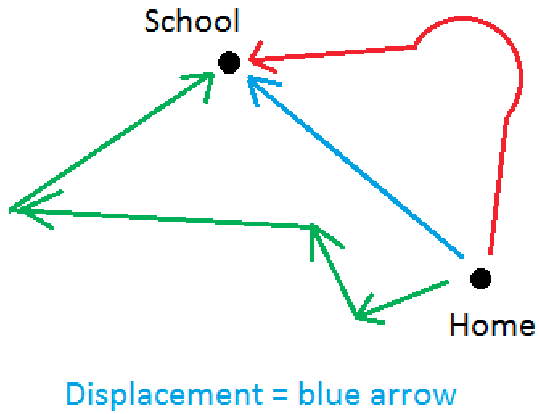
- Understanding of Velocity
- Computation of Velocity
- Measurement of Velocity

Velocity of a rigid body is related to position through displacement which is different from distance …



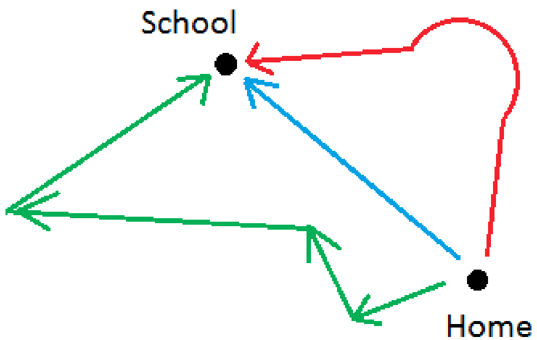




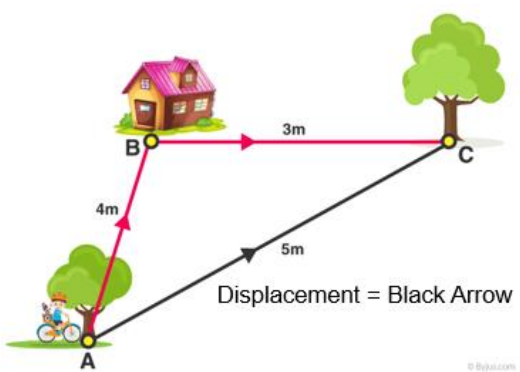


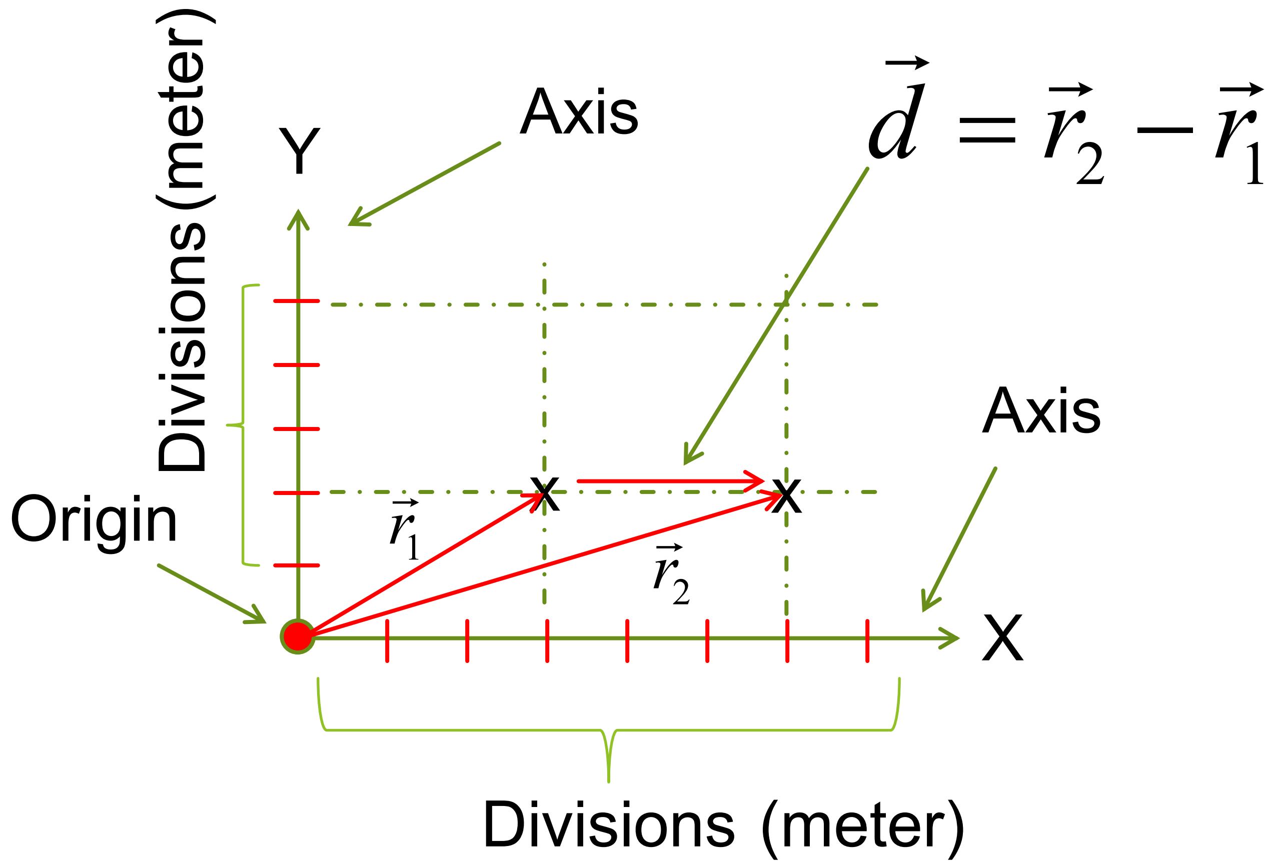
Definition of Displacement

- When an object in solid state changes its position, the difference between the final position and the initial position is called Displacement.



Displacement = blue arrow





Definition of Velocity

- The time rate change of positions is called Velocity.
- Velocity is a vector, which is equal to displacement vector divided by time.
- Amplitude of velocity is called Speed.

$$\vec{d} = \vec{r}_2 - \vec{r}_1 \quad \Rightarrow \quad \vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}(\vec{r}_2 - \vec{r}_1)$$

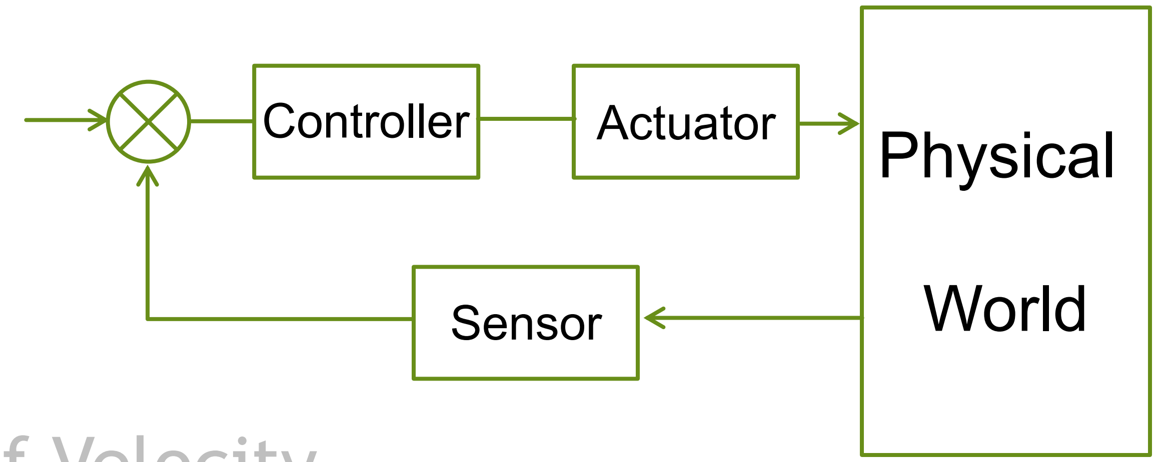
More examples of velocity …

Velocity of Solid

Velocity of Liquid

Velocity of Gas

Outline



- Understanding of Velocity
- Computation of Velocity
- Measurement of Velocity

Arbitrary Linear Motions

- If we know the time function of positions, then:

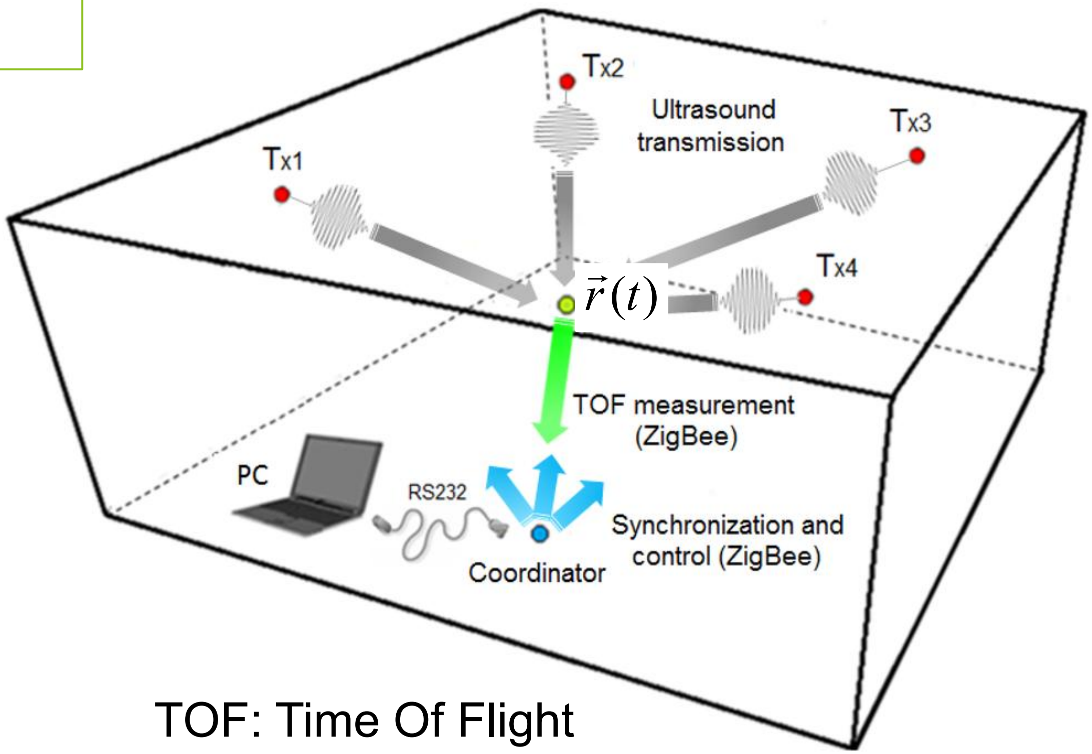
$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$

- If we know the acceleration function of positions, then:

$$\vec{v}(t) = \int_{t_0}^{t} \vec{a}(t)dt$$

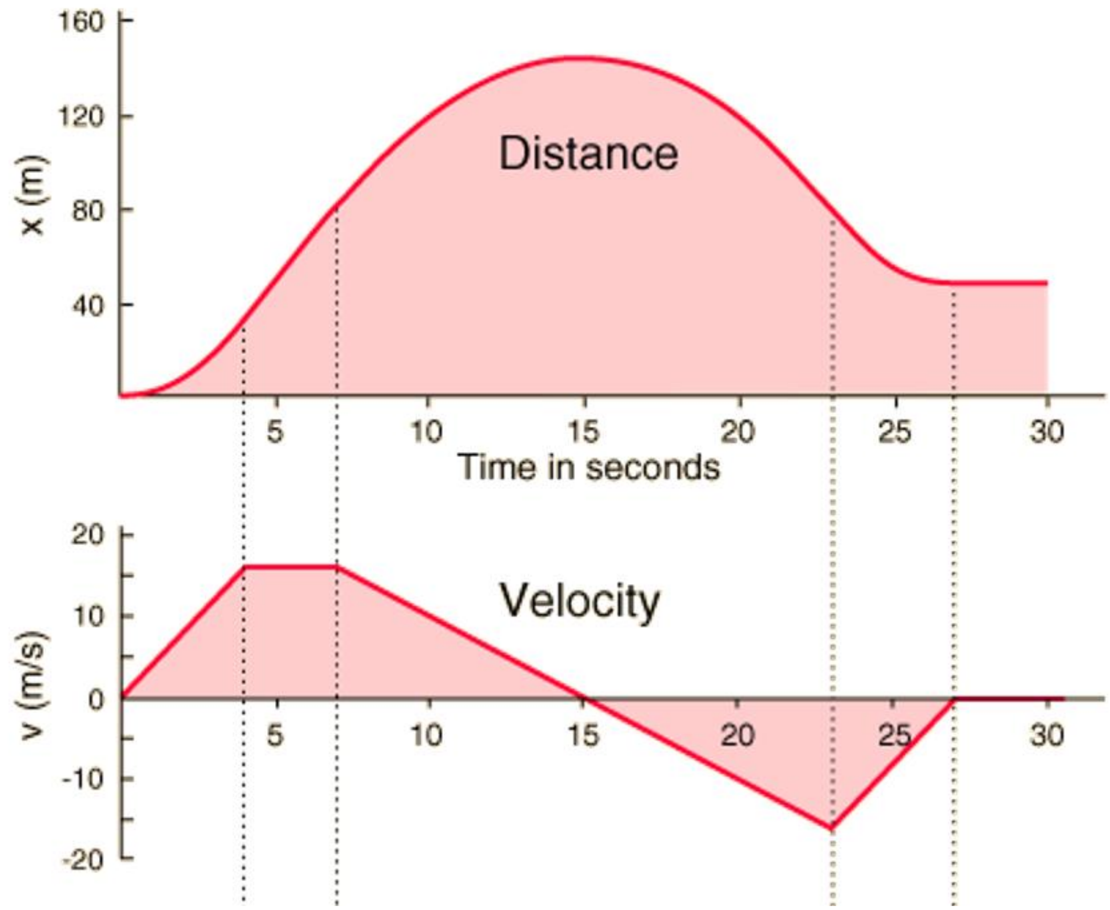
Example of Using Indoor GPS Data

$$\bar{v} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$



TOF: Time Of Flight

Example of Using Recorded Distance Data



Arbitrary Angular Motions

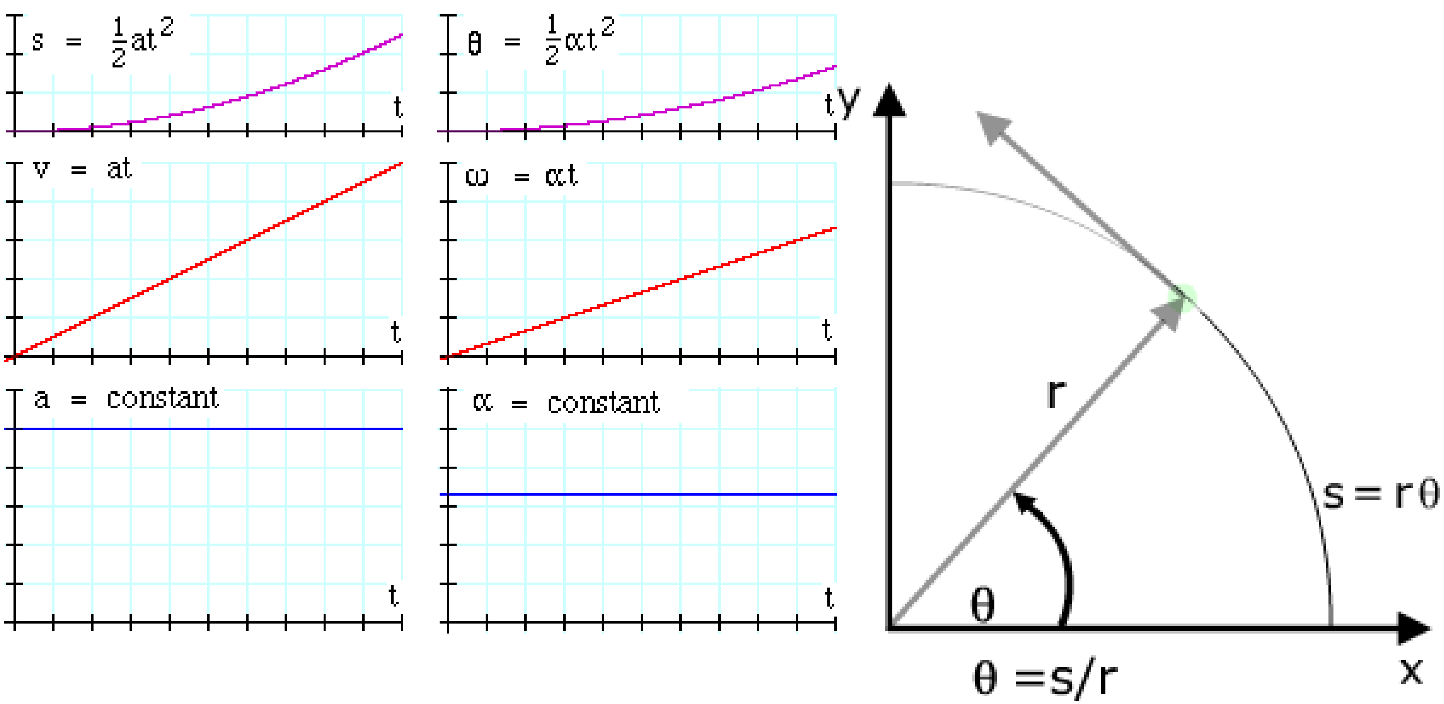
- If we know the time function of angular positions, then:

$$\omega(t) = \frac{d}{dt}\theta(t)$$

- If we know the acceleration function of angular positions, then:

$$\omega(t) = \int_{t_0}^{t} \alpha(t)dt$$

Example of Angular Motion versus Circular Motion



$$\omega = \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt} = \frac{1}{r}\frac{dv}{dt} = \frac{a}{r}$$

Special Linear Motion: Motion with Constant Linear Acceleration

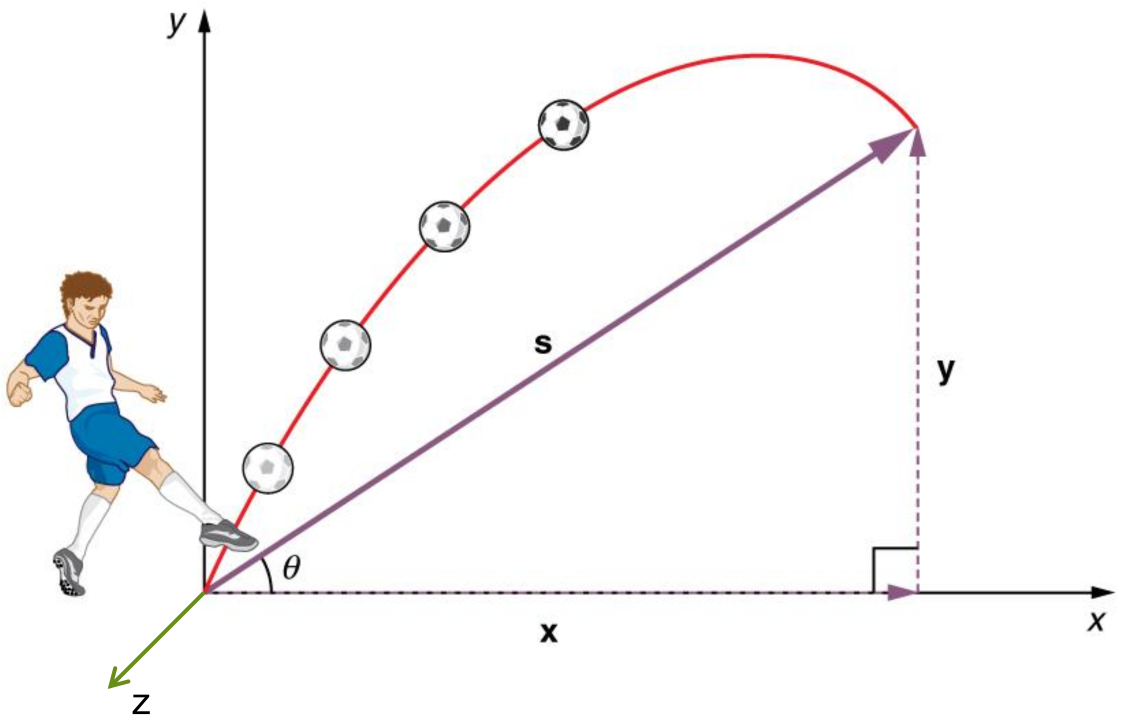
- Basic Equations of Kinematics:

$$\vec{r}(t) = \vec{v}(0)t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}(t) = \vec{v}(0) + \vec{a}t$$

Example

- If a target moves with constant accelerations, then we have:



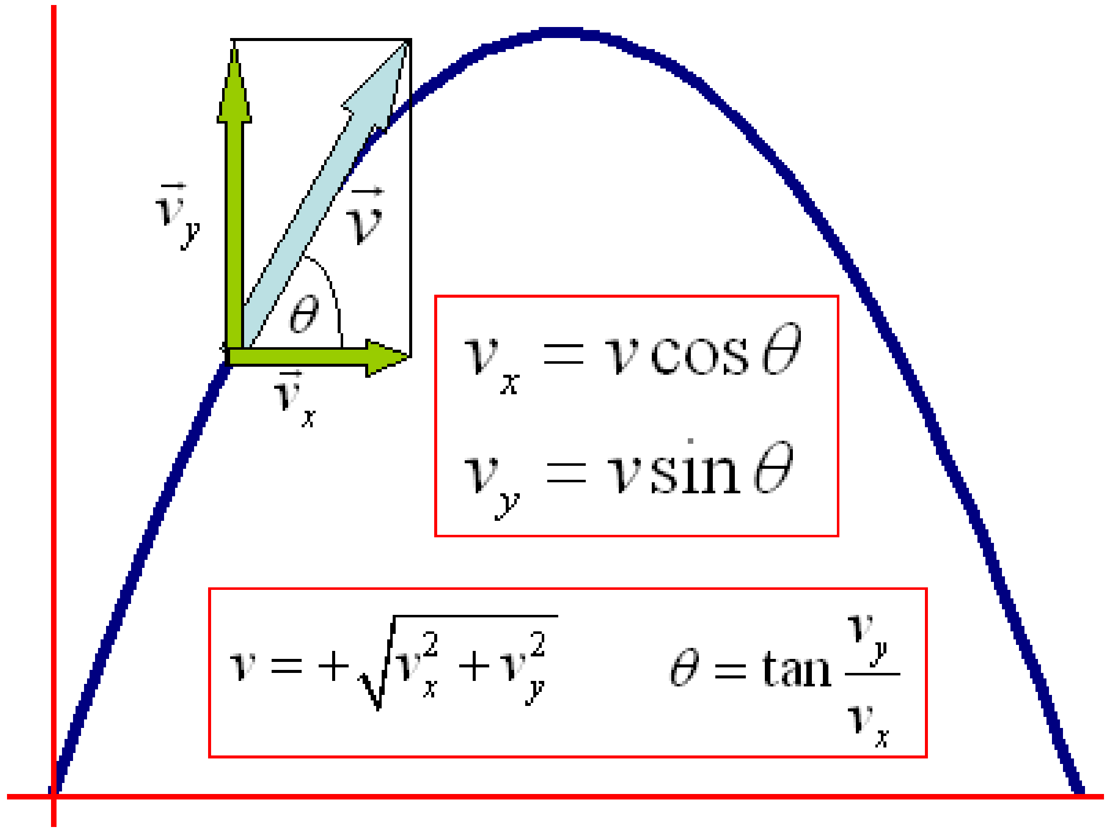
$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$$v_x(t) = v_x(0) + a_x t$$

$$v_y(t) = v_y(0) + a_y t$$

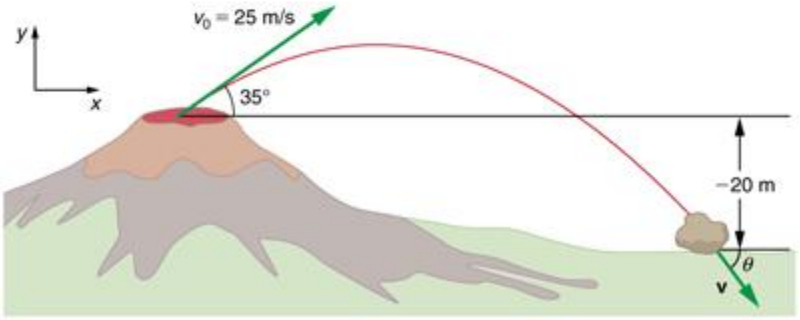
$$v_z(t) = v_z(0) + a_z t$$

Example


$\vec{v}_y$
$\vec{v}$
θ
$\vec{v}_x$
$v_x = v\cos\theta$
$v_y = v\sin\theta$
$v = +\sqrt{v_x^2 + v_y^2}$
$\theta = \tan \frac{v_y}{v_x}$

Example

- As shown in the figure, what is the velocity when t = 1.5 seconds?



- Answer:

$$v_x(0) = 25 \bullet \cos(35^0) = 20.48 m/s$$

$$v_y(0) = 25 \bullet \sin(35^0) = 14.34 m/s$$

$$v_x(1.5) = v_x(0) = 20.48 m/s$$

$$v_y(1.5) = 14.34 + (-9.8)\times 1.5 = -0.36 m/s$$

$$\vec{v}(1.5) = 20.48\vec{i} - 0.36\vec{j} \ \ \text{(m/s)}$$

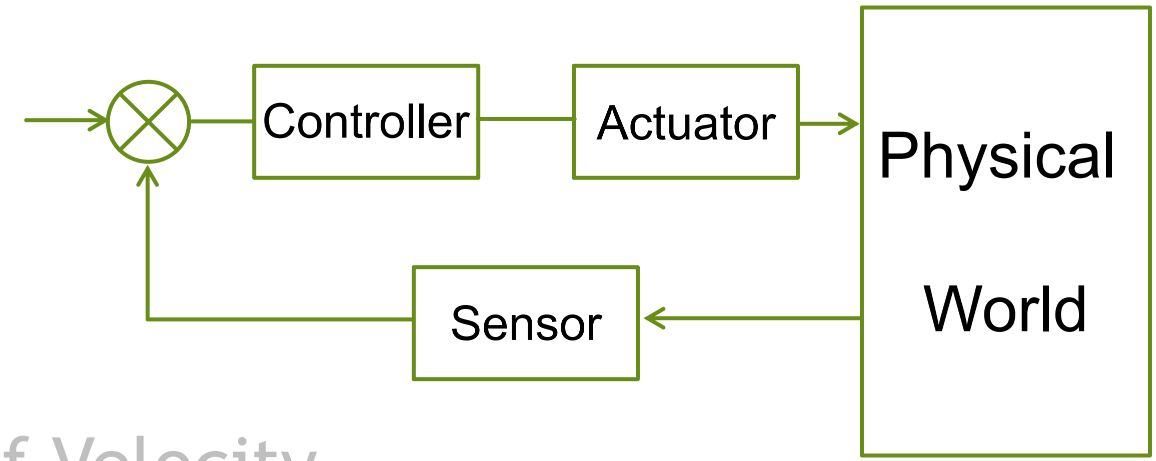
Special Angular Motion: Motion with Constant Angular Acceleration

- Basic Equations of Kinematics:

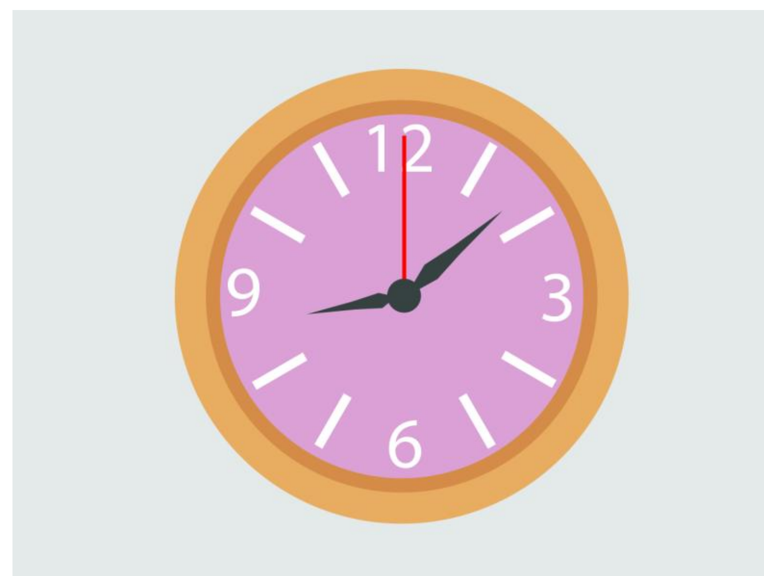
$$\theta(t) = \omega(0)t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega(0) + \alpha t$$

Outline



- Understanding of Velocity
- Computation of Velocity
- Measurement of Velocity



Applications: Robot's Velocity Control

Applications: Vehicle's Velocity Control

Observation

- Amplitude of a rigid body's velocity is the ratio of displacement and time duration.

$$\bar{v} = \pm\frac{\Delta d}{\Delta t} \qquad \bar{\omega} = \pm\frac{\Delta \theta}{\Delta t}$$

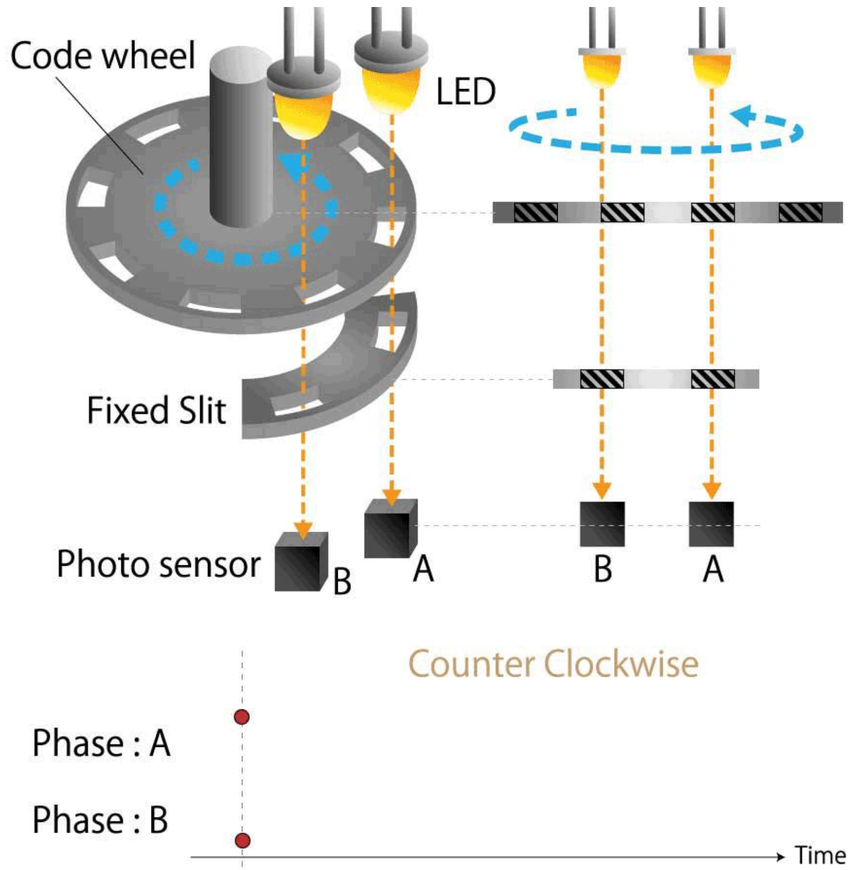
- Hence, we could measure a rigid body's velocity if it is possible to measure:
  - a) displacement
  - b) time duration
  - c) direction of motion

Could be converted into digital signals

(a series of binary digits or bits)

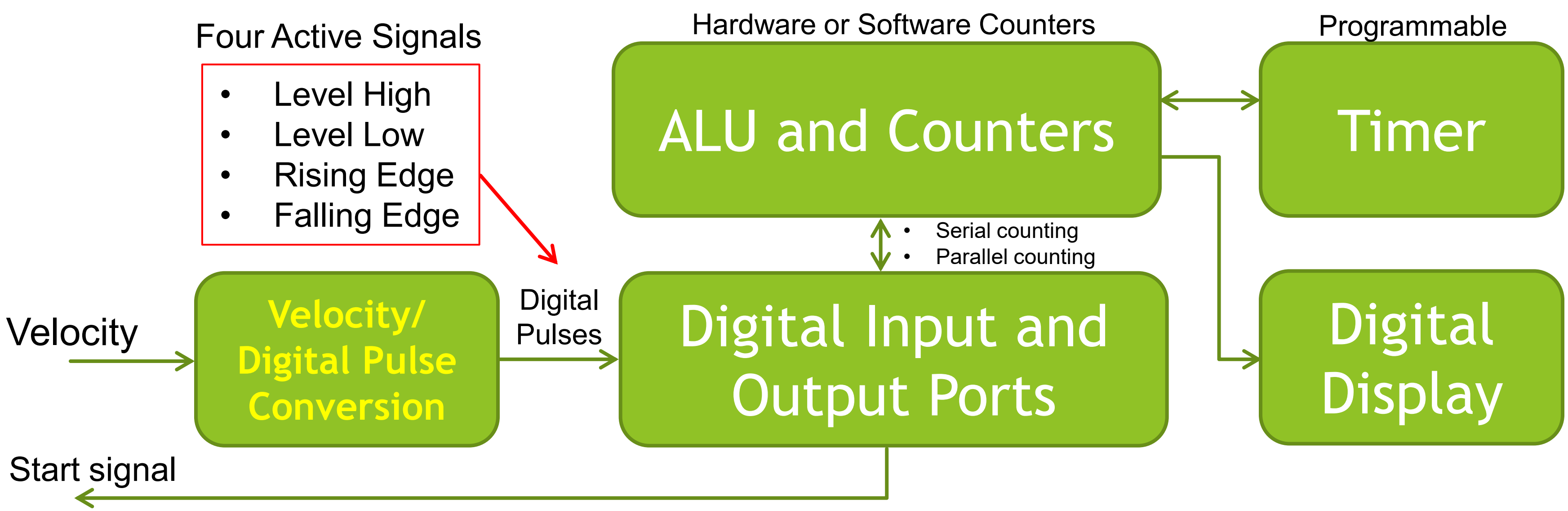
Design Principle: Using Microcontrollers as Sensors of Digital Signals

- Digital signals (i.e., bits) could be directly measured.
- Velocities could be converted into digital signals in series.
- Hence, velocities could be digitally measured.



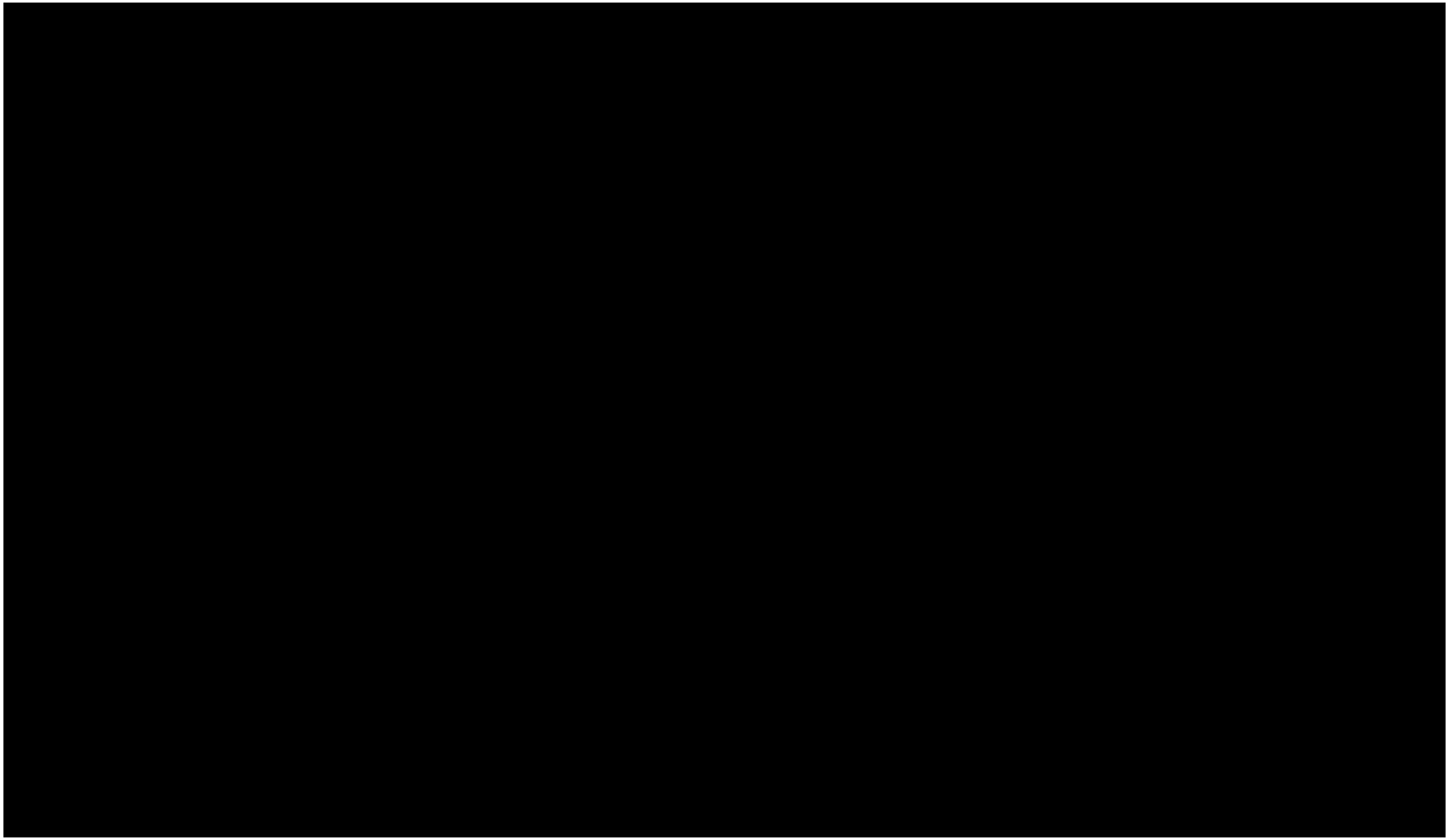
How to design digital measurement and sensing systems for velocities?

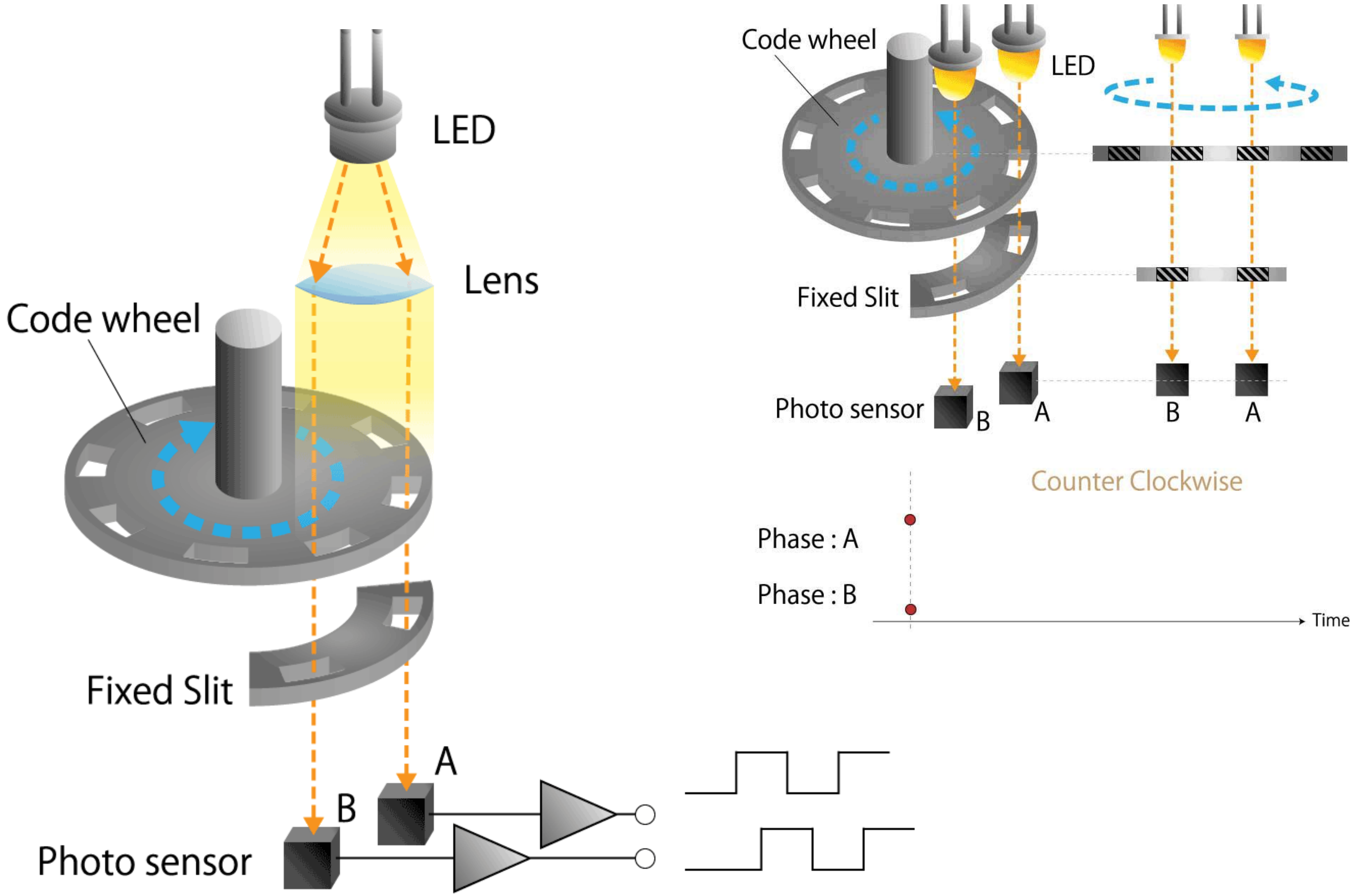
- Velocity is converted to digital signals (i.e. binary digits or bits) which are to be measured by microcontrollers.



All microcontrollers are programmable sensors of digital signals!

How to convert angular velocities into digital signals?

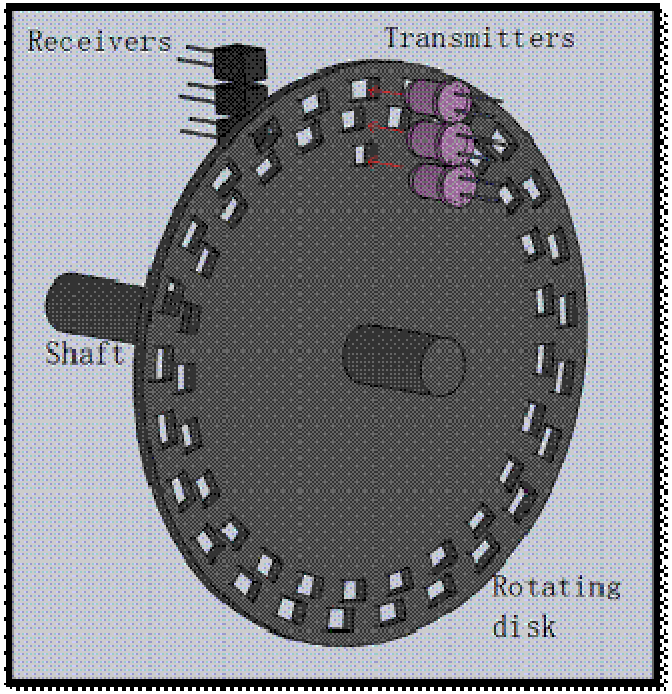




LED
Lens
Code wheel
Fixed Slit
A
B
Photo sensor
Code wheel
LED
Fixed Slit
Photo sensor
B
A
B
A
Counter Clockwise
Phase : A
Phase : B
Time

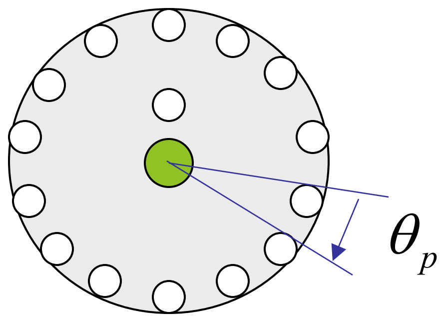
How to convert angular velocities into digital signals?

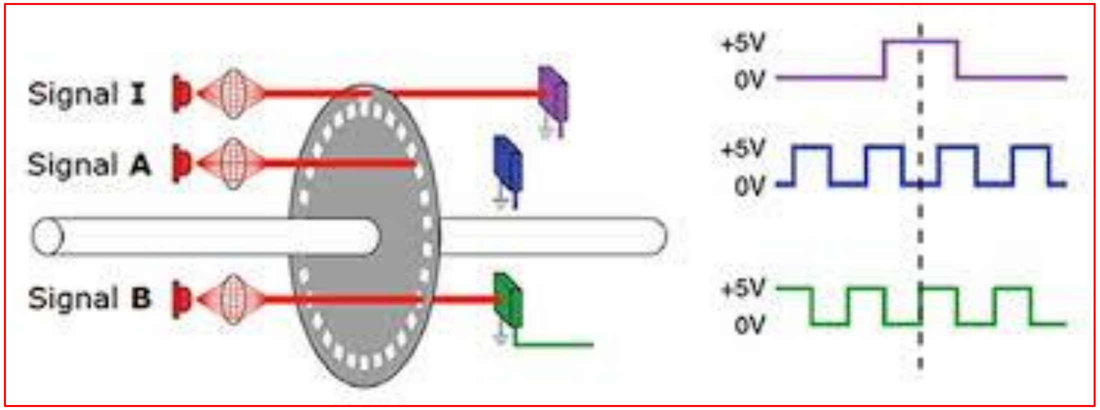
- Details of Hardware:
  - A code disk has a ring of holes or slits.
  - A pair of light and photo cell is to convert a displacement into a series of pulses of logic 1 and 0.
  - The pitch angle between two adjacent holes or slits is equal to an angular displacement.
  - The counting of the pulses within a time interval allows to determine the speed.



How to convert angular velocities into digital signals?

- Details of Equations:





$$\overline{\omega} = \frac{N_p \times \theta_p}{\Delta t}$$

N_p : Number of counted pulses

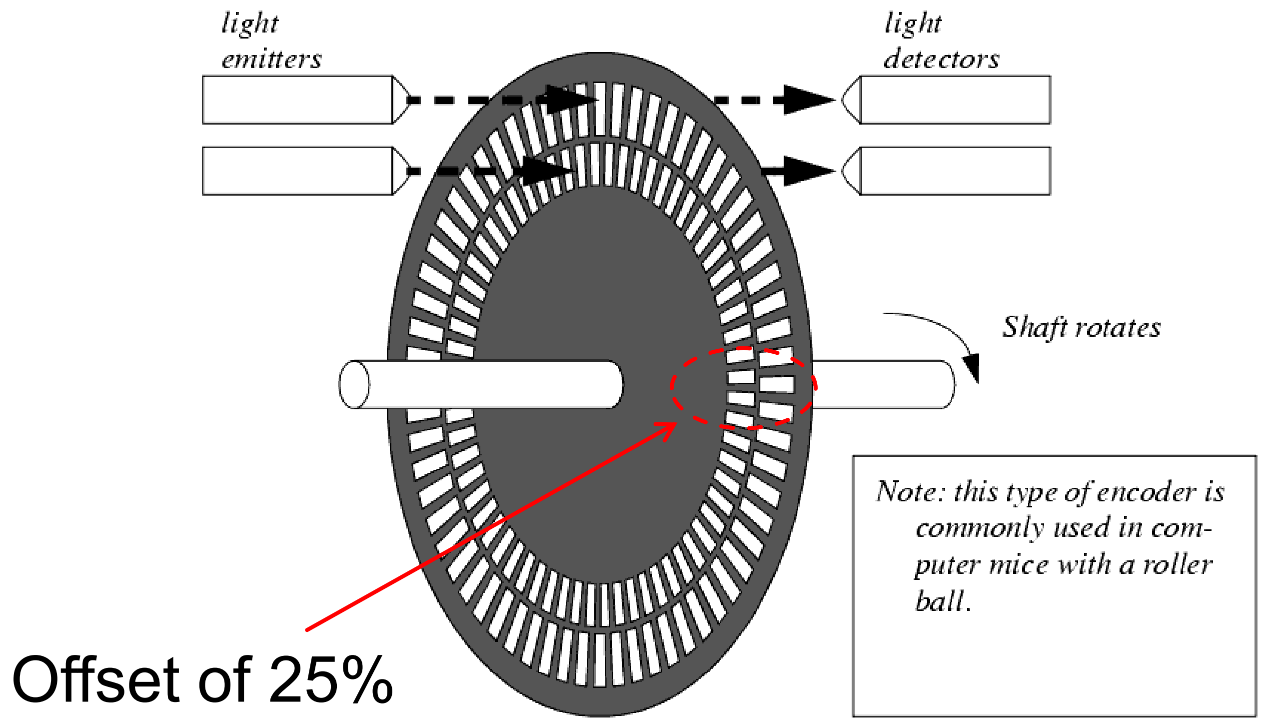
Example

- A velocity sensor can produce 200 pulses when the code disk of the sensor makes a full rotation. What is the angular displacement corresponding to one pulse?

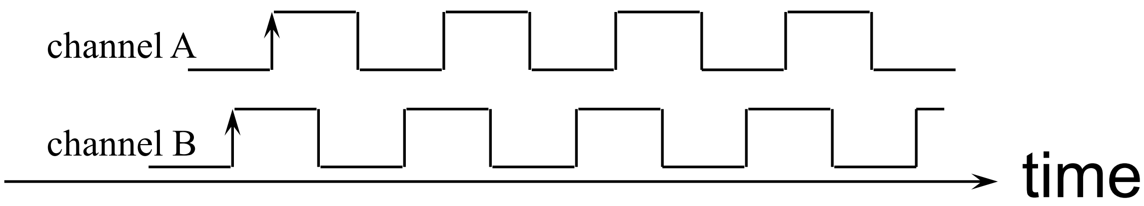
- Answer:

$$\theta_p = \frac{360^0}{200} = 1.8^0$$

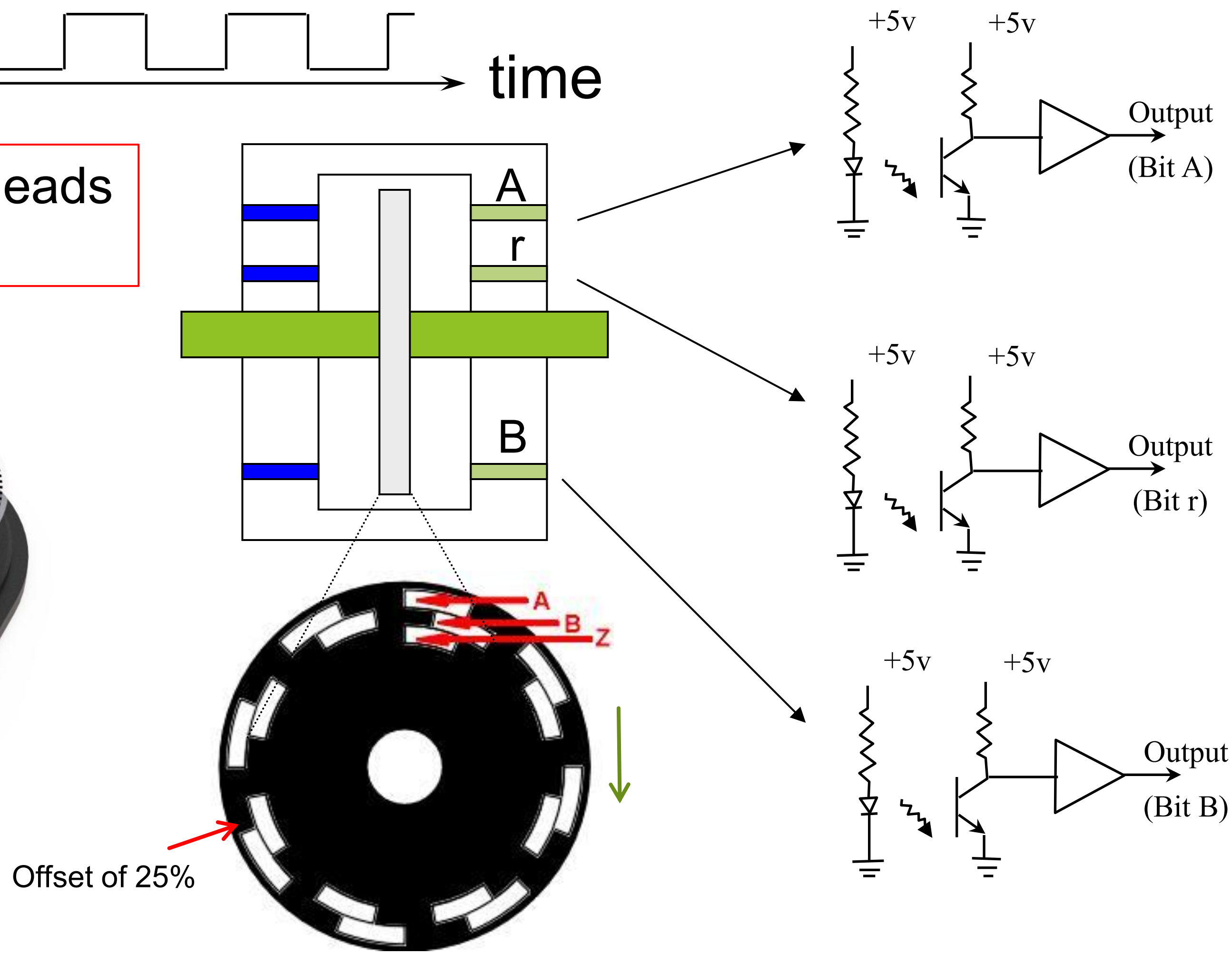
Discussion:
How to know direction of motion?



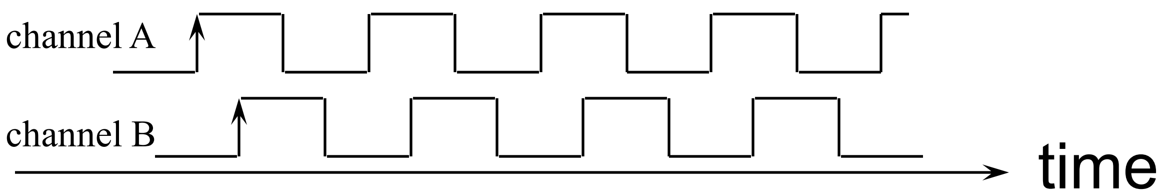
Pulse Waveforms of Clockwise Rotation



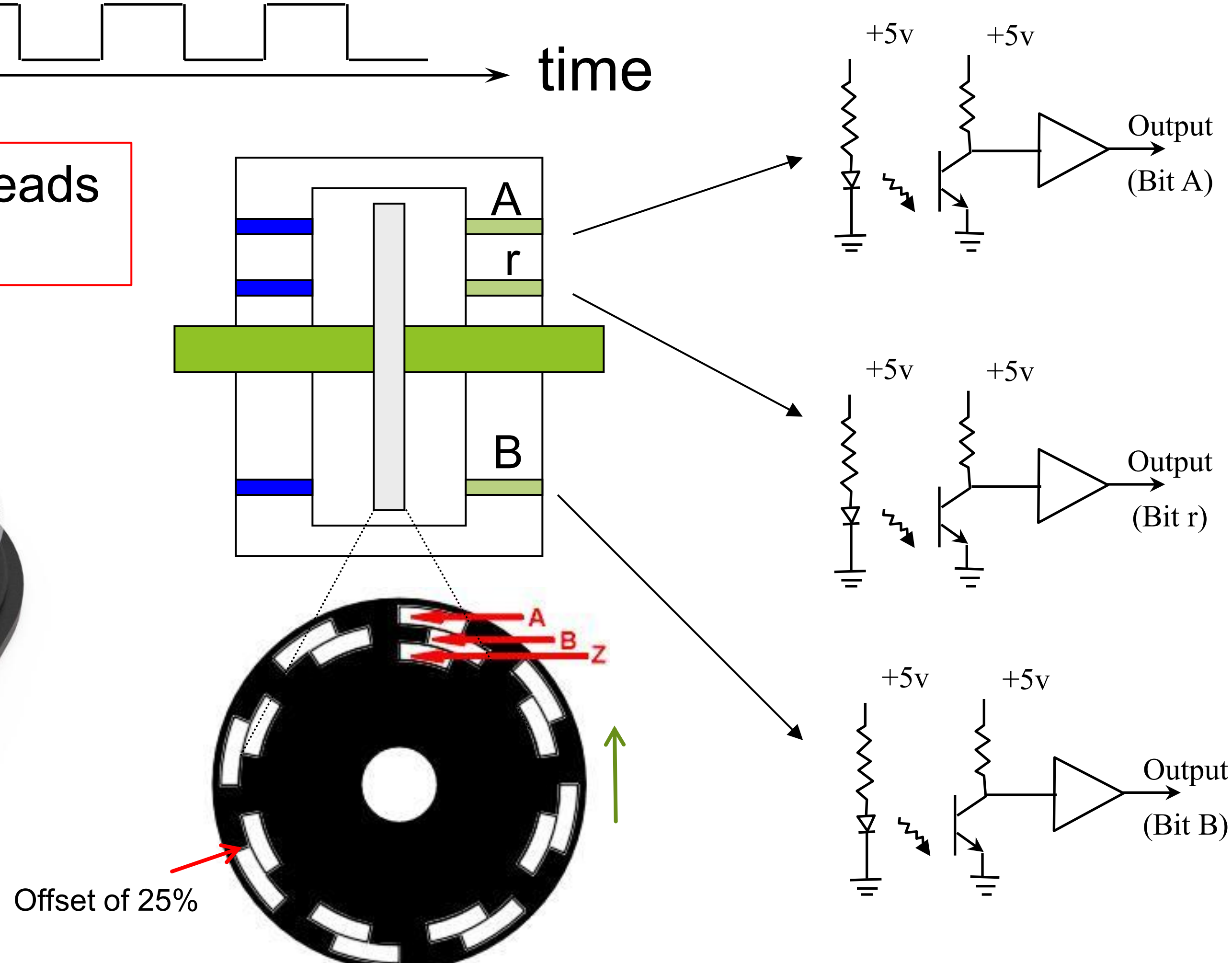
Active signal of B leads active signal of A



Pulse Waveforms of Counter-clockwise Rotation

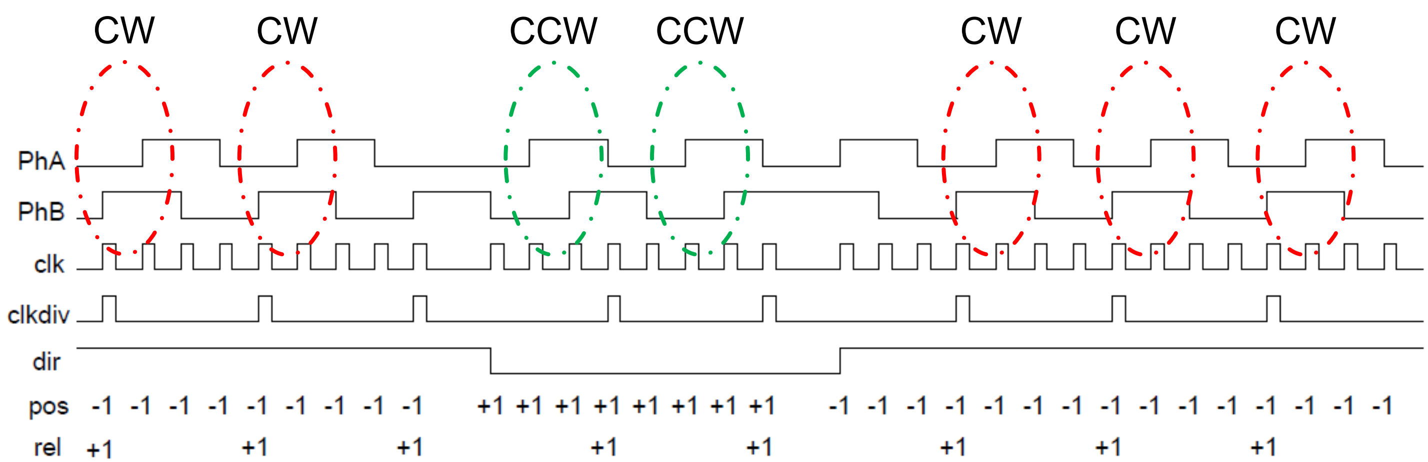


Active signal of A leads active signal of B



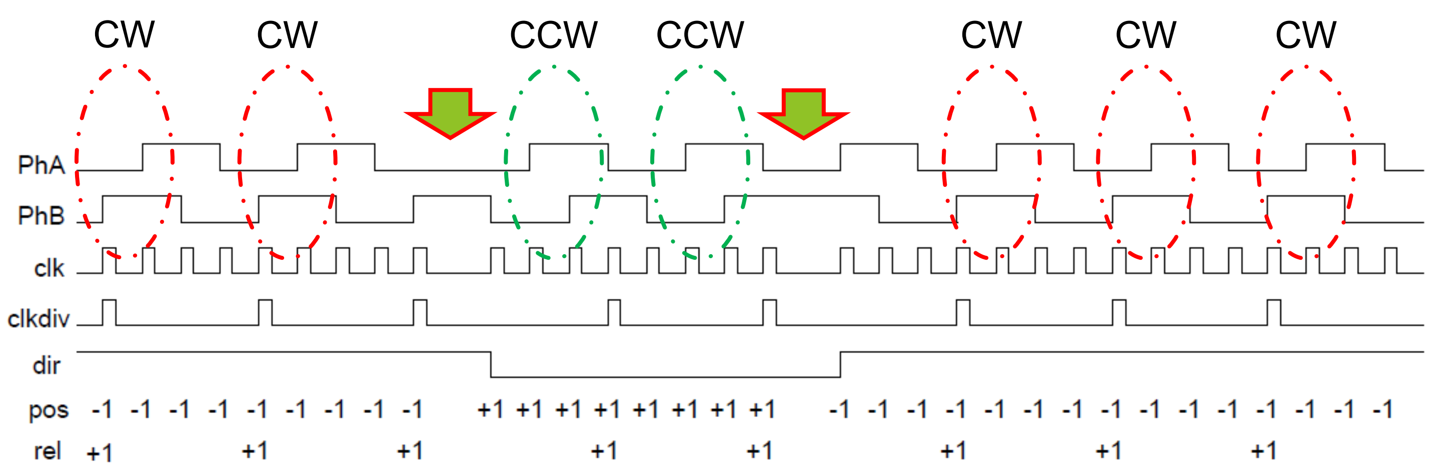
Rule 1: Direction of Motion

- If pulse B leads pulse A by 90 degrees, the direction is clockwise.
- If pulse A leads pulse B by 90 degrees, the direction is counter-clockwise.



Rule 2: Change of Direction of Motion

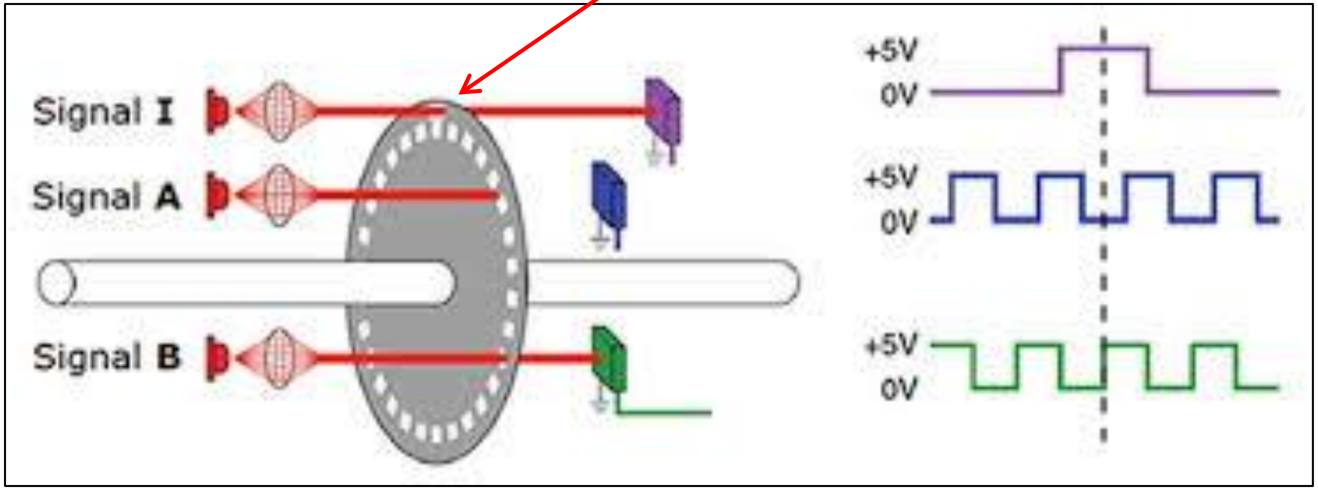
- Case 1: When pulse B makes two transitions while pulse A remains unchanged, CW is changed to CCW.
- Case 2: When pulse A makes two transitions while pulse B remains unchanged, CCW is changed to CW.



How to enable the calculation of angular positions from angular velocities?

- The solution is to include an additional track which is called I track or Z track. I stands for index of zero point.

Position of Zero or Origin



How to calculate the angular positions from the readings of velocities?

- Details of Equations:

Equation of Velocity Definition

$$\overline{\omega} = \frac{\theta - \theta_i}{\Delta t}$$

Equation of Velocity Measurement

$$\overline{\omega} = \frac{C \times \theta_p}{\Delta t}$$

$$\frac{\theta - \theta_i}{\Delta t} = \frac{C \times \theta_p}{\Delta t}$$

$$\theta = \theta_i + C \times \theta_p$$

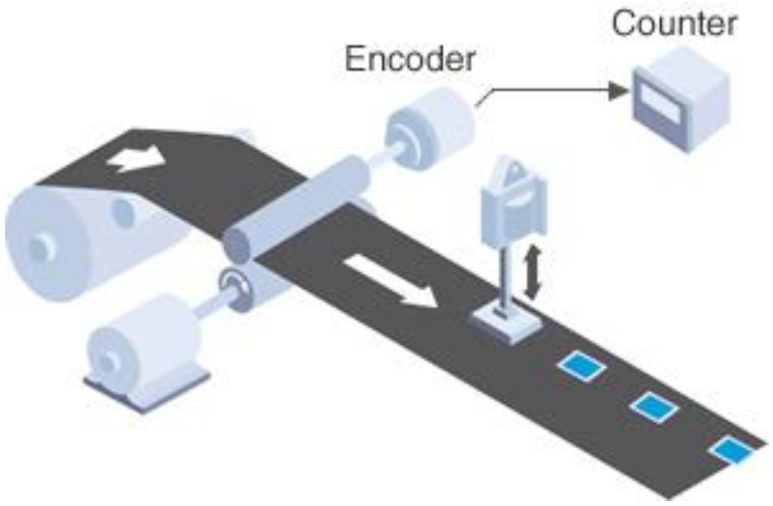
when I or Z signal appears

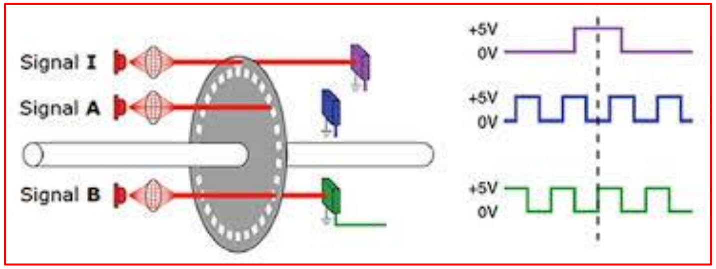
$$\theta_r = \theta_i + C_r \times \theta_p$$

$$\theta - \theta_r = (C - C_r) \times \theta_p$$

Example

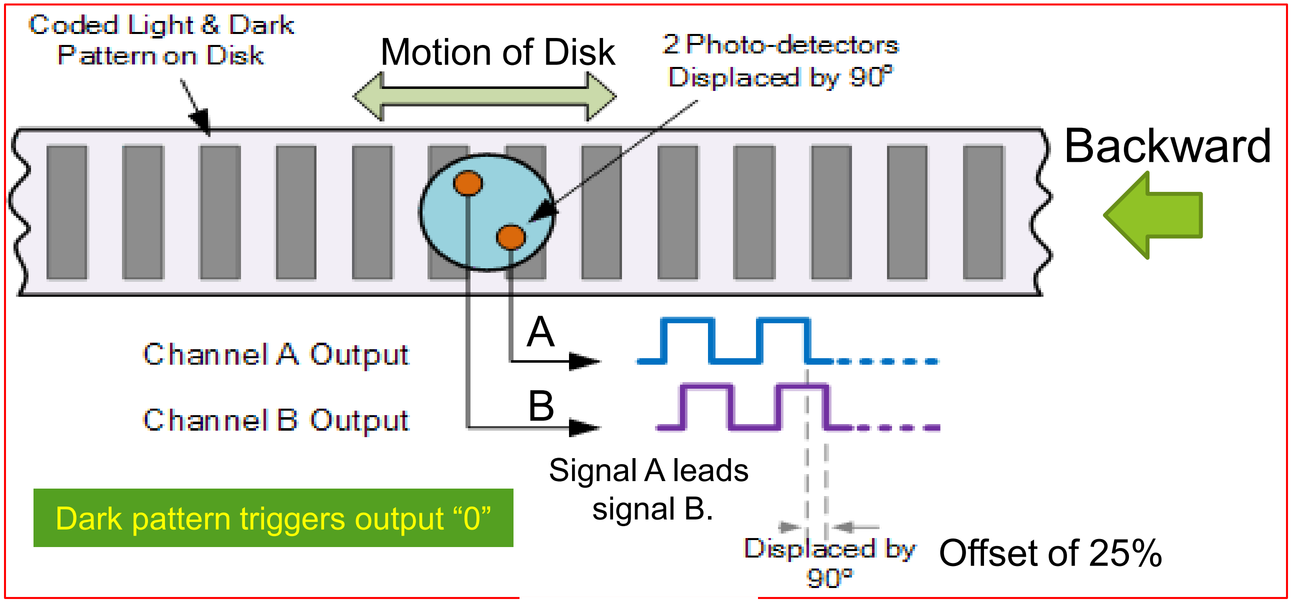
- A velocity sensor's pitch angle is 1.8 degrees. When the velocity sensor is powered on, we rotate the motor's shaft until the pulse from the photo cell of the reference appears. And, the reading of channel A is +650 counts. Now, we let the motor to make a movement. If the reading from channel A is -1190 counts, what is the angular position of the motor's shaft?
- Answer:

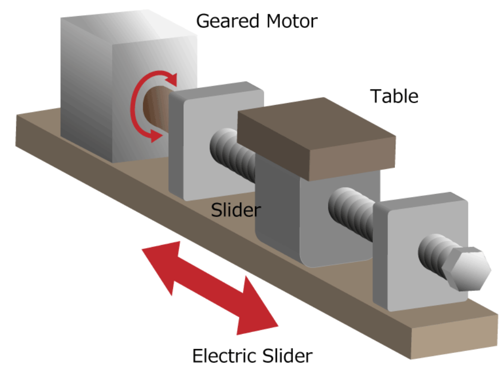


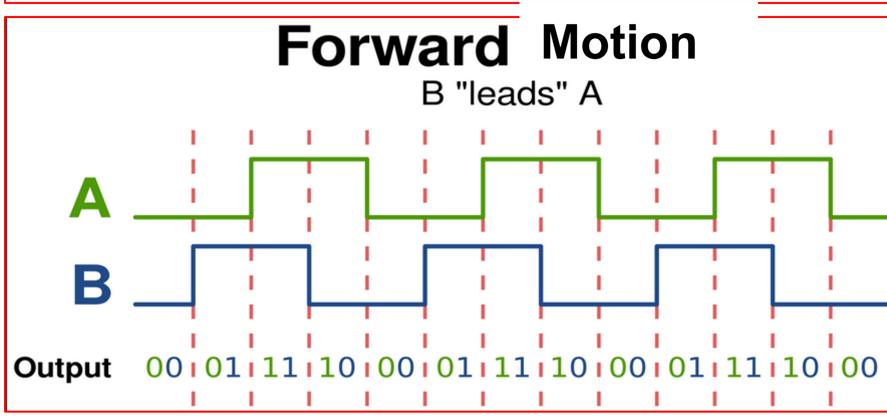


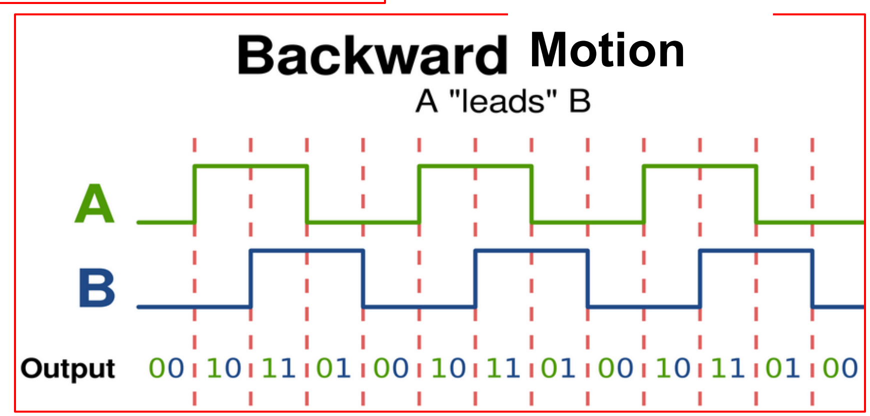
$$\theta - \theta_r = (C - C_r) \times \theta_p = (-1190 - 650) \times 1.8^0 = -3312^0$$

How to convert linear velocities into digital signals?



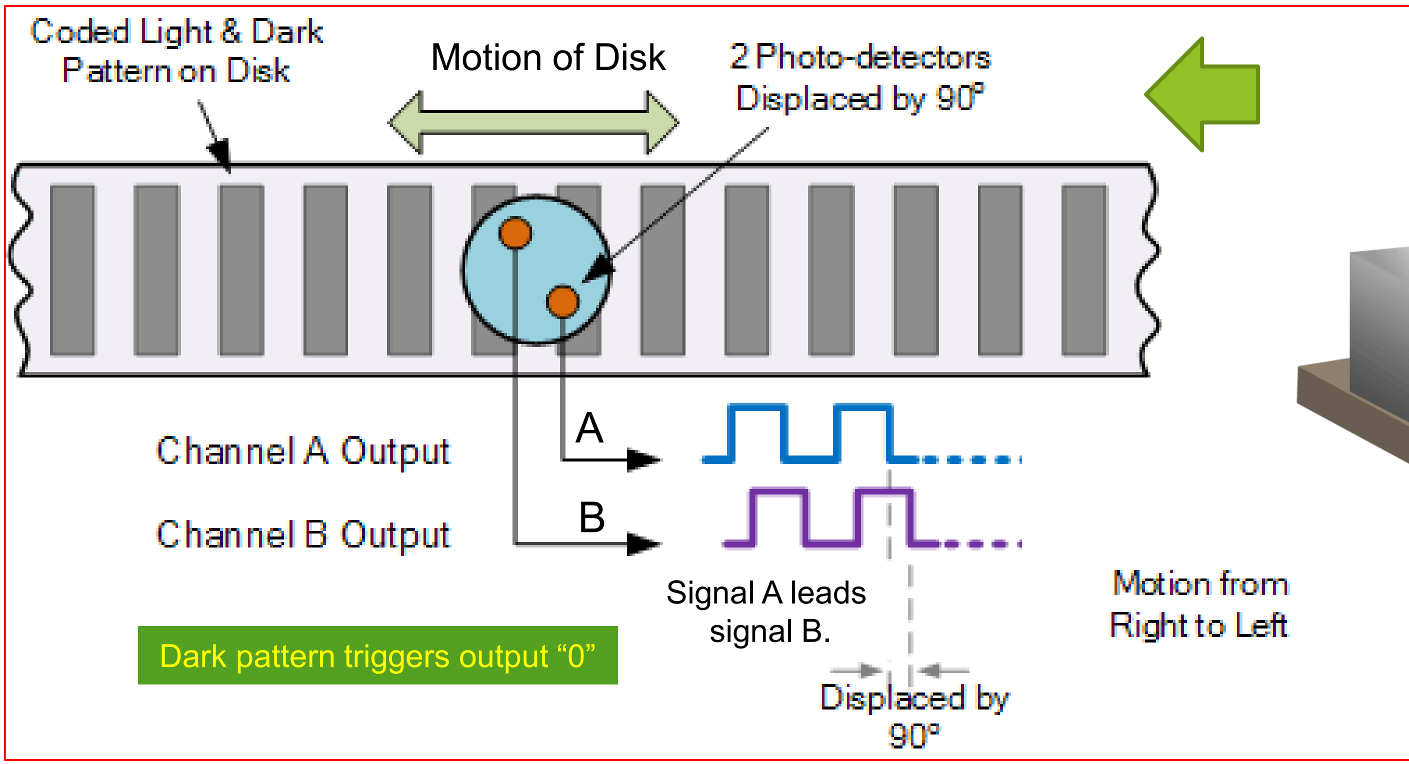


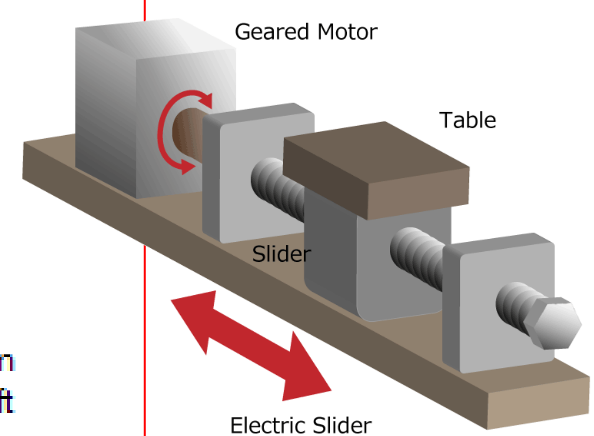




Example

- In a linear incremental encoder, the distance between two adjacent empty slits is 0.1mm. If the counted number of pulses is 7000counts/s, what is the linear speed?
- Answer: $$v = 0.1 \times 10^{-3} \times 7000 = 0.7 m/s$$

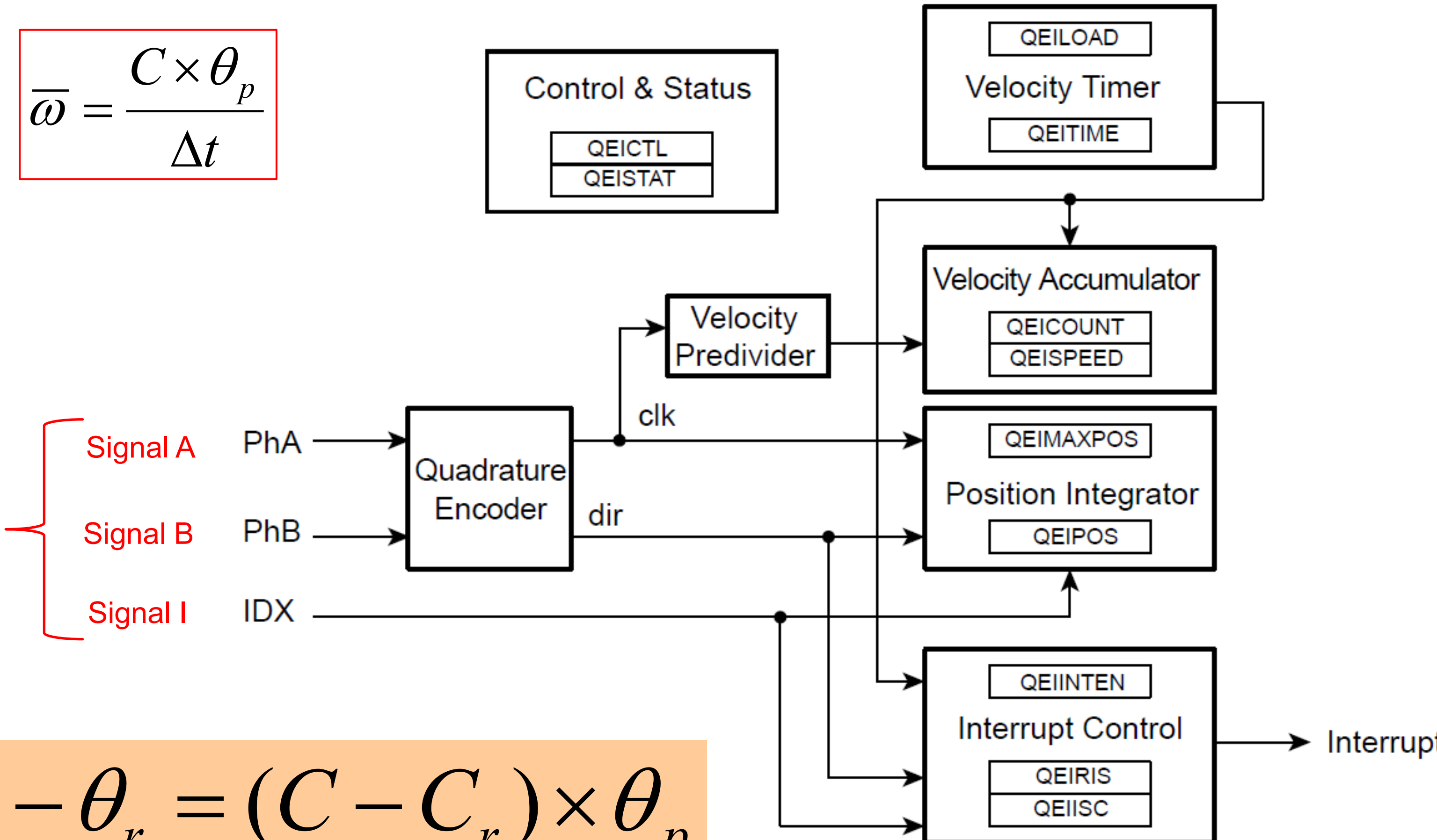




Good News: Some Microcontrollers provide advanced support to the implementation of velocity sensors ...

- ARM Cortex Microcontroller's QEI Block Diagram

$$\overline{\omega} = \frac{C \times \theta_p}{\Delta t}$$



$$\theta - \theta_r = (C - C_r) \times \theta_p$$

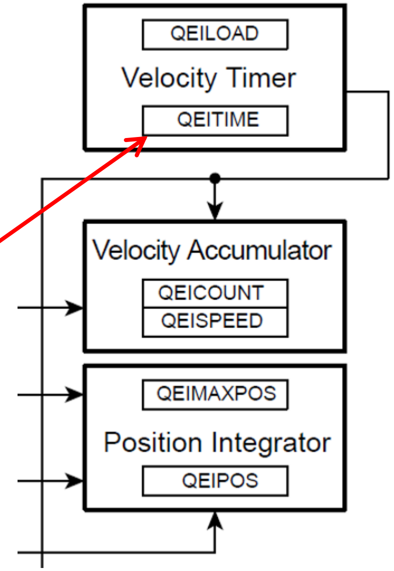
Configure QEI's Input

Number of cycles per unit of time

- Setting of Load Value of QEI's Velocity Timer

Cycle Time = 1/Frequency

Δt = QEITIME x 1/Frequency



QEI Timer Load (QEILOAD)

QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x010
Type RW, reset 0x0000.0000

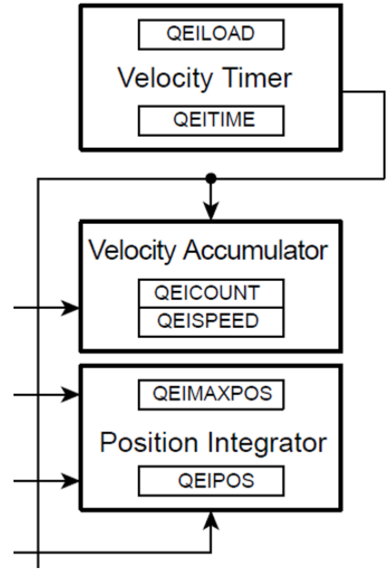
	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	LOAD															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	LOAD															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bit/Field	Name	Type	Reset	Description
31:0	LOAD	RW	0x0000.0000	Velocity Timer Load Value The load value for the velocity timer.

Configure QEI's Output

- Setting of Maximum Value of Position Integrator



QEI Maximum Position (QEIMAXPOS)

QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x00C
Type RW, reset 0x0000.0000

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	MAXPOS															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

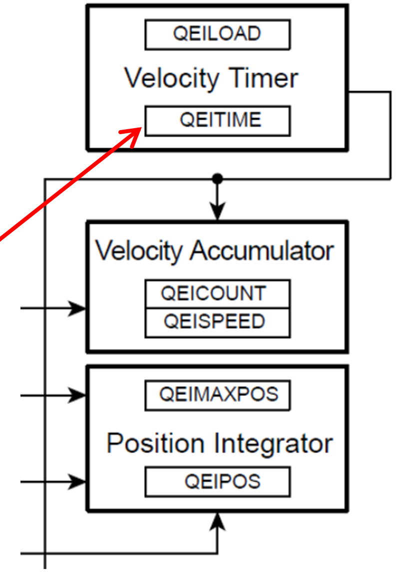
	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	MAXPOS															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bit/Field	Name	Type	Reset	Description
31:0	MAXPOS	RW	0x0000.0000	Maximum Position Integrator Value The maximum value of the position integrator.

Run QEI to Read Time Duration

- Current Value of QEI's Velocity Timer

$$\overline{\omega} = \frac{C \times \theta_p}{\Delta t}$$



QEI Timer (QEITIME)

QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x014
Type RO, reset 0x0000.0000

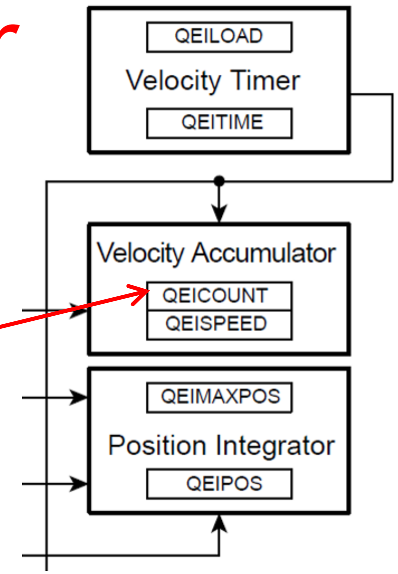
	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	TIME															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	TIME															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bit/Field	Name	Type	Reset	Description
31:0	TIME	RO	0x0000.0000	Velocity Timer Current Value The current value of the velocity timer.

Run QEI to Read Pulse Number

- Current Number of QEI's Velocity Counter



$$\overline{\omega} = \frac{C \times \theta_p}{\Delta t}$$

QEI Velocity Counter (QEICOUNT)

QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x018
Type RO, reset 0x0000.0000

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	COUNT															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	COUNT															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

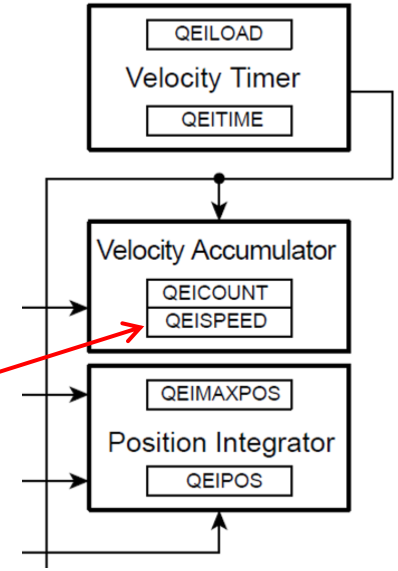
Bit/Field	Name	Type	Reset	Description
31:0	COUNT	RO	0x0000.0000	Velocity Pulse Count The running total of encoder pulses during this velocity timer period.

Run QEI to Read Velocity

- Current Value of QEI's Estimation of Speed (pulses/period)

$$\bar{\omega} = \frac{C}{\Delta t} \times \theta_p$$

$$\frac{C}{\Delta t} = \text{QEISPEED}$$



QEI Velocity (QEISPEED)

QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x01C
Type RO, reset 0x0000.0000

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	SPEED															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	SPEED															
Type	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO	RO
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

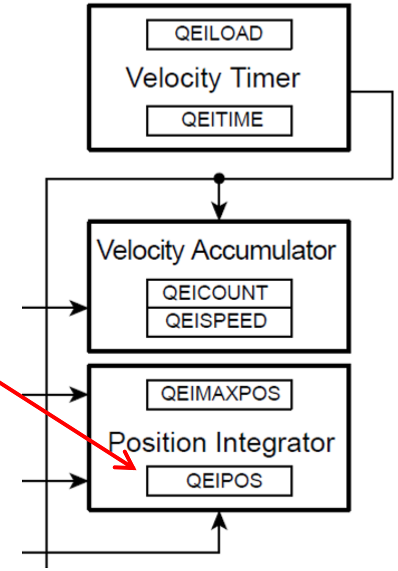
Bit/Field	Name	Type	Reset	Description
31:0	SPEED	RO	0x0000.0000	Velocity The measured speed of the quadrature encoder in pulses per period.

Run QEI to Read Position

- Current Value of Position Integrator (pulses)

$$\text{QEIPOS} = (C - C_r)$$

$$\theta - \theta_r = (C - C_r) \times \theta_p$$



QEI Position (QEIPOS)

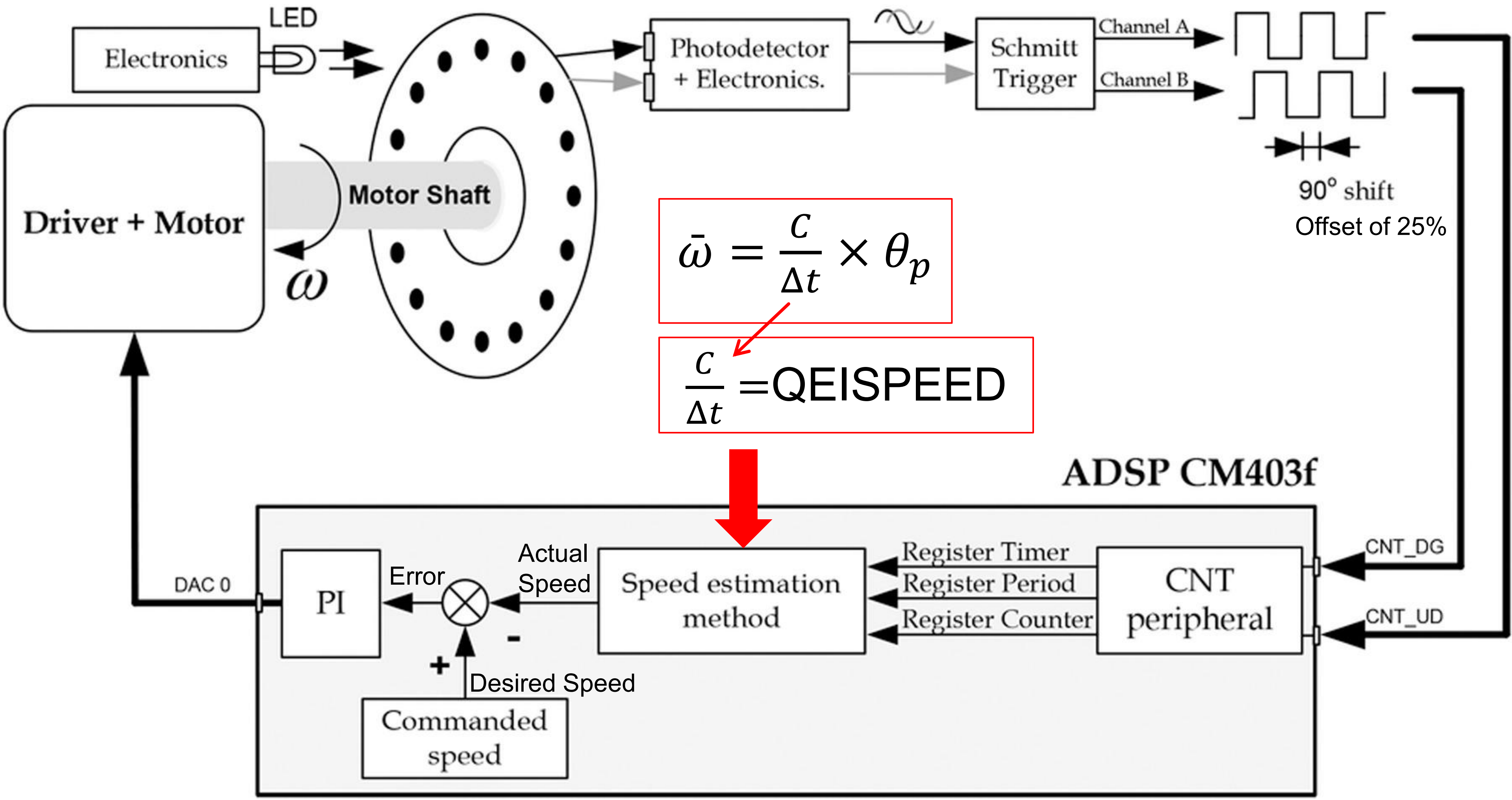
QEI0 base: 0x4002.C000
QEI1 base: 0x4002.D000
Offset 0x008
Type RW, reset 0x0000.0000

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
	POSITION															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	POSITION															
Type	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW	RW
Reset	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bit/Field	Name	Type	Reset	Description
31:0	POSITION	RW	0x0000.0000	Current Position Integrator Value The current value of the position integrator.

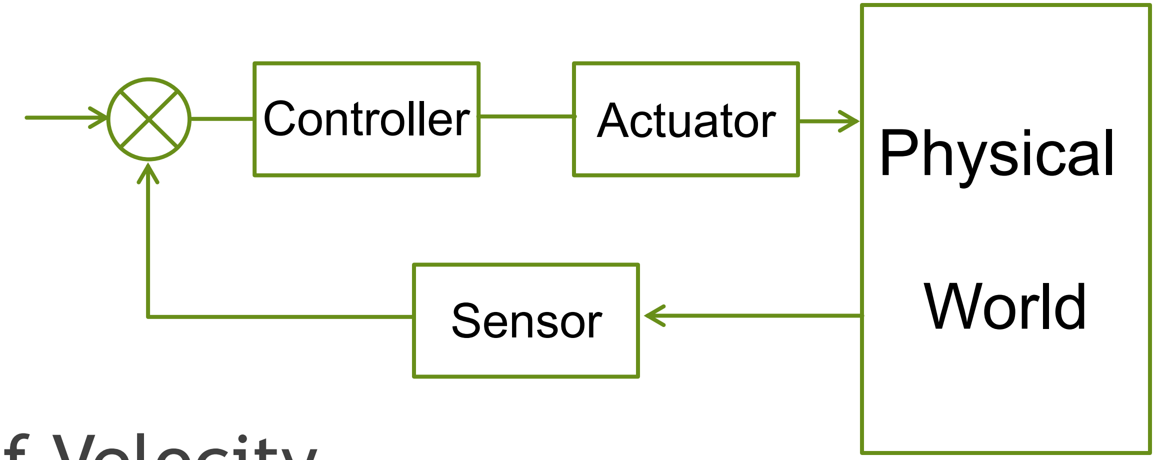
Example of Speed Control Loop …



$$\bar{\omega} = \frac{C}{\Delta t} \times \theta_p$$

$$\frac{C}{\Delta t} = \text{QEISPEED}$$

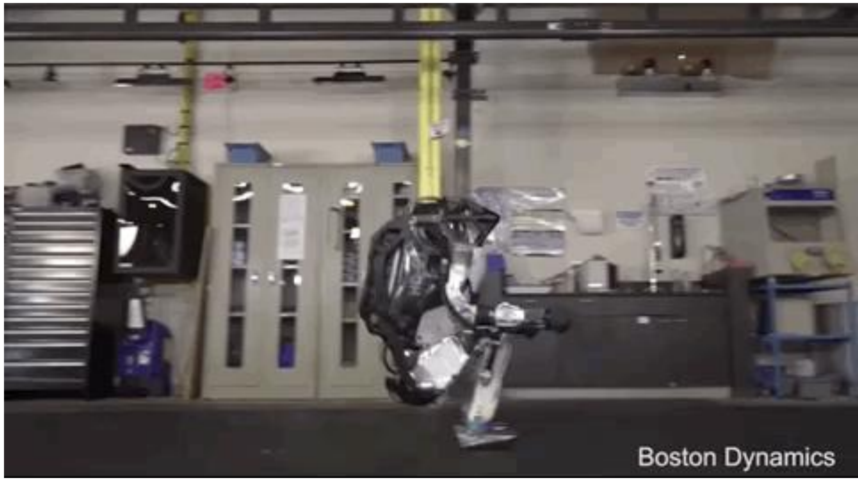
Summary



- Understanding of Velocity
- Computation of Velocity
- Measurement of Velocity

Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



Module 3 Lecture 3

MA4822

Measurement of Acceleration

Xie Ming, PhD (France)

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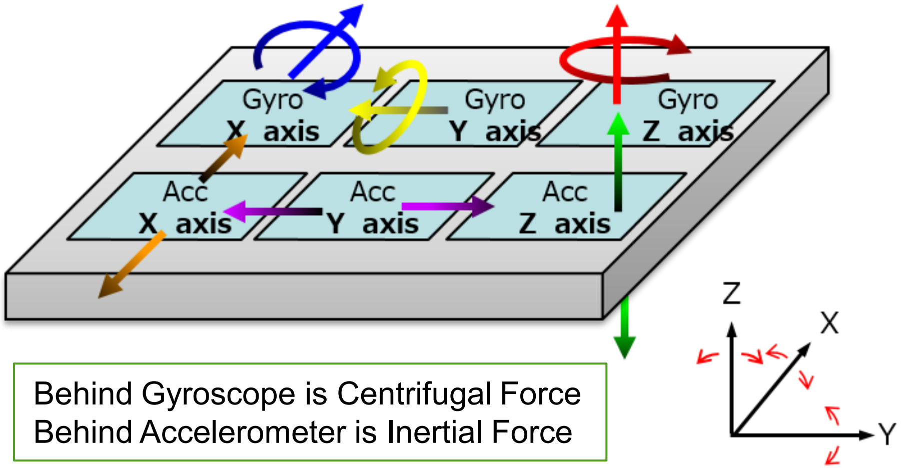
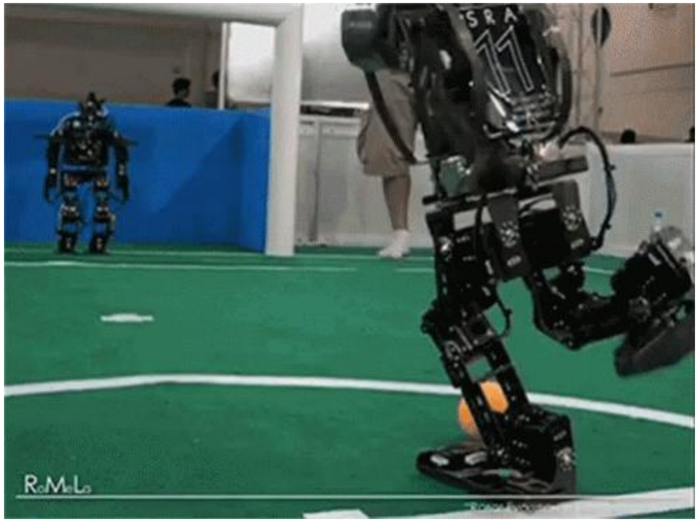
http://personal.ntu.edu.sg/mmxie

Outline

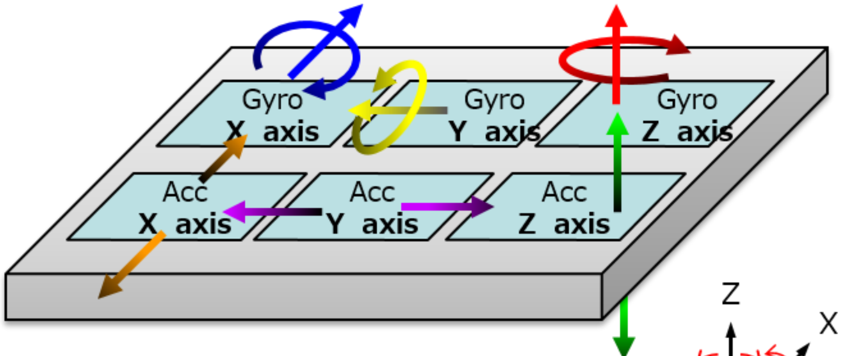
- Understanding of Acceleration
- Computation of Acceleration
- Measurement of Acceleration



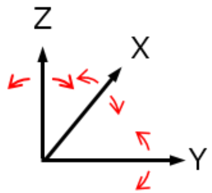


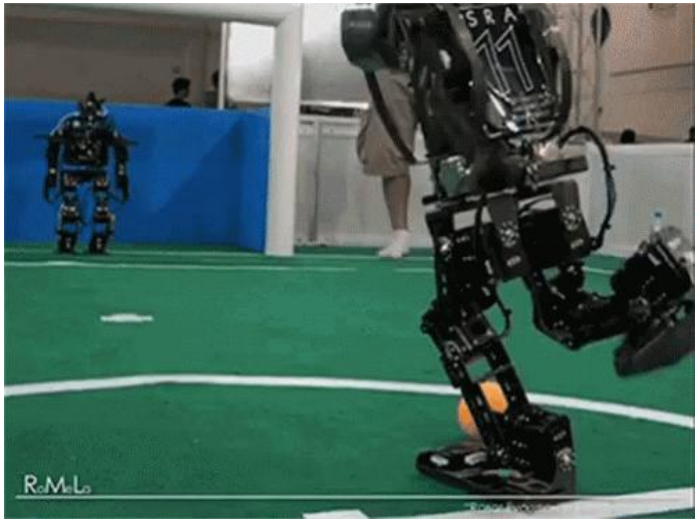
Outline

- Understanding of Acceleration
- Computation of Acceleration
- Measurement of Acceleration



Behind Gyroscope is Centrifugal Force
Behind Accelerometer is Inertial Force

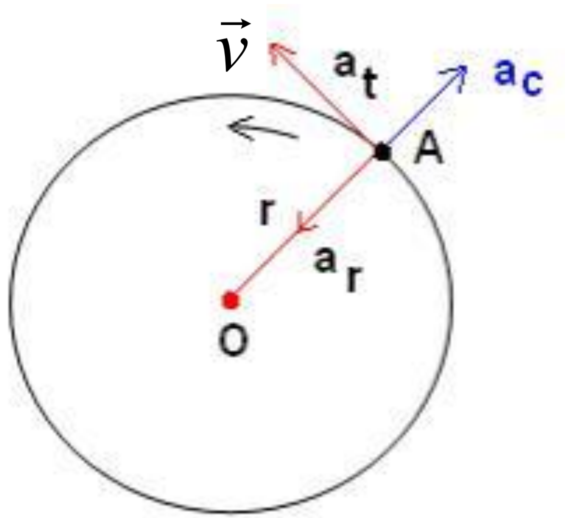




There Are Three Types of Basic Motions

- Linear Motion
- Angular Motion
- Circular Motion

Their Combinations Create Complex Motions



Definition of Acceleration

- The time change rate of velocity is called Acceleration.

$$\vec{a} = \frac{d}{dt}\vec{v}$$

Linear Motion

$$\alpha = \frac{d}{dt}\omega$$

Angular Motion

Cause behind Acceleration

- Acceleration is the change of motion state.
- The change of motion state is caused by net force acting on a moving target.

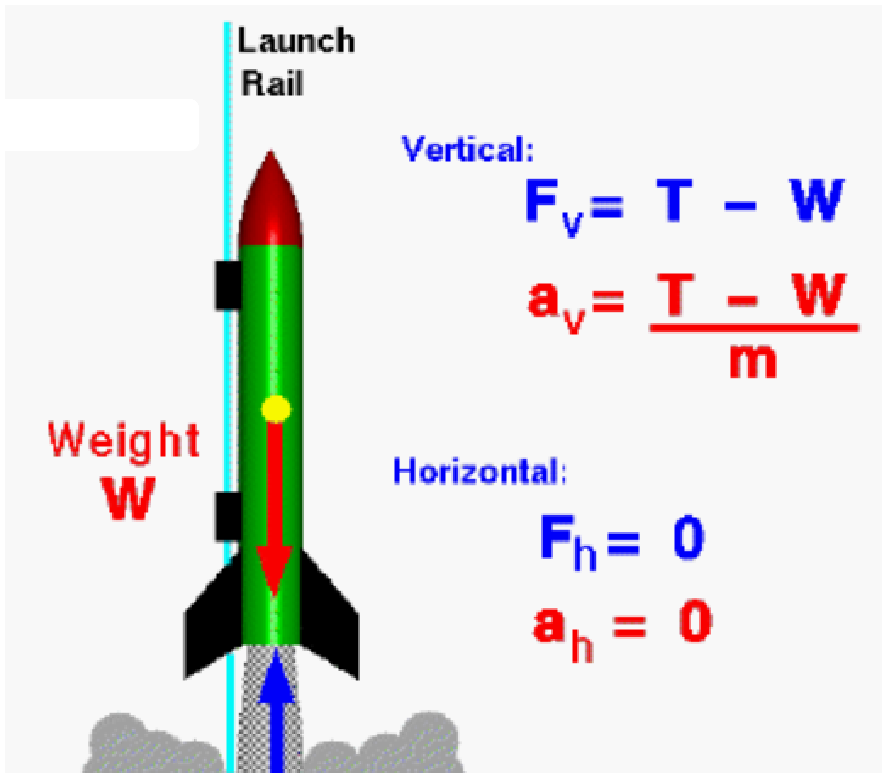
$$a = \frac{F}{m}$$

Linear Motion

$$\alpha = \frac{\tau}{I}$$

Angular Motion

Example of Linear Acceleration



T
Thrust

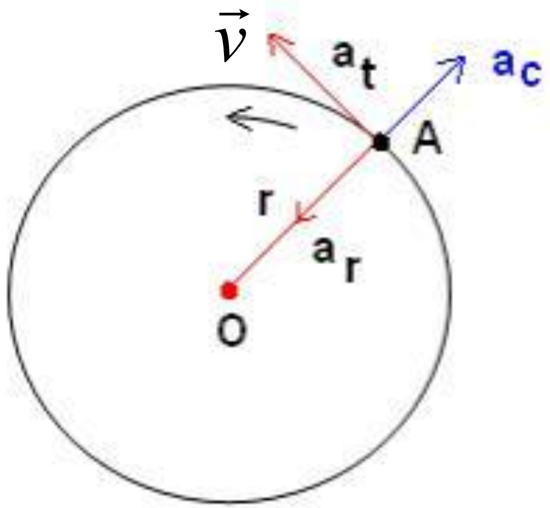
Example of Angular/Circular Acceleration

- Radial Acceleration and Centripetal Force:

$$a_r = \frac{v^2}{r} \qquad F_r = m\frac{v^2}{r}$$

- Centrifugal Acceleration and Centrifugal Force:

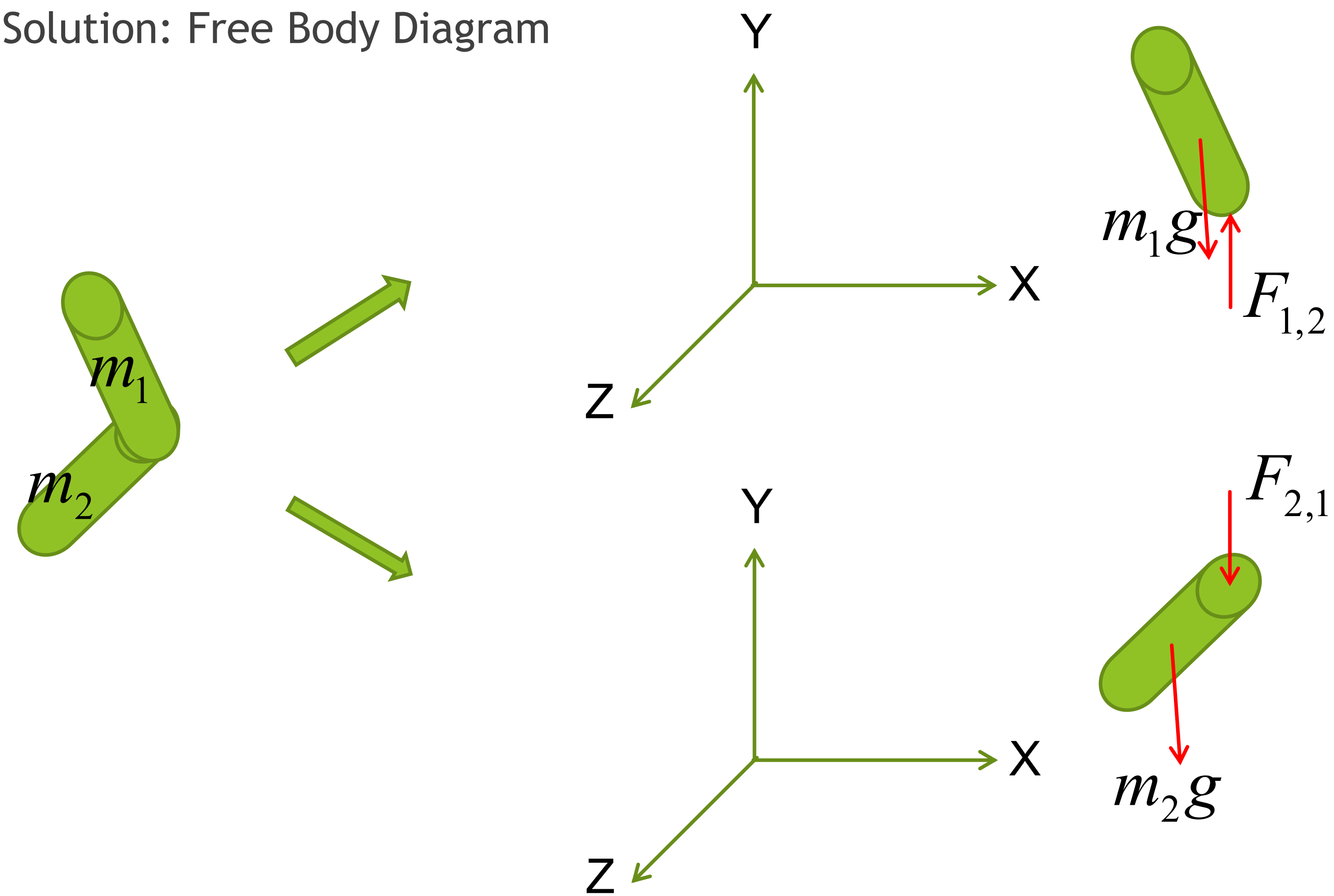
$$a_c = \frac{v^2}{r} \qquad F_c = m\frac{v^2}{r}$$



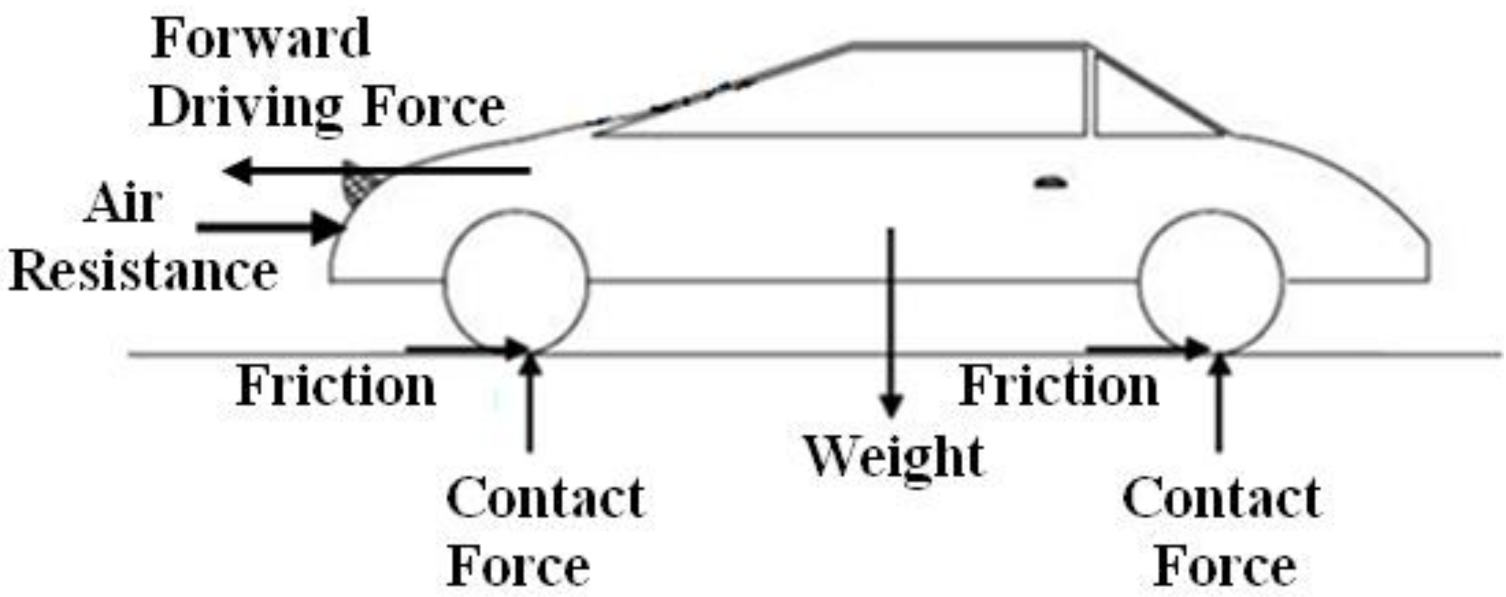
Angular acceleration systematically induces radial acceleration, which is equivalent to linear acceleration in radial direction

Determination of Net Force

- Solution: Free Body Diagram



Example of Determining Net Force



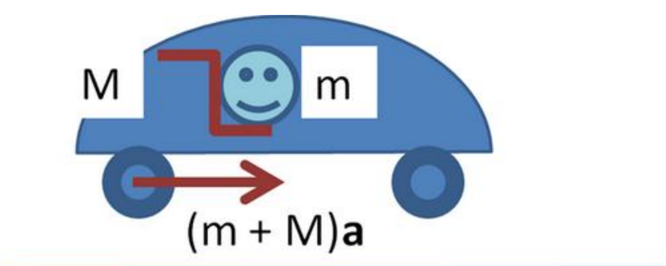
$$F_{y,net} = F_{weight} - F_{contact,front} - F_{contact,back}$$

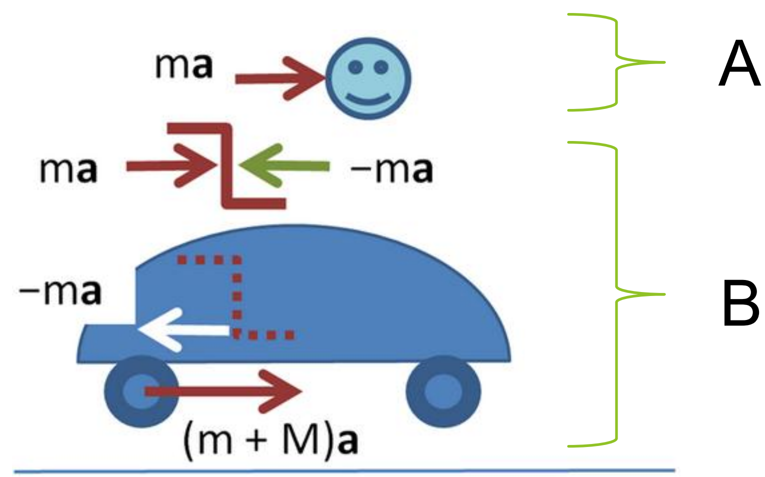
$$F_{x,net} = F_{driving} - F_{resistance} - F_{friction}$$

Inertial Force due to Acceleration

- Object A is placed on object B. When object B accelerates, object A will exert a so-called inertial force on object B if both objects stay together.
- Reacting force from A is equal to inertial force from B.

$$F - m \bullet a = 0$$

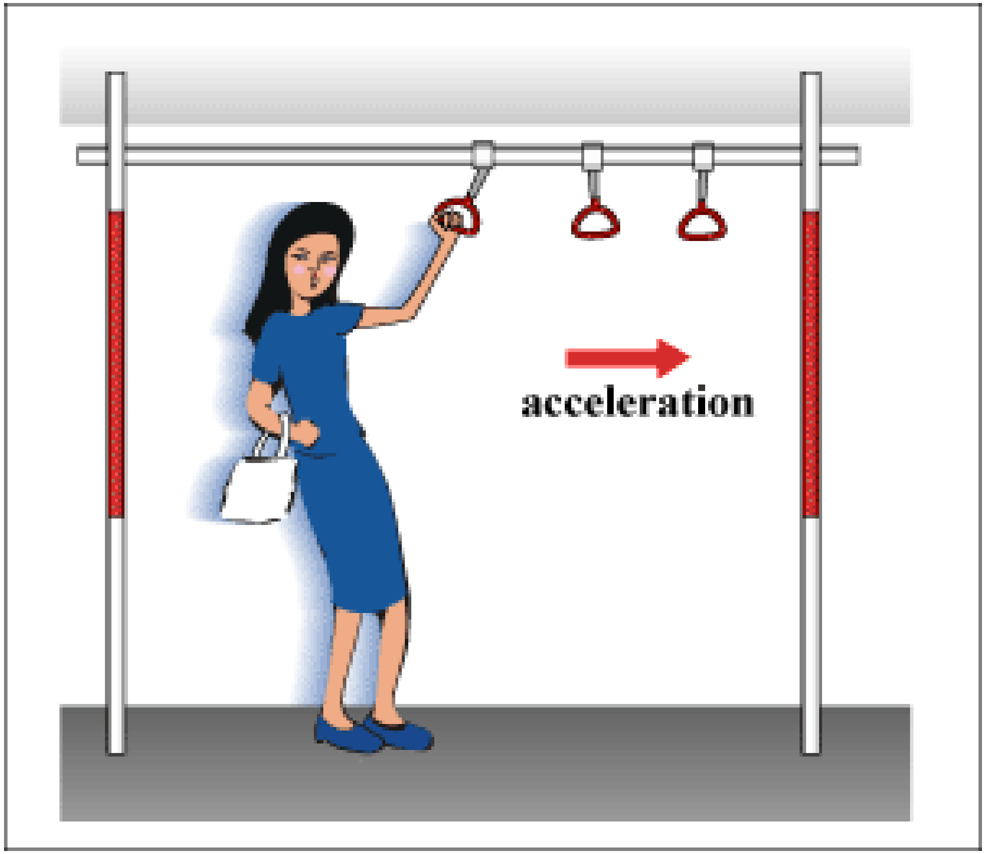




Example with Reacting Force

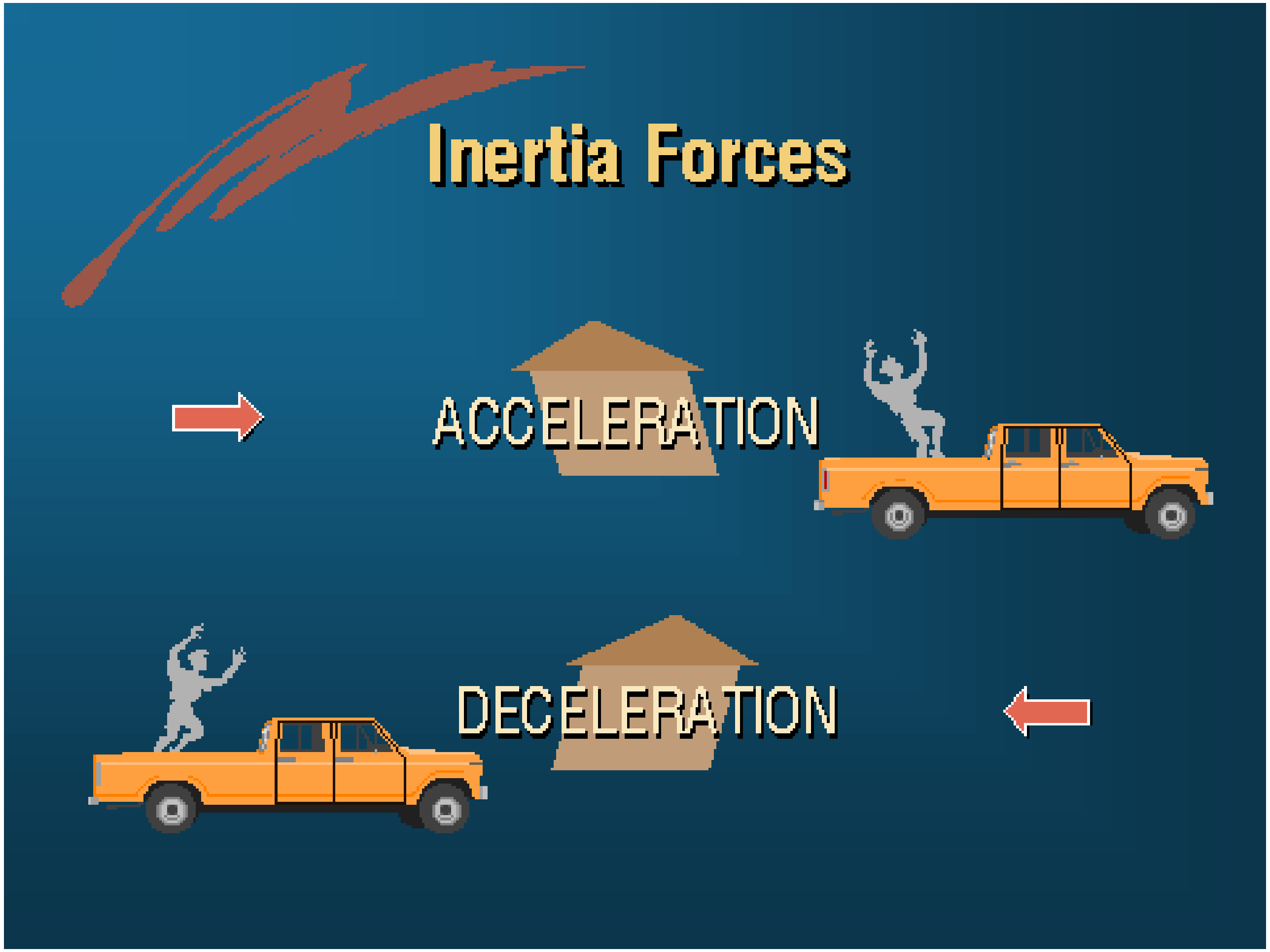
Reacting Force – Inertial Force = 0

$$F - m \bullet a = 0$$



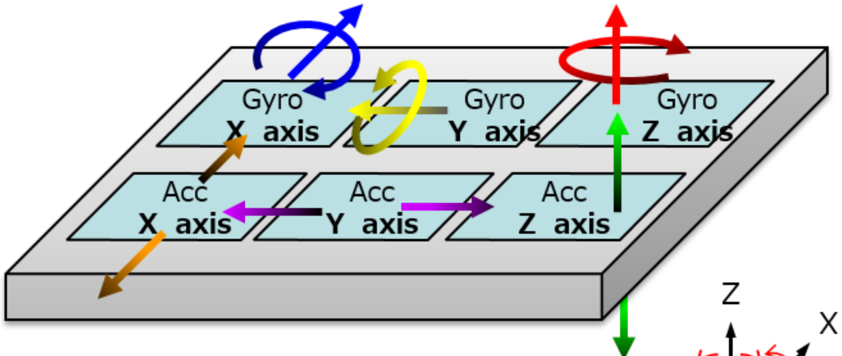
Example without Reacting Force

$$0 - m \bullet a \neq 0$$

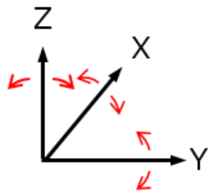


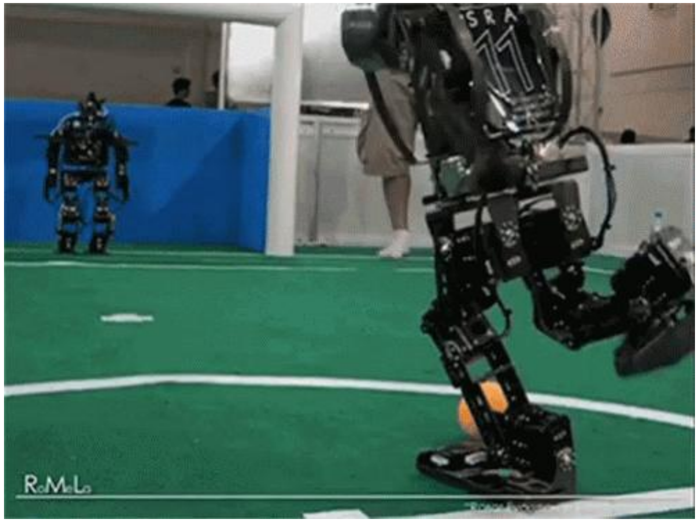
Outline

- Understanding of Acceleration
- Computation of Acceleration
- Measurement of Acceleration



Behind Gyroscope is Centrifugal Force
Behind Accelerometer is Inertial Force





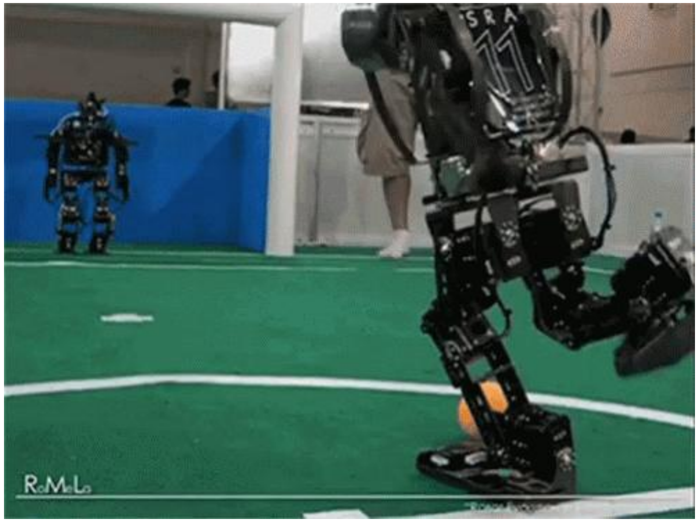
Linear Acceleration Could Be Computing from Linear Position and Velocity …

- If we know the time functions of linear positions, we have:

$$\vec{a}(t) = \frac{d^2}{dt^2}\vec{r}(t)$$

- If we know the time functions of linear velocities, we have:

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$



Example

Starting from rest at position zero

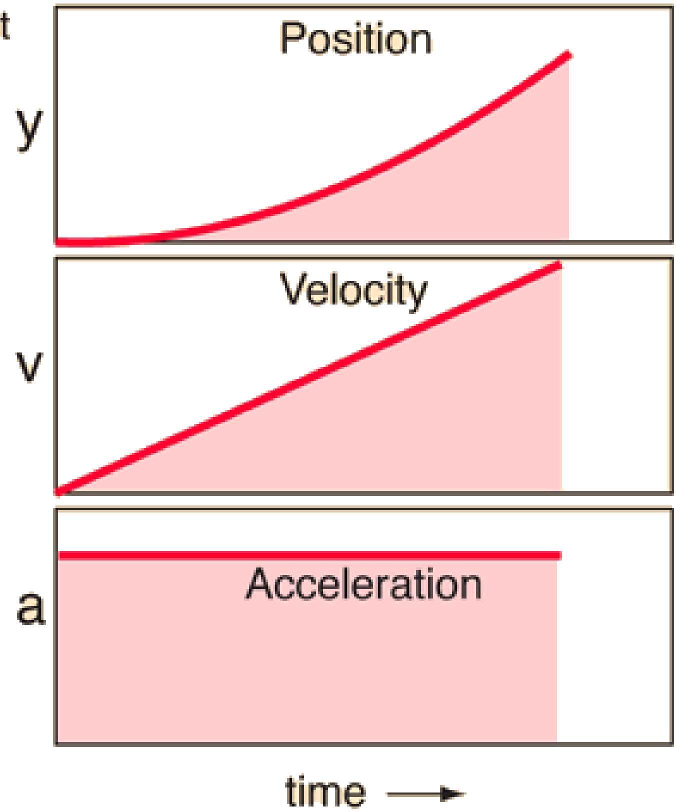
$$y = \frac{1}{2}at^2$$

$$v = at$$

$$a = \text{constant}$$

accelerating at

$$9.8\ \text{m/s}^2$$



More generally

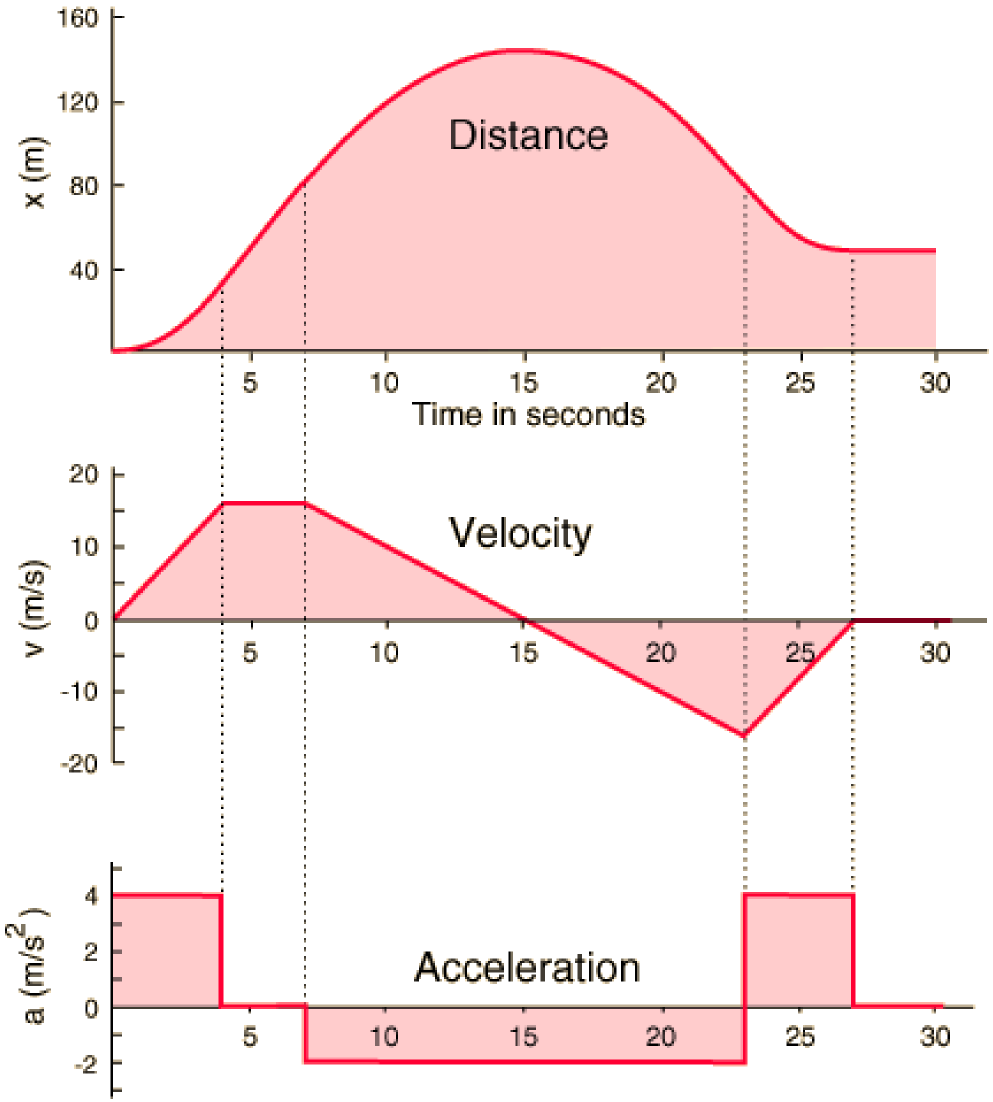
$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

Velocity is equal to the slope of the position curve.

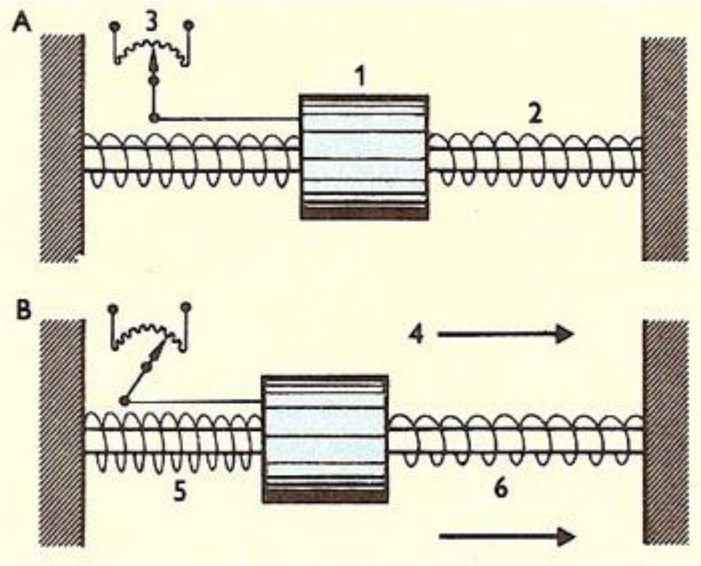
Acceleration is equal to the slope of the velocity curve.

Example


160
120
80
40
x (m)
Distance
5
10
15
20
25
30
Time in seconds
20
10
0
-10
-20
v (m/s)
Velocity
4
2
0
-2
a (m/s^2)
Acceleration

Linear Acceleration Could Be Computed from Linear Displacement of Spring …

- In a mass-spring system, the mass will undergo a linear displacement when an acceleration occurs in the direction of the spring's motion axis.



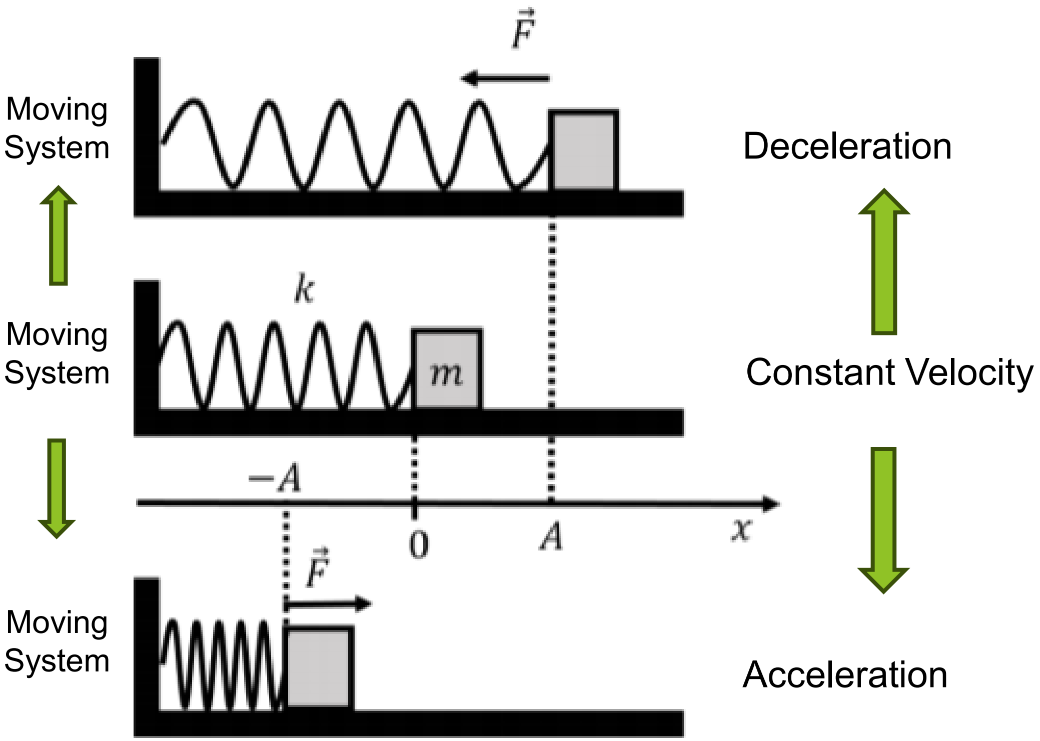
Without acceleration

With acceleration

Acceleration

Linear Acceleration Could Be Computed from Linear Displacement of Spring …

- Equations:



Inertial Force:

$$F_{\text{inertial}} = -m \times a$$

Displacement of Spring:

$$\left|F_{\text{inertial}}\right| = k\Delta d$$

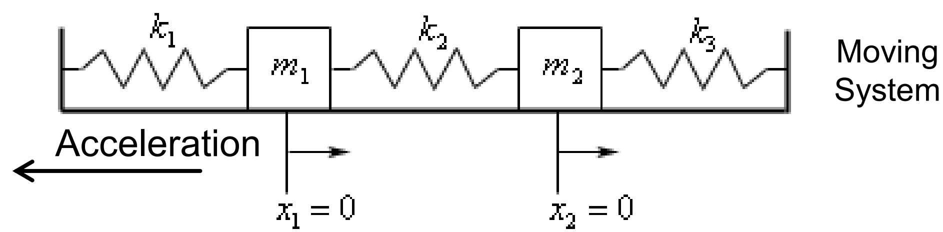
$$\Delta d = \frac{m \bullet a}{k}$$

Equation for Calculating a:

$$a = \frac{k\Delta d}{m}$$

Example of Computing Linear Acceleration from Spring's Linear Displacement …

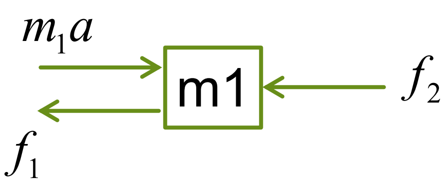
- We have a friction-free mass-spring system as shown in the figure below. What is the equation for computing acceleration from the displacement of mass 1?



- Answer:

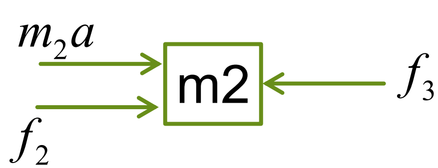
$$f_1 + f_2 = m_1 a$$

$$f_3 - f_2 = m_2 a$$



$$k_1 x_1 + k_2 (x_1 - x_2) = m_1 a$$

$$k_3 x_2 - k_2 (x_1 - x_2) = m_2 a$$



$$k_1 x_1 + k_2(x_1 - x_2) = m_1 a$$

$$k_3 x_2 - k_2(x_1 - x_2) = m_2 a$$

$$(k_1 + k_2)x_1 - k_2 x_2 = m_1 a \qquad (1)$$

$$-k_2 x_1 + (k_3 + k_2)x_2 = m_2 a \qquad (2)$$

$$(1) \bullet (k_2 + k_3) + (2) \bullet k_2$$

$$a = \frac{k_1 k_2 + k_1 k_3 + k_2 k_3}{m_1(k_2 + k_3) + m_2 k_2} x_1$$

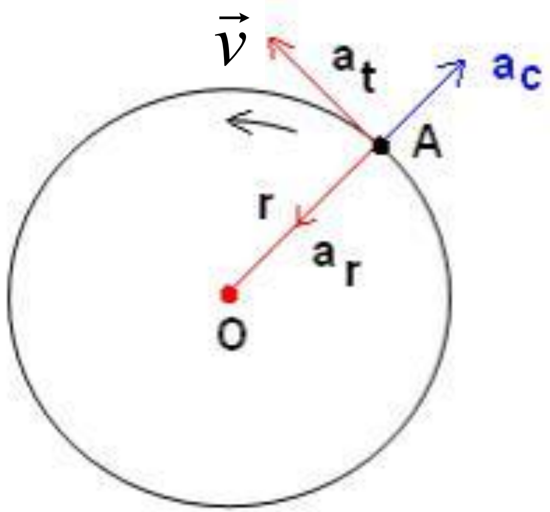
Angular Acceleration Could Be Computing from Angular Position and Velocity …

- If we know the time function of angular positions, then:

$$\alpha(t) = \frac{d^2}{dt^2}\theta(t)$$

- If we know the time function of angular velocities, then:

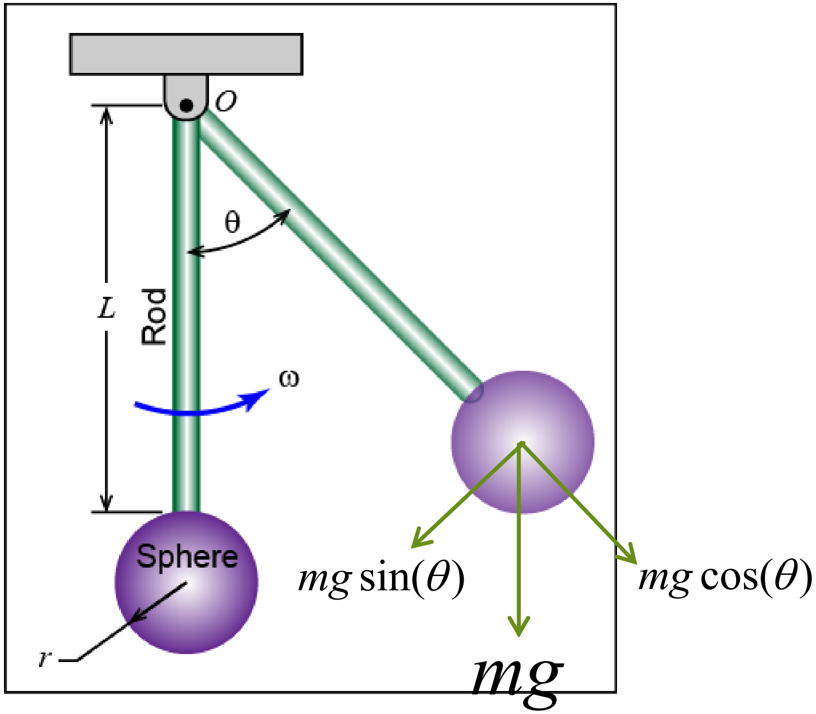
$$\alpha(t) = \frac{d}{dt}\omega(t)$$



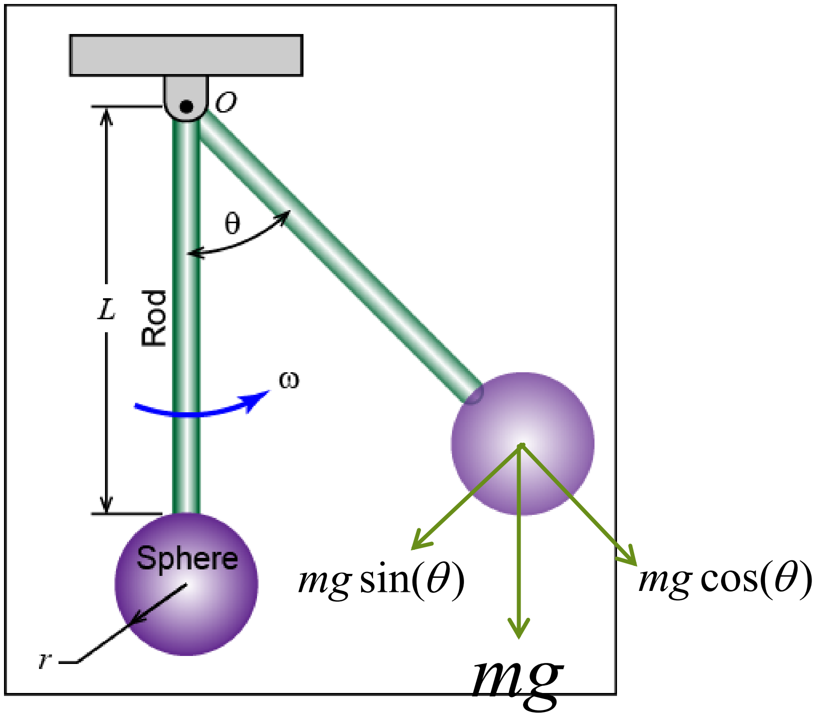
Angular acceleration systematically induces radial acceleration, which is equivalent to linear acceleration in radial direction.

Example of Computing Angular Acceleration

- What is the radial acceleration of the ball?
- What is the tangential acceleration of the ball?



Solution:



- What is the radial acceleration?

Answer:

$$a_r = \frac{F_r}{m}$$

$$= \frac{mg\cos(\theta)}{m} = g\cos(\theta)$$

- What is the tangential acceleration?

Answer:

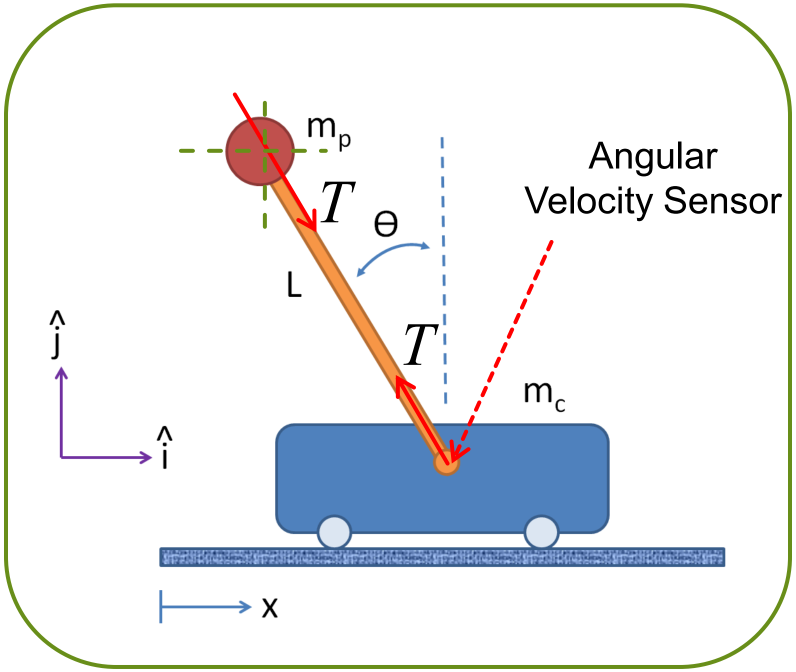
$$a_t = \frac{F_t}{m}$$

$$= \frac{mg\sin(\theta)}{m} = g\sin(\theta)$$

Example of Relating Linear Acceleration to Angular Acceleration

- Assume that there is a velocity sensor at the joint of the rotating arm.

- What is the relationship between the angular acceleration of the ball and the linear acceleration of the cart?



$$f(\ddot{\theta}, \ddot{x}) = 0$$

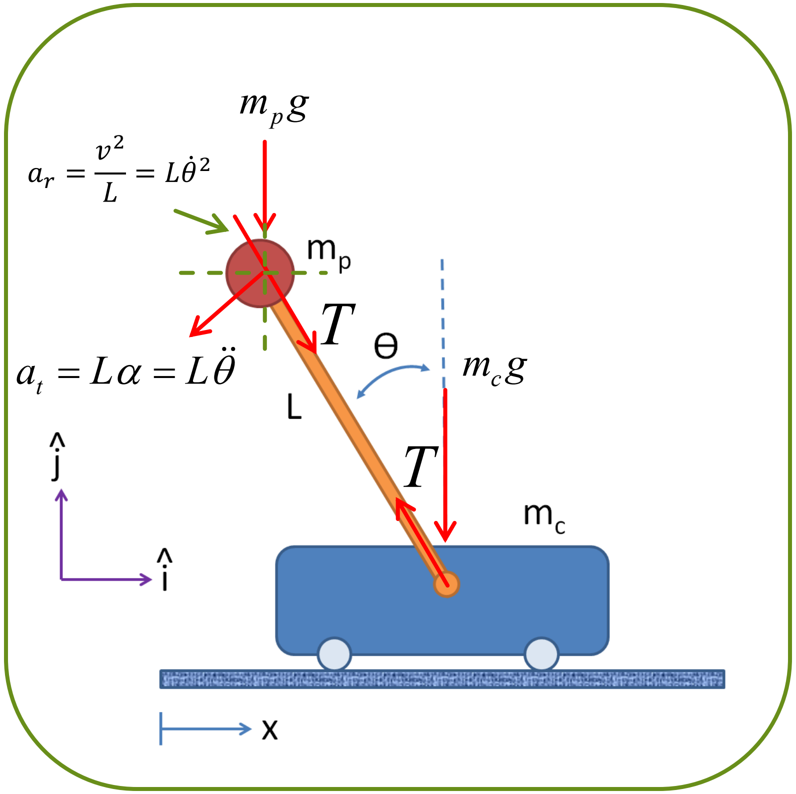
Solution

Angular velocity measured by velocity sensor

$$\omega = \dot{\theta} = \frac{v}{L}$$

$$a_r = \frac{v^2}{L} = L\dot{\theta}^2$$

$$a_t = L\alpha = L\ddot{\theta}$$



Solution (continued)

$$a_r = \frac{v^2}{L} = L\dot{\theta}^2$$

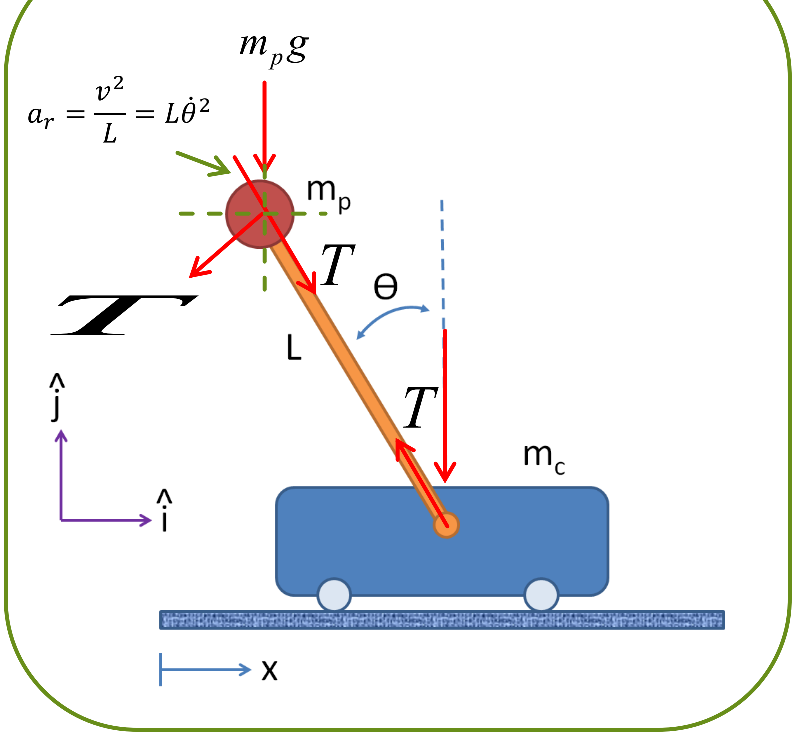
$$a_t = L\alpha = L\ddot{\theta}$$

- If mass c is at rest, the ball's acceleration in X direction is:

$$\ddot{x}_{p/c} = -a_t\cos(\theta) + a_r\sin(\theta)$$

⇩

$$\ddot{x}_{p/c} = -L\cos(\theta)\ddot{\theta} + L\sin(\theta)\dot{\theta}^2$$



Solution (continued)

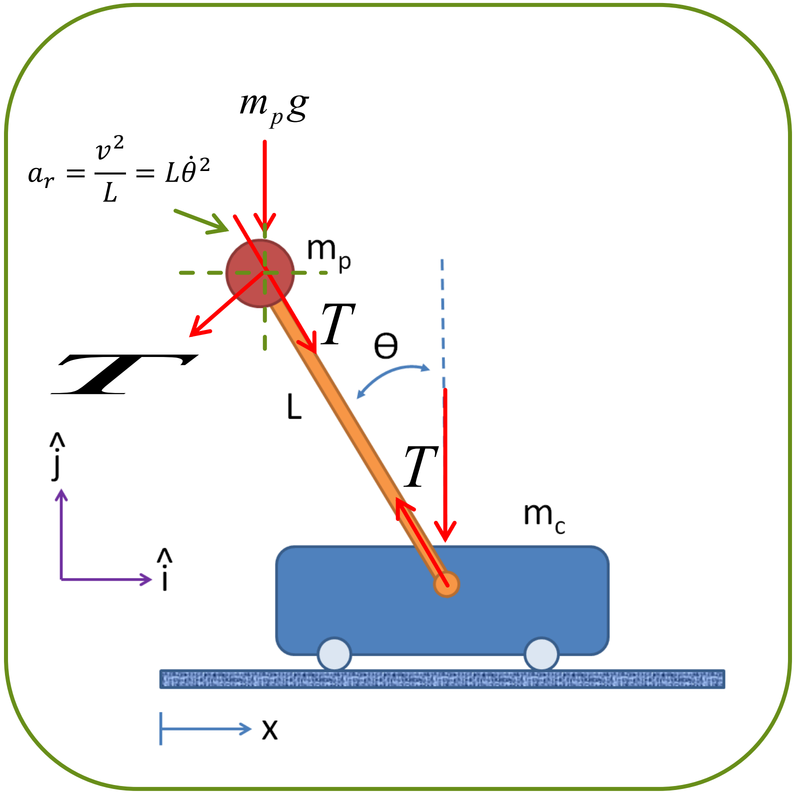
$$a_r = \frac{v^2}{L} = L\dot{\theta}^2$$

$$a_t = L\alpha = L\ddot{\theta}$$

- If mass c is at rest, the ball's acceleration in Y direction is:

$$\ddot{y}_{p/c} = -a_t \sin(\theta) - a_r \cos(\theta)$$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$



Solution (continued)

Now, mass c has acceleration …

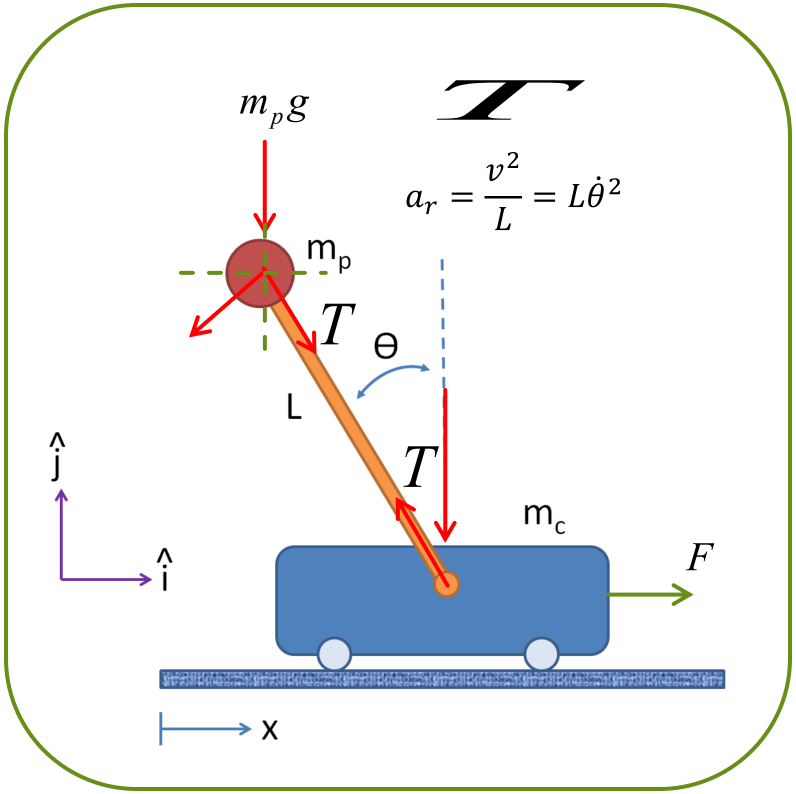
- From the free-body diagram of cart's mass in X direction:

$$F - T\sin(\theta) = m_c \ddot{x}_c$$

- From the free-body diagram of ball's mass in X direction:

$$T\sin(\theta) = m_p \ddot{x}_p$$

$$\ddot{x}_p = \ddot{x}_c + \ddot{x}_{p/c}$$



$$\ddot{x}_{p/c} = -L\cos(\theta)\ddot{\theta} + L\sin(\theta)\dot{\theta}^2$$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

Solution (continued)

Now, mass c has acceleration …

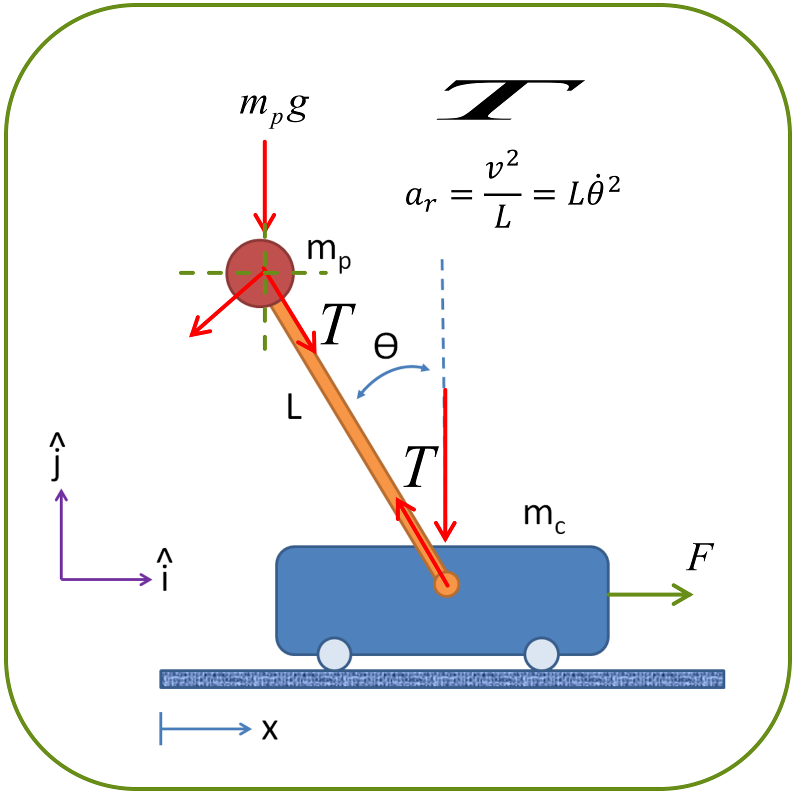
- From the free-body diagram of cart's mass in X direction:

$$F - T\sin(\theta) = m_c \ddot{x}_c$$

- From the free-body diagram of ball's mass in Y direction:

$$-T\cos(\theta) - m_p g = m_p \ddot{y}_p$$

$$\ddot{y}_p = \ddot{y}_{p/c}$$



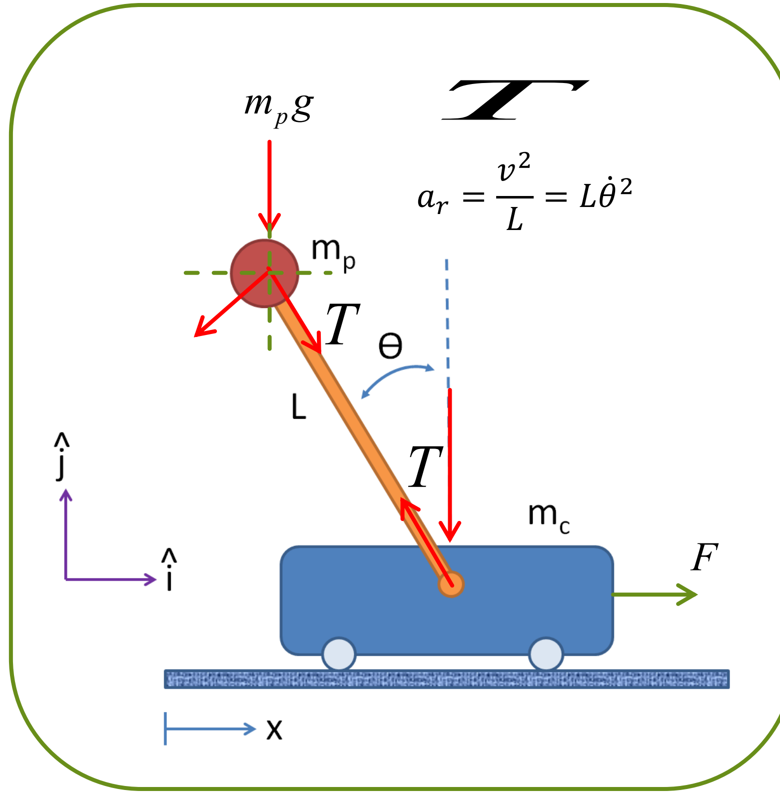
$$\ddot{x}_{p/c} = -L\cos(\theta)\ddot{\theta} + L\sin(\theta)\dot{\theta}^2$$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

Solution (continued)

$$T\sin(\theta) = m_p \ddot{x}_p = m_p \ddot{x}_c + m_p \ddot{x}_{p/c} \qquad (1)$$

$$-T\cos(\theta) - m_p g = m_p \ddot{y}_p = m_p \ddot{y}_{p/c} \qquad (2)$$



$$(1)\bullet\cos(\theta) + (2)\bullet\sin(\theta)$$

$$\ddot{x}_{p/c} = -L\cos(\theta)\ddot{\theta} + L\sin(\theta)\dot{\theta}^2$$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

$$L\ddot{\theta} = \cos(\theta)\ddot{x}_c + g\sin(\theta)$$

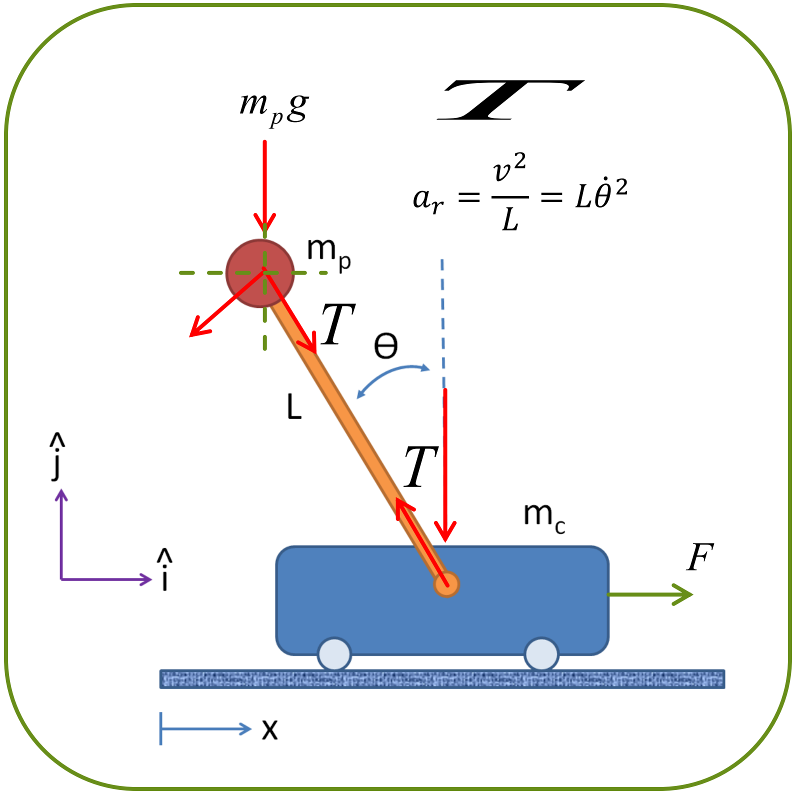
Solution (continued)

- Angular acceleration could be computed from linear acceleration.

$$L\ddot{\theta} = \cos(\theta)\ddot{x}_c + g\sin(\theta)$$

- Linear acceleration could be computed from angular acceleration.

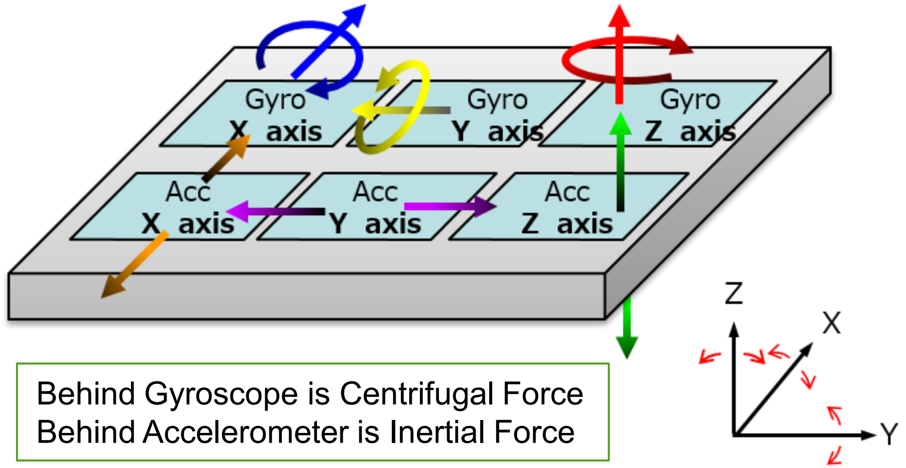
$$L\ddot{\theta} = \cos(\theta)\ddot{x}_c + g\sin(\theta)$$

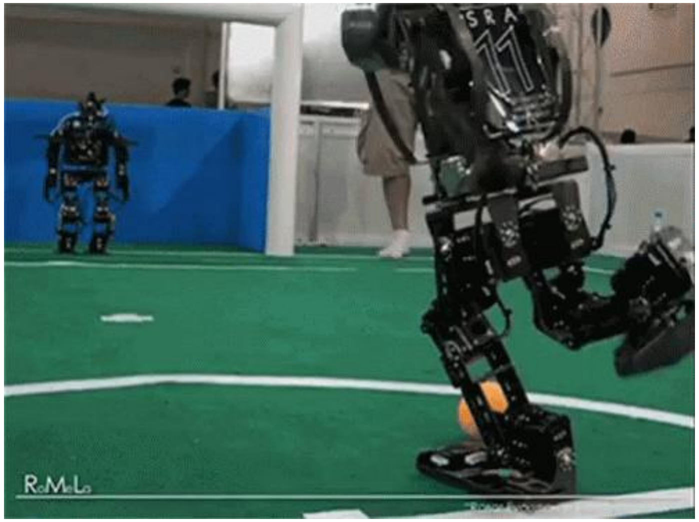


How to measure acceleration?

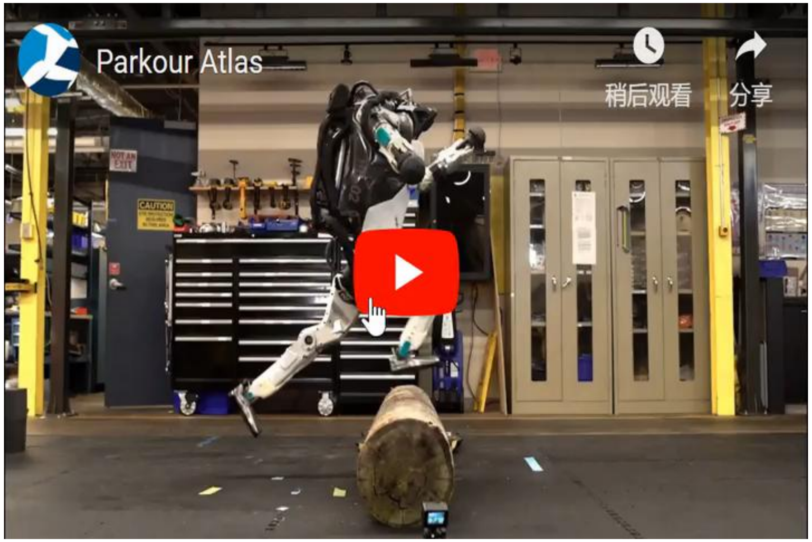
Outline

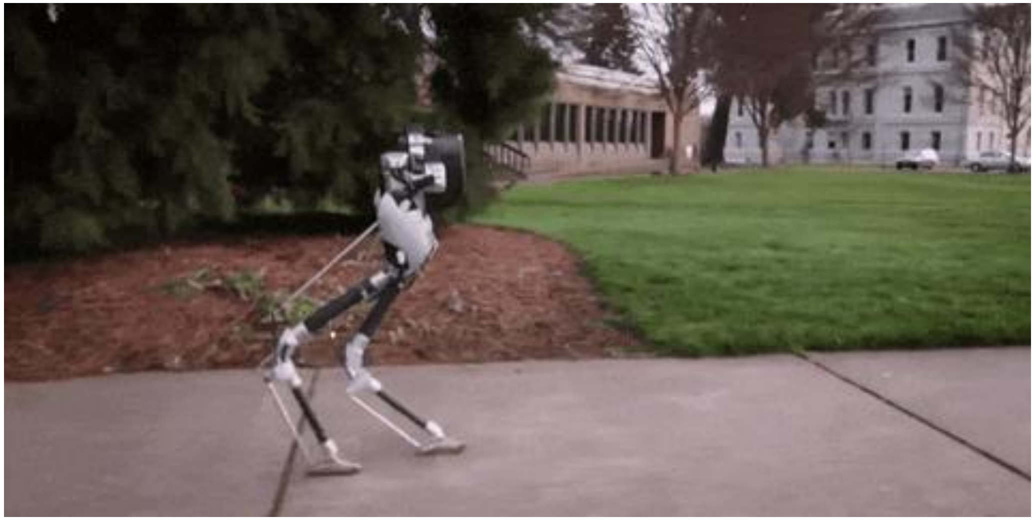
- Understanding of Acceleration
- Computation of Acceleration
- Measurement of Acceleration





Applications in Robotics



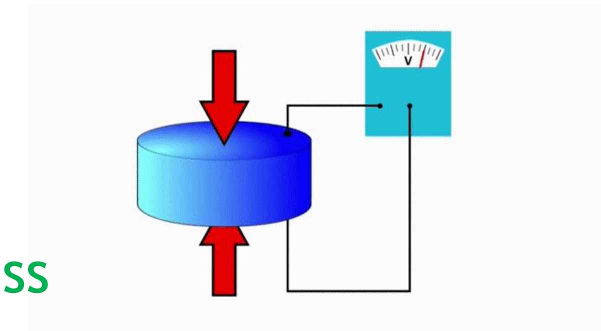


Applications in Automobile

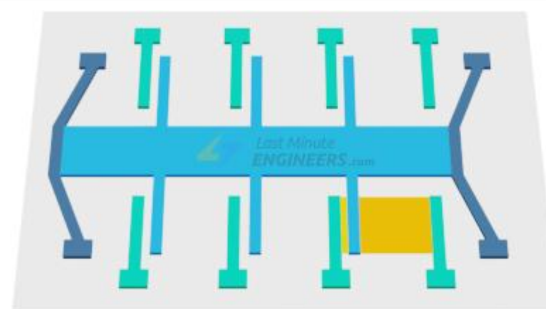
Principles of Measurement

$$a = \frac{1}{m} F$$

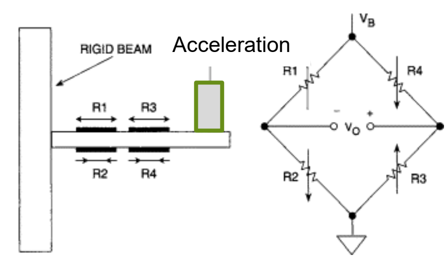
- Principle 1: Acceleration can induce inertial force or centrifugal force on a mass, which could output voltage.



- Principle 2: Acceleration can cause a mass to undergo displacement, which can be sensed and measured by doing displacement to capacitance conversion.

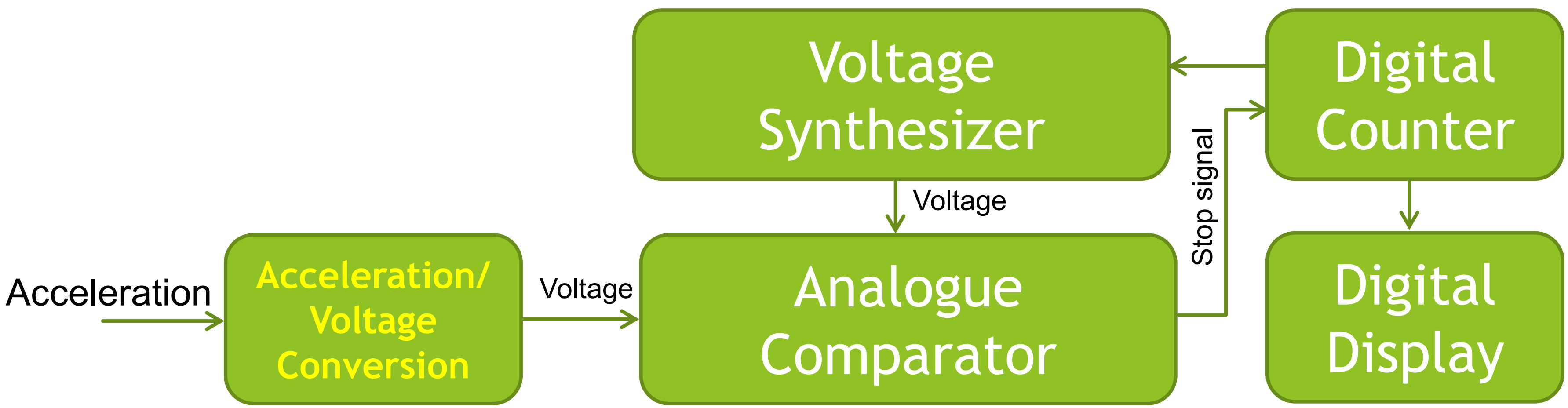


- Principle 3: Acceleration can cause a mass to undergo deformation, which can be sensed and measured by doing deformation to resistance conversion.



How to apply principle 1 to design digital measurement and sensing systems for acceleration?

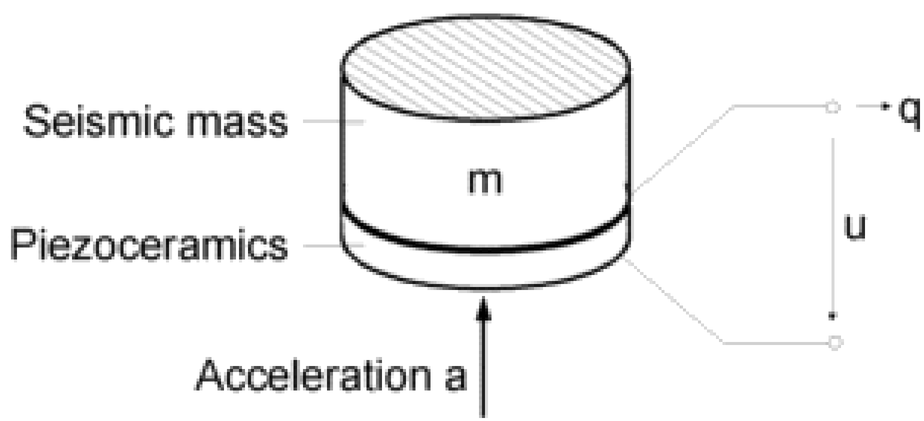
- Acceleration is converted to voltage which is measured by digital voltmeter (e.g. microcontrollers).



All microcontrollers are programmable digital sensors of voltage!

How to convert acceleration to voltage?

- A piezo-electric material is placed underneath a mass. When there is an acceleration, the induced inertial force (or centrifugal force) compresses the piezo-electric material. The piezo-electric material in turn outputs electrical charges or potentials.



$F = m \cdot a$

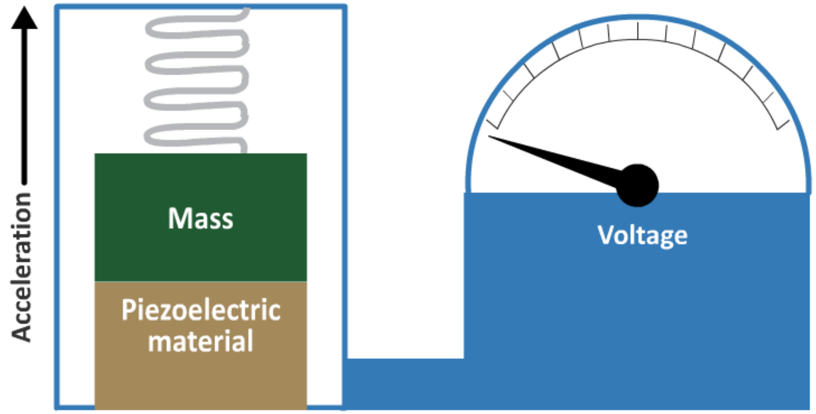
charge sensitivity:

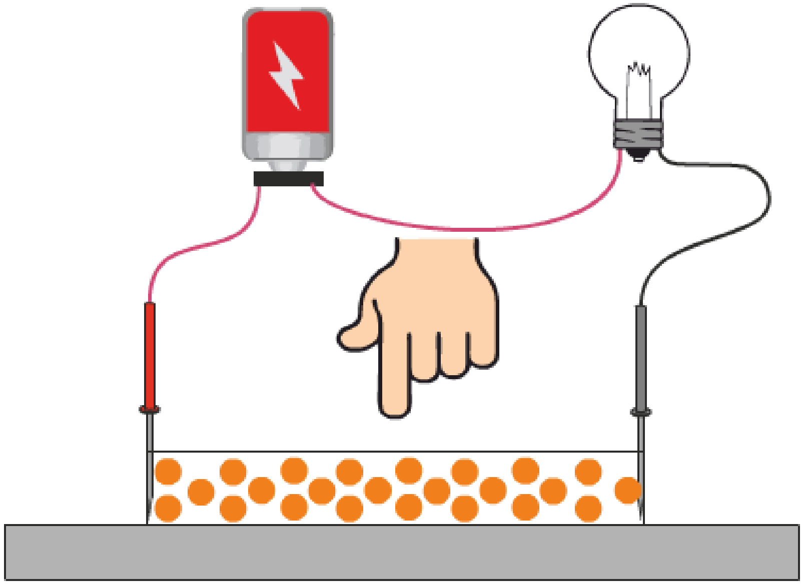
$$Bqa = \frac{q}{a}$$ Charge/Acceleration

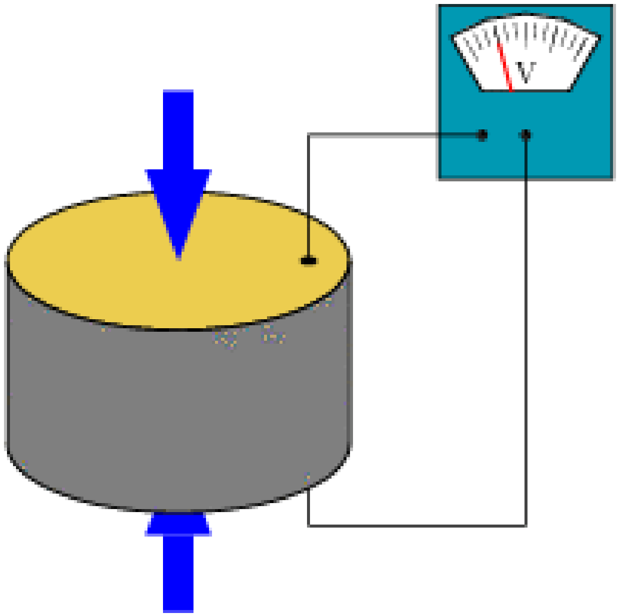
voltage sensitivity:

$$Bua = \frac{u}{a}$$ Voltage/Acceleration

Examples of Illustration

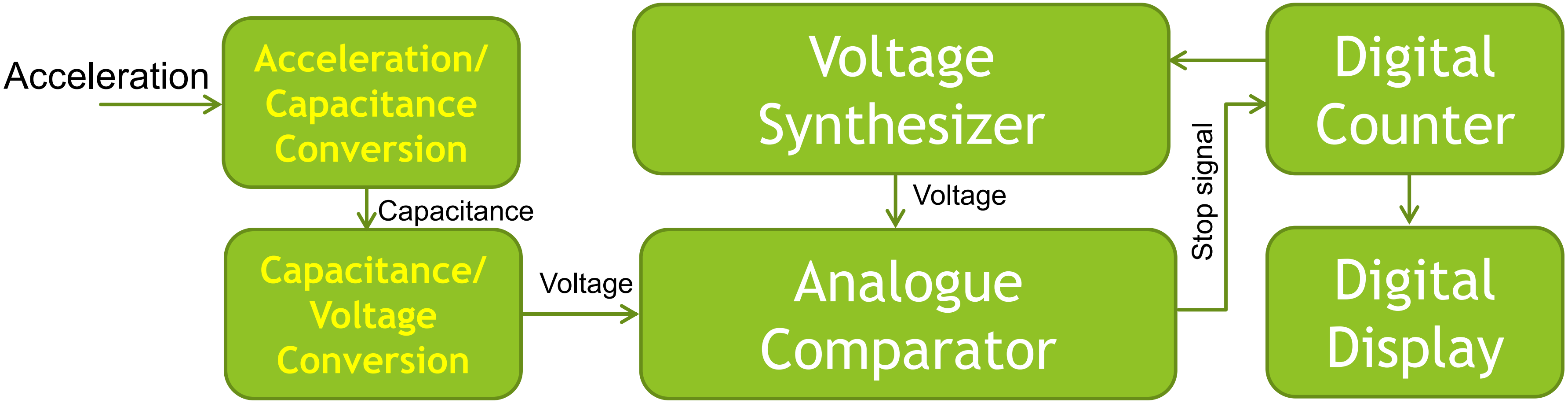






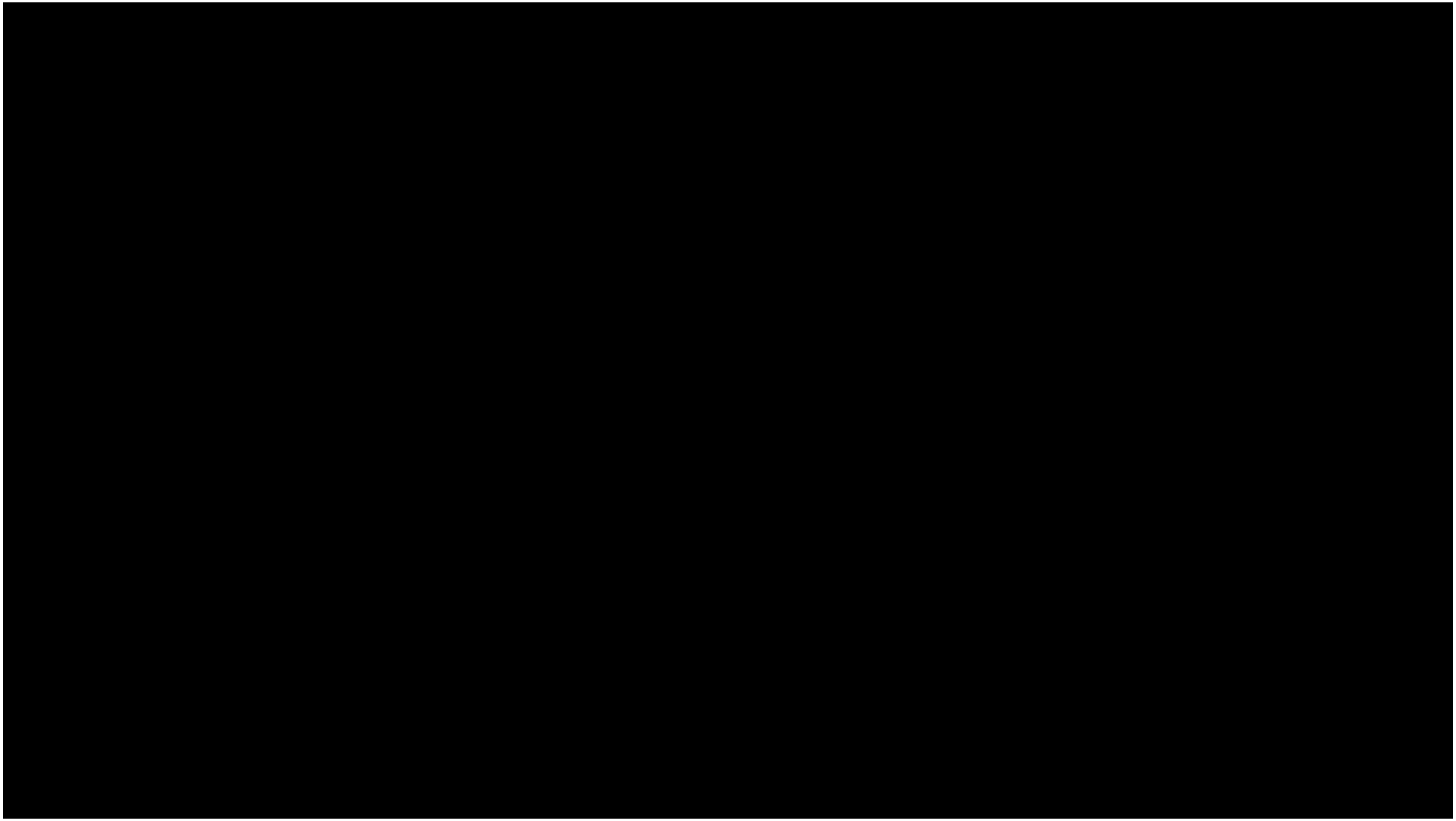
How to apply principle 2 to design digital measurement and sensing systems for acceleration?

- Acceleration is converted to capacitance which is then converted to voltage. Finally, the voltage is measured by digital voltmeter (e.g. microcontrollers).



All microcontrollers are programmable digital sensors of voltage!

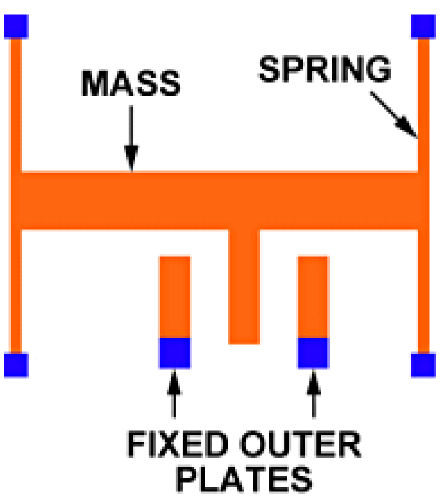
How to convert acceleration to capacitance?

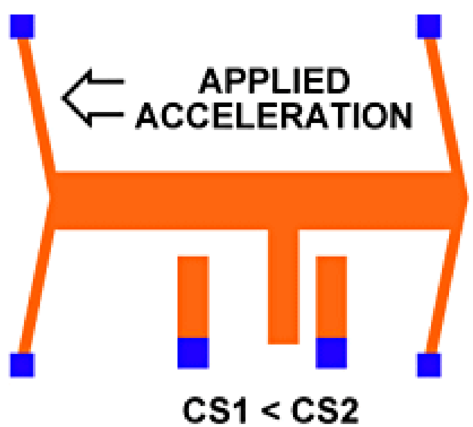


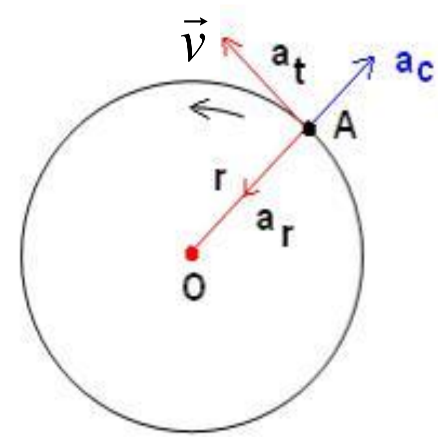
How to convert acceleration to capacitance?

- Hardware Design:
  - When acceleration occurs, the inertial force (or centrifugal force) causes a moving mass to change its position, which in turn causes the changes of capacitances of capacitors.

$$F - m \bullet a = 0$$







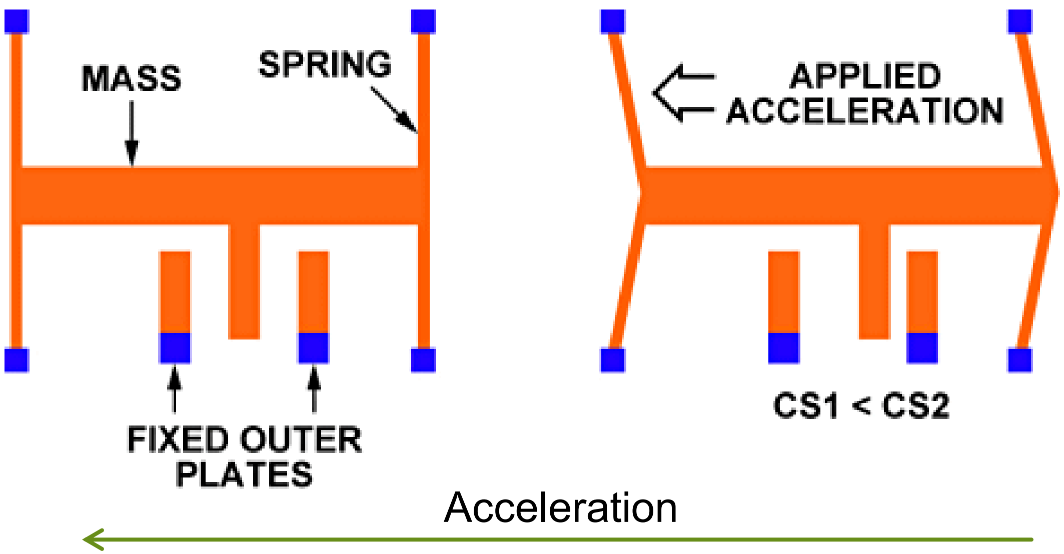
Acceleration

How to convert acceleration to capacitance?

- Equations of Measurement:

$$C = \varepsilon_0 \frac{A}{d} \Rightarrow \Delta C = -\varepsilon_0 \frac{A}{d^2} \Delta d$$

$$C = \varepsilon_0 \frac{A}{d \pm \Delta d}$$



Inertial Force:

$$F_{\text{inertial}} = -m \times a$$

Displacement of Plate:

$$F_{\text{inertial}} = k\Delta d$$

$$\Delta d = \frac{-m \times a}{k}$$

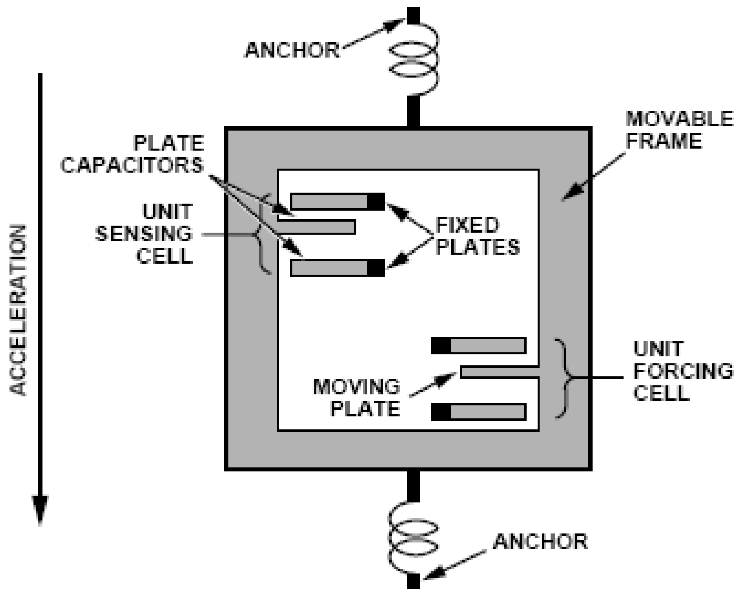
Equation of Acceleration:

$$a = \frac{k\Delta d}{m} = \frac{k \times d^2}{m \times A \times \varepsilon_0} \Delta C$$

Equation of Capacitance:

$$C = \varepsilon_0 \frac{A}{d \pm m \times a/k}$$

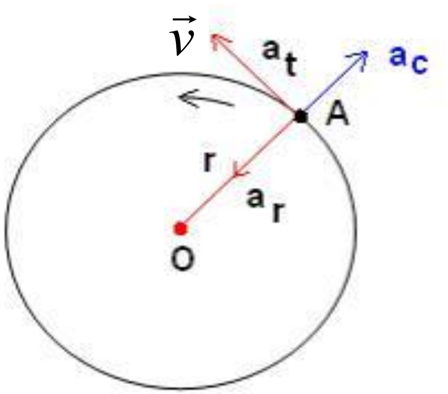
Example of 1-Axis Accelerometer

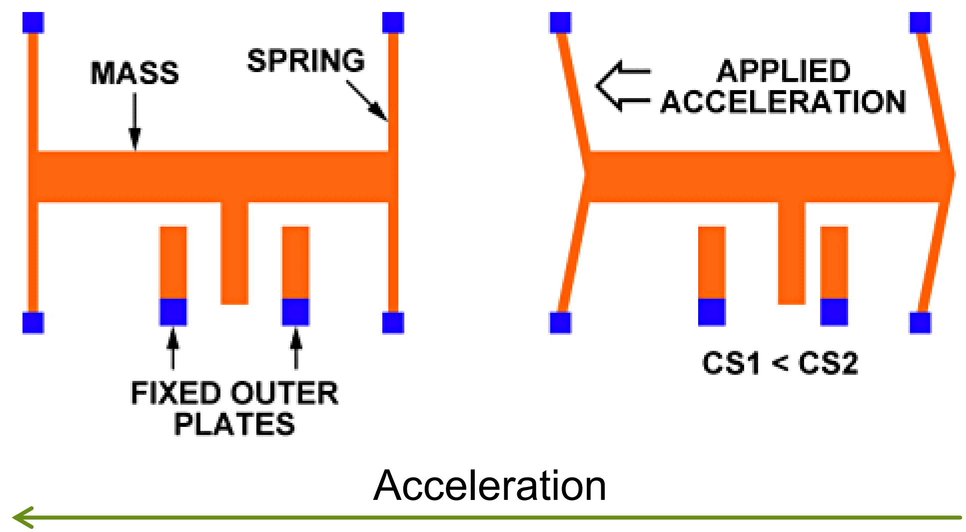


$$F - m \bullet a = 0$$

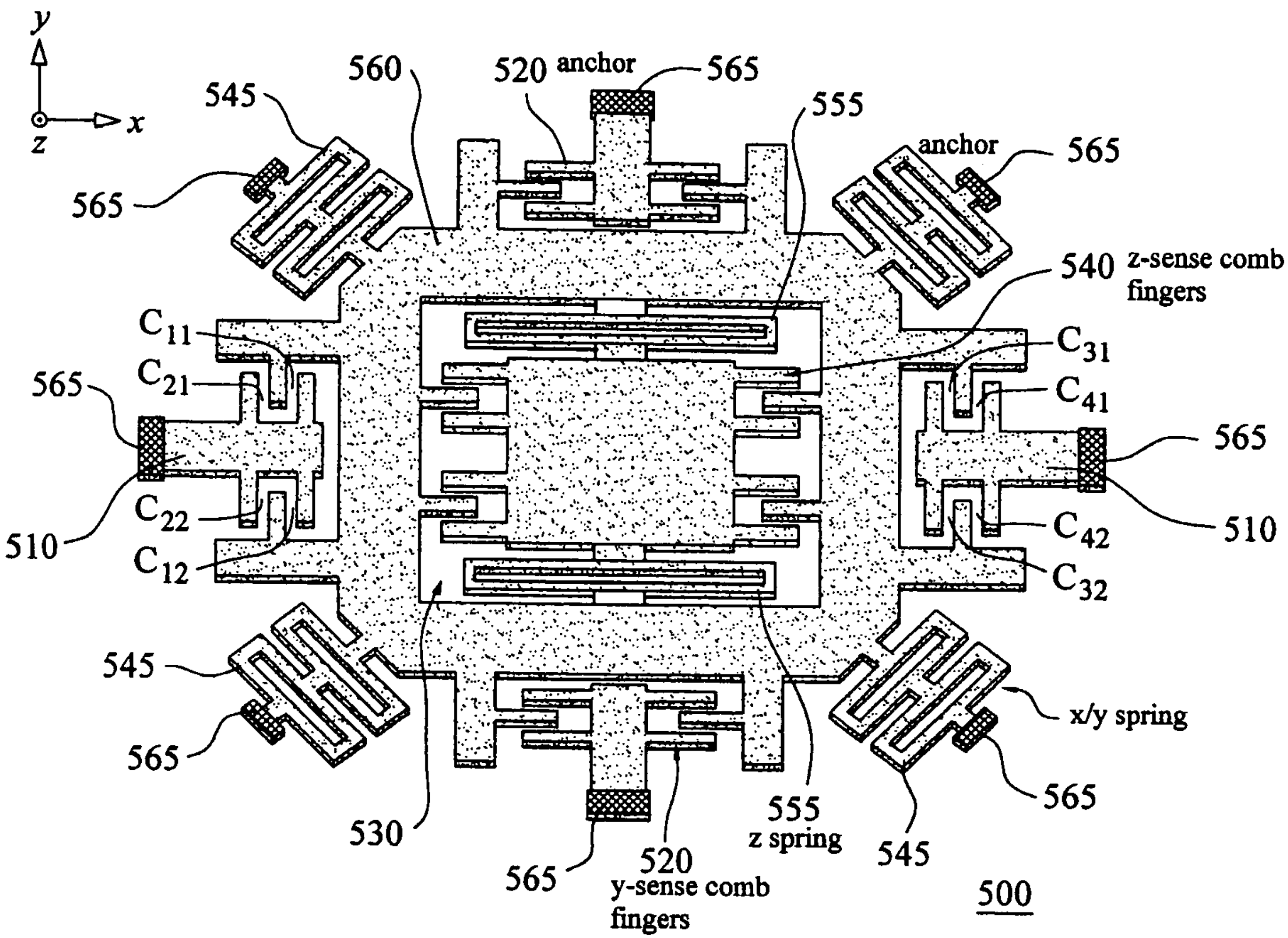
$$C = \varepsilon_0 \frac{A}{d}$$

$$C = \varepsilon_0 \frac{A}{d \pm \Delta d}$$



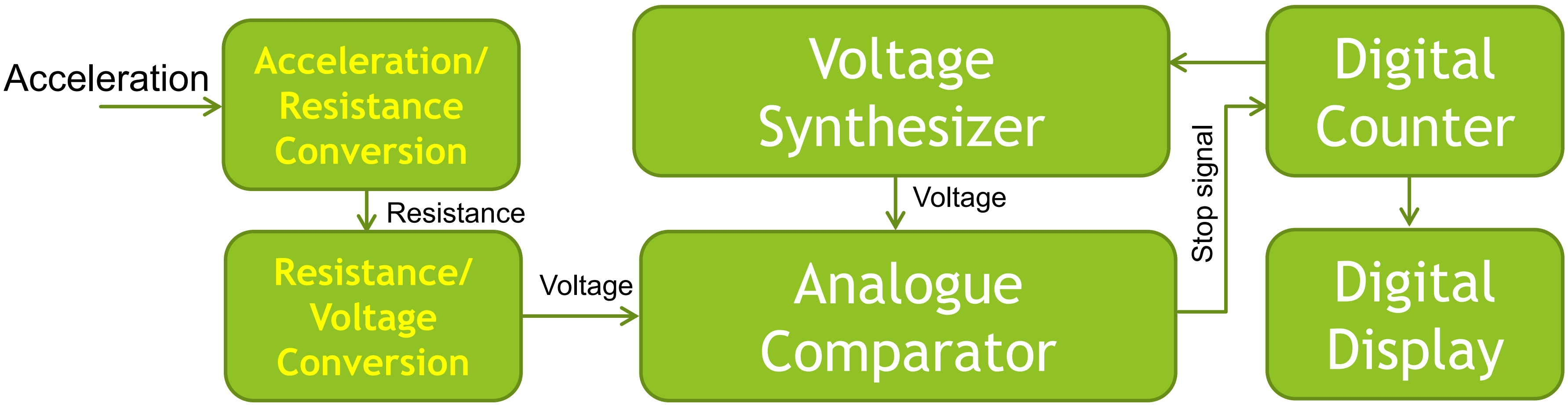


Example of 2-Axis Accelerometer



How to apply principle 3 to design digital measurement and sensing systems for acceleration?

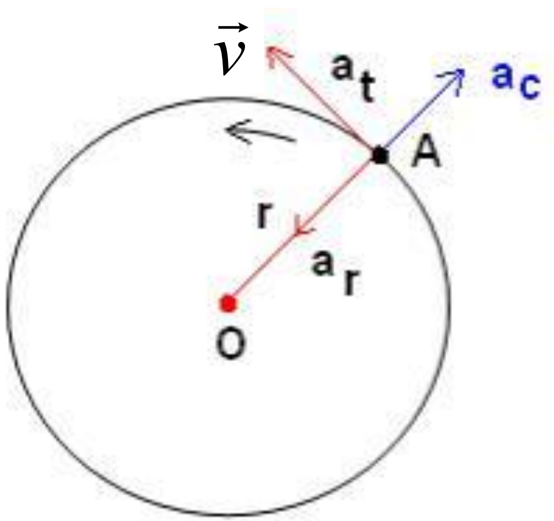
- Acceleration is converted to resistance which is then converted to voltage. Finally, the voltage is measured by digital voltmeter (e.g. microcontrollers).

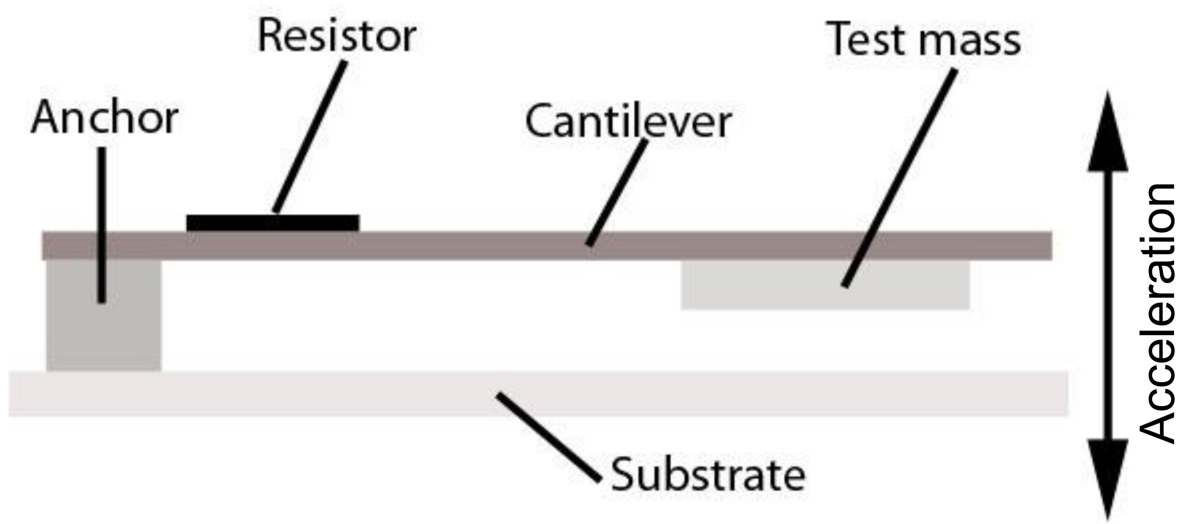


All microcontrollers are programmable digital sensors of voltage!

How to convert acceleration to resistance?

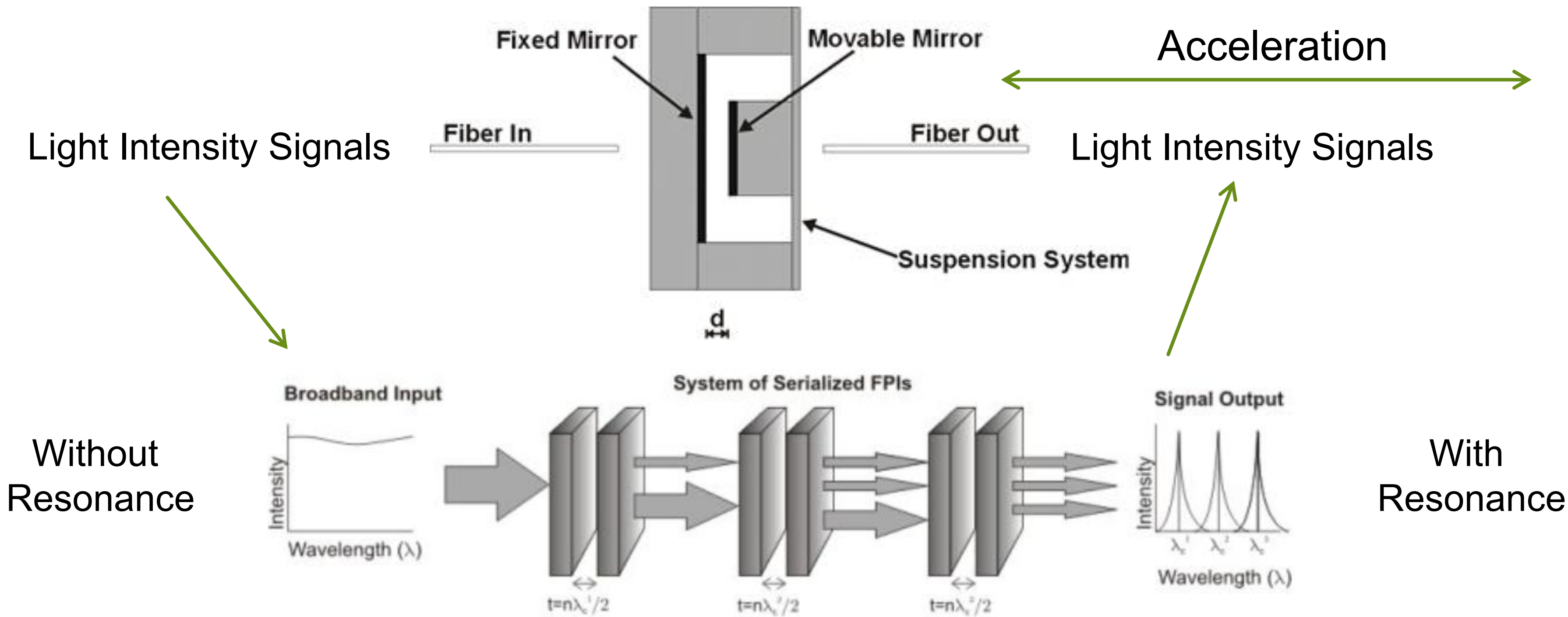
- A resistive material is placed onto a deformable beam. When there is an acceleration, the induced inertial force (or centrifugal force) causes the beam to deform. The deformation in turn causes the change of resistance of the resistive material.





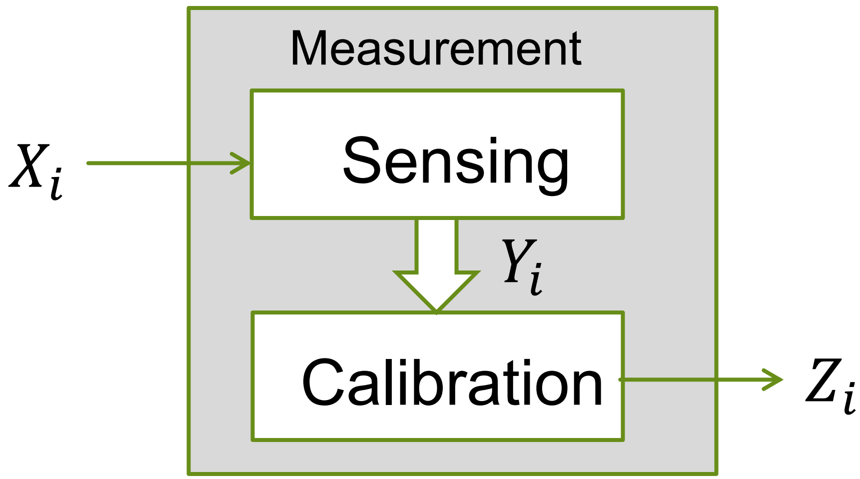
Other Principle of Designing Measurement and Sensing Systems for Acceleration …

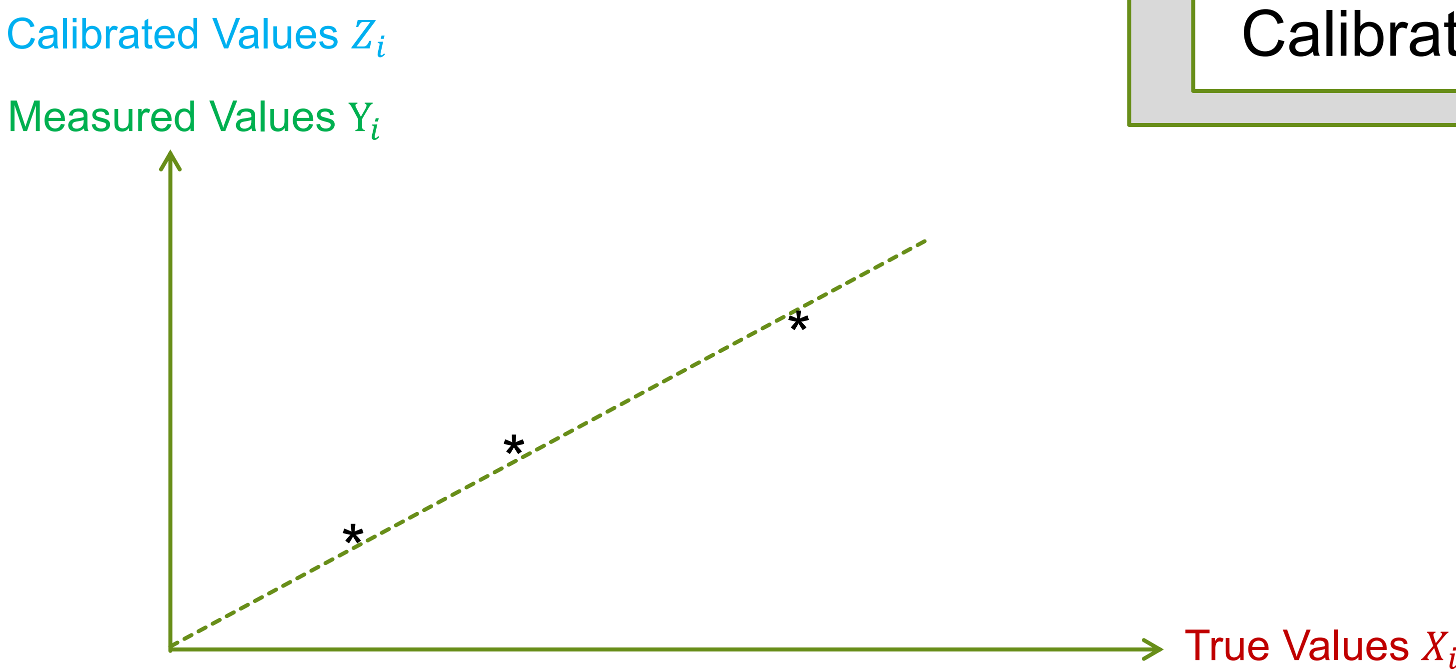
- Two parallel mirrors in a chamber form a resonator of lights. The frequencies of resonance depend on the distances between the two mirrors. Hence, one movable mirror is fixed on a movable mass. When an acceleration occurs, the movable mirror-mass causes the change of frequencies of light’s resonance.



Remember to Do Calibration

- Curve fitting for calibration:
  - Yi is produced by Xi
  - Zi is computed from Yi
  - Zi must be equal to Xi

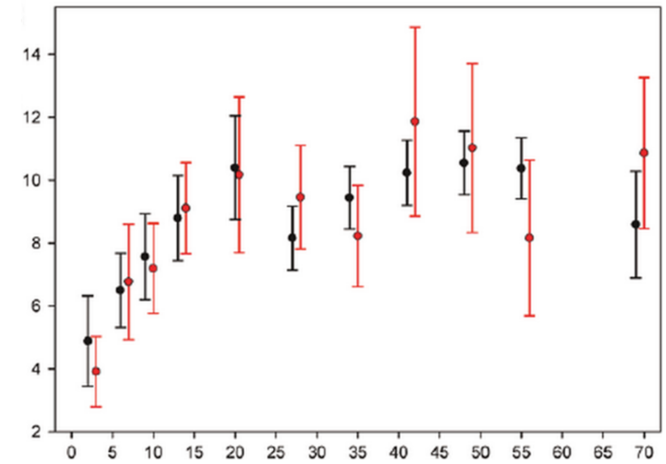


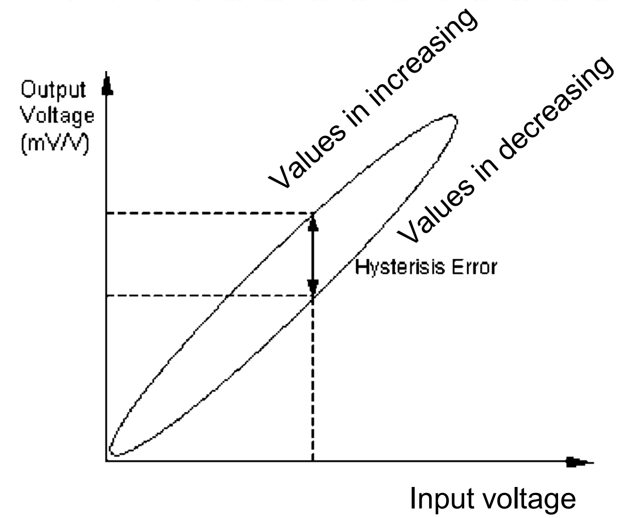


Remember to Do Error Analysis

- Systematic error = mean value – true value
- Repeatability error = value with maximum error – mean value
- Accuracy = value with minimum error – mean value
- Hysteresis error = |measured value in increasing – measured value in decreasing|

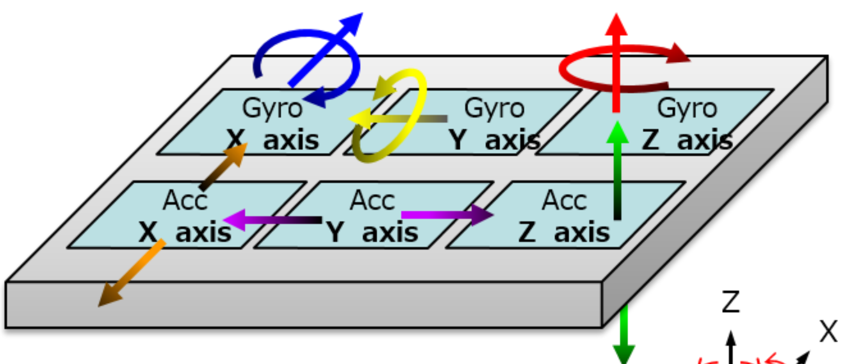
For each true value, we can do error analysis

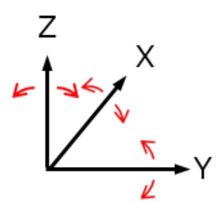




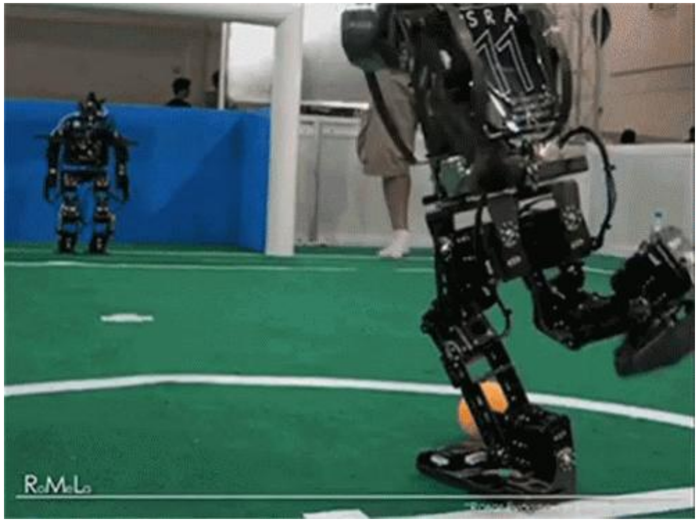
Summary

- Understanding of Acceleration
- Computation of Acceleration
- Measurement of Acceleration



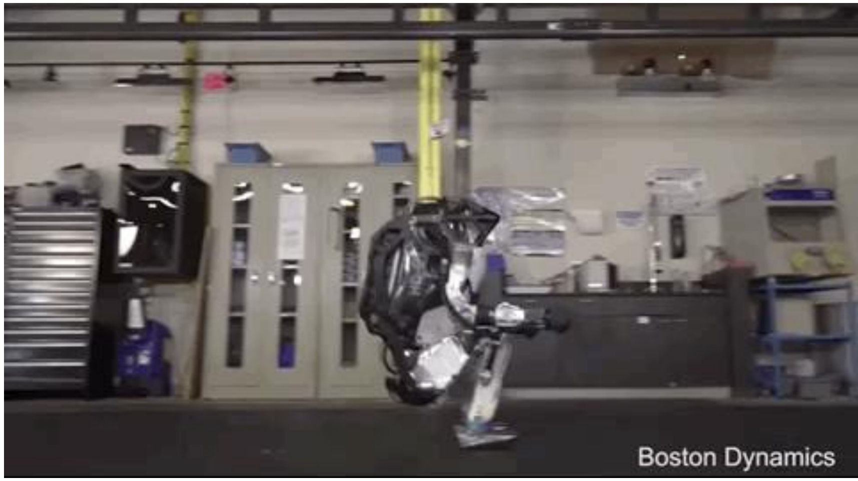


Behind Gyroscope is Centrifugal Force
Behind Accelerometer is Inertial Force



Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



Module 3 Lecture 4

MA4822

Measurement of Force

Xie Ming, PhD (France)

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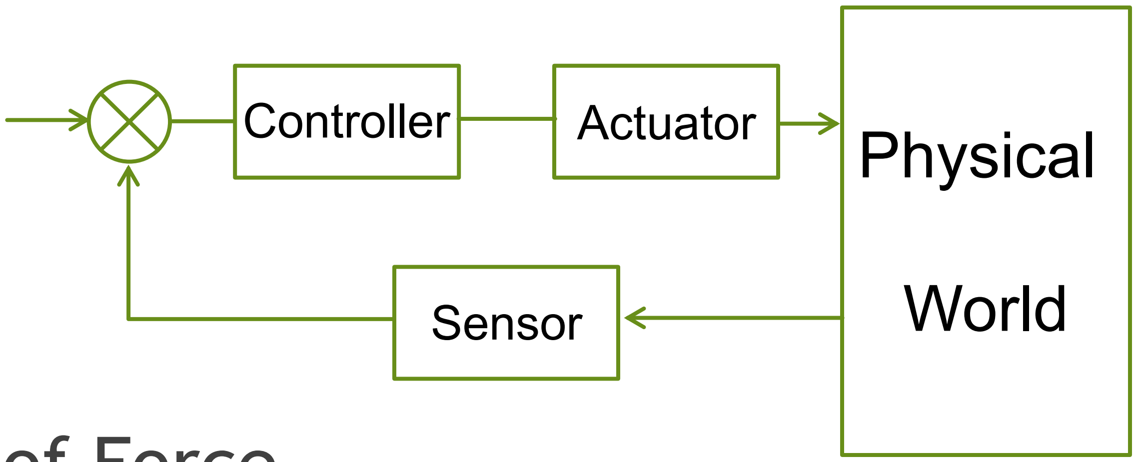
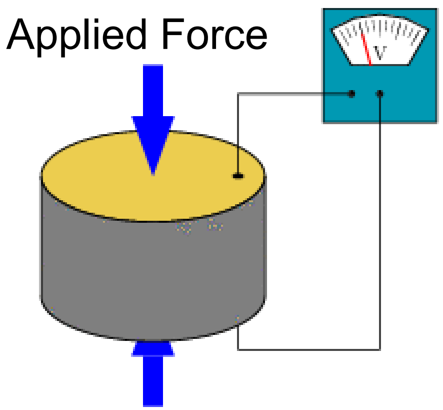
http://personal.ntu.edu.sg/mmxie

Outline

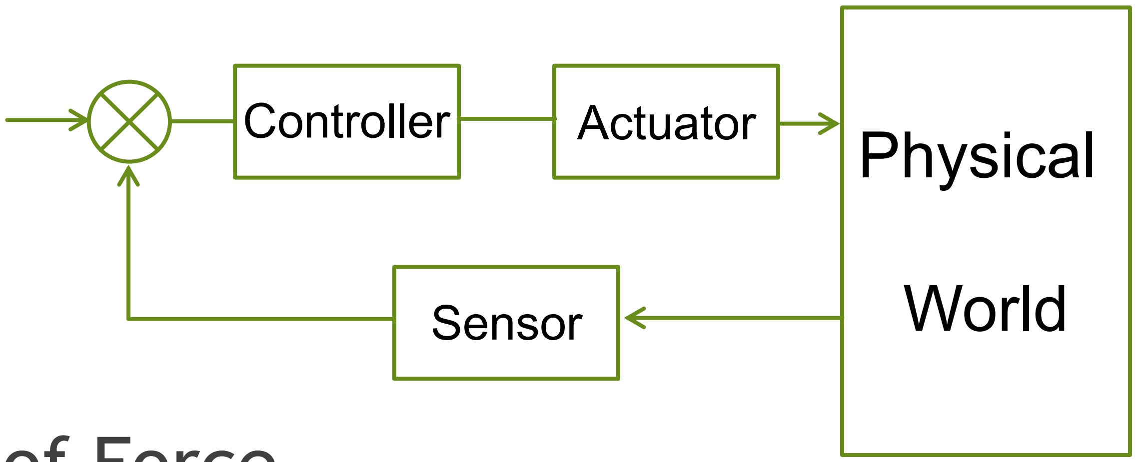


- Understanding of Force
- Computation of Force
- Measurement of Force

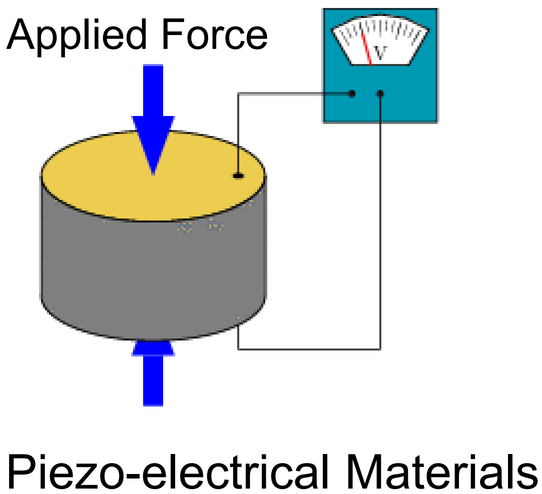


Piezo-electrical Materials

Outline

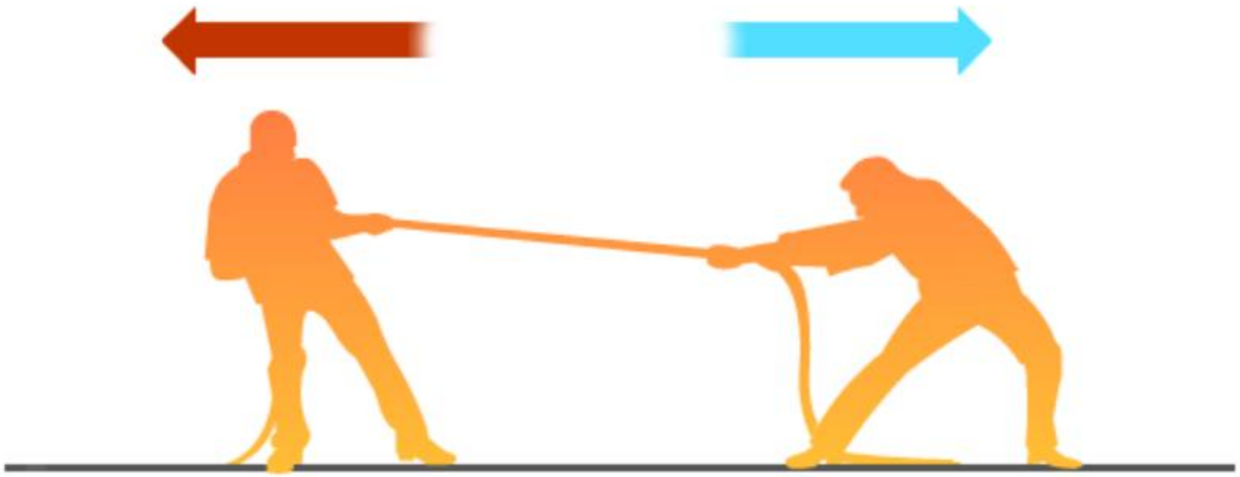


- Understanding of Force
- Computation of Force
- Measurement of Force



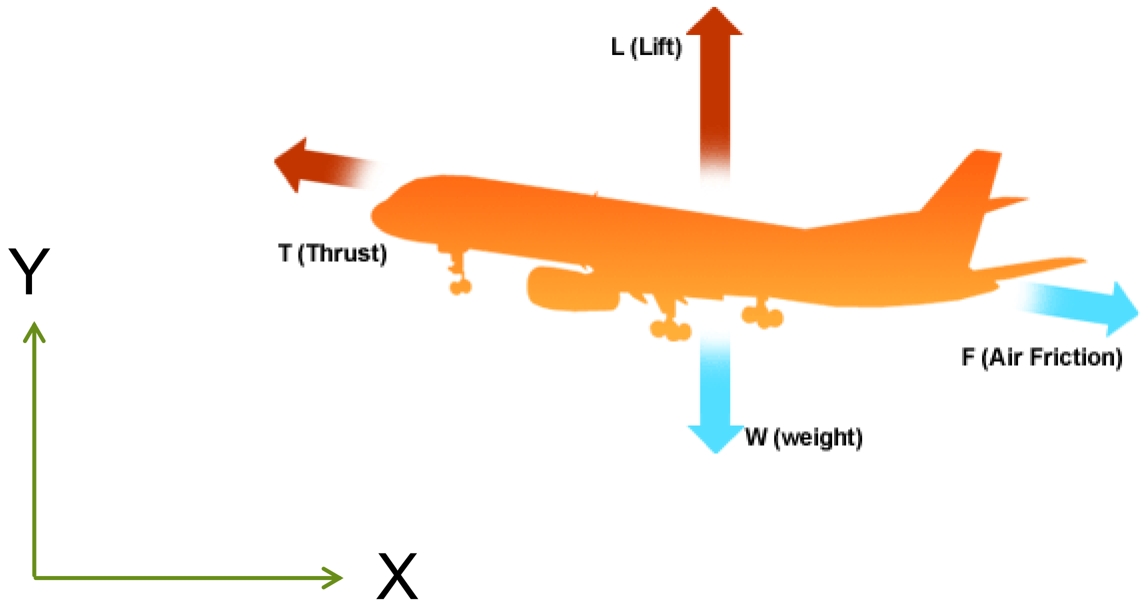
Understanding Force (1)

- 1. Force is the cause which changes the state of linear motion.



Example

$$\left|F_{net,x}\right| > 0 \qquad F_{net,y} = 0$$

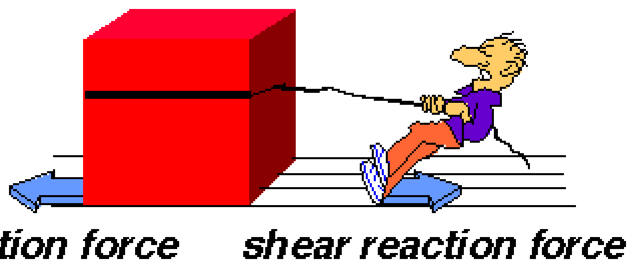


Understanding Force (2)

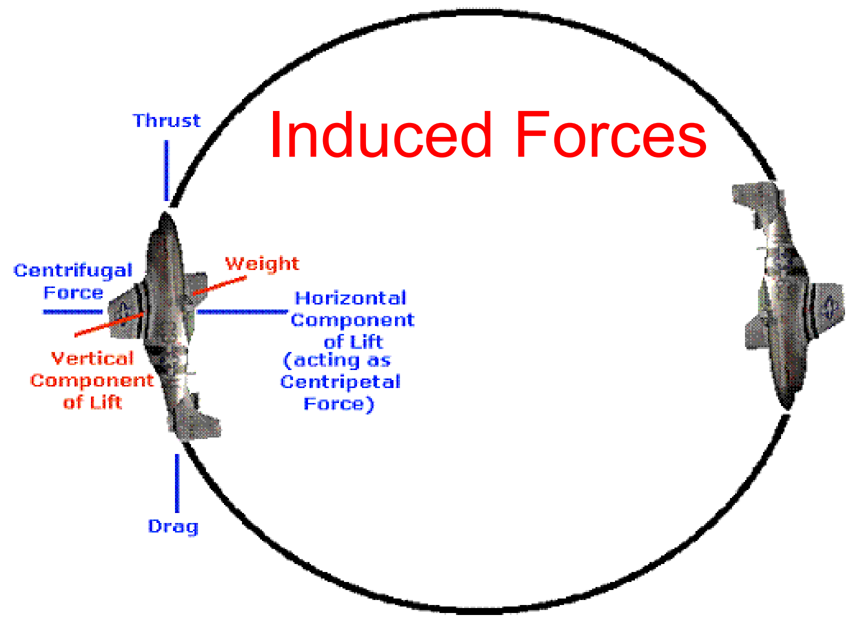
- 2. There are three types of forces.

Field Forces

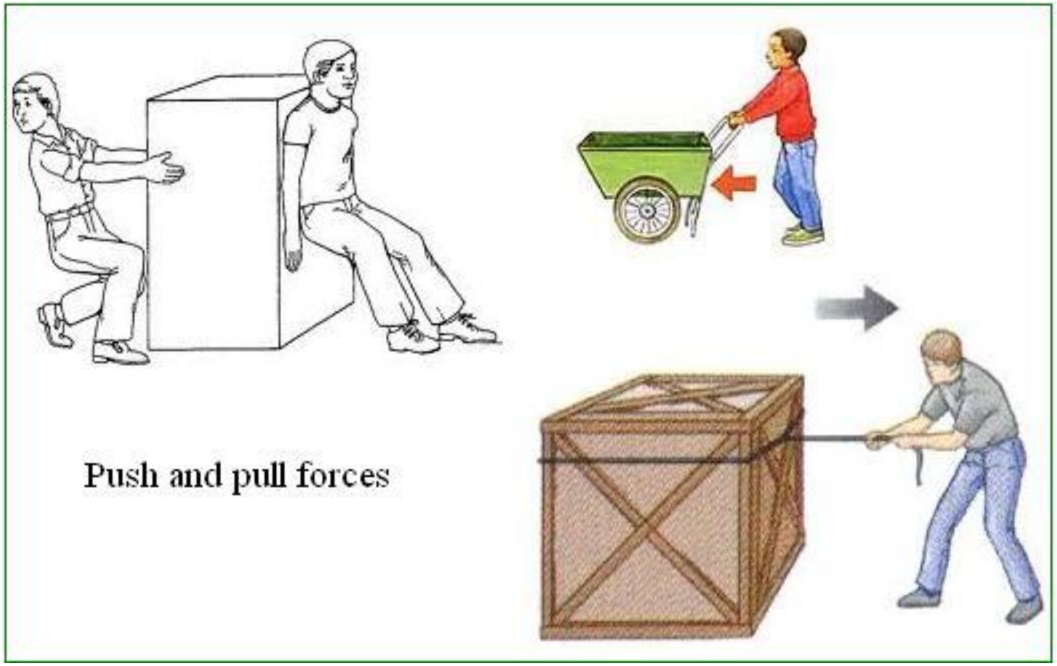
Contact Forces

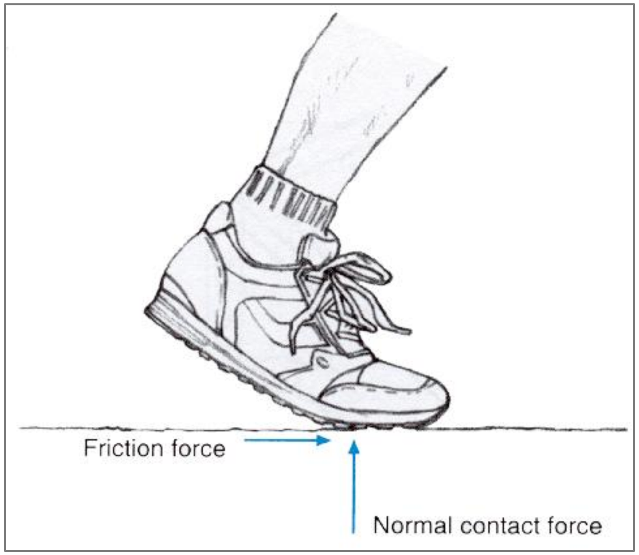


Induced Forces



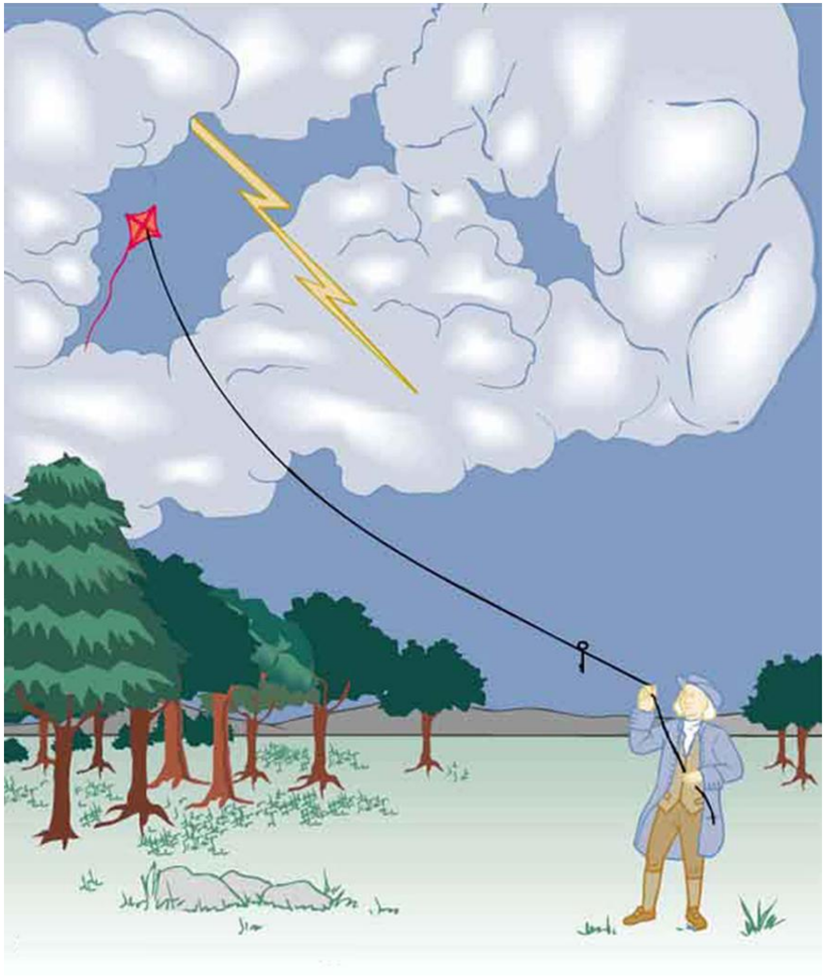
Example of Contact Forces





Example of Fluid's Resistive Force

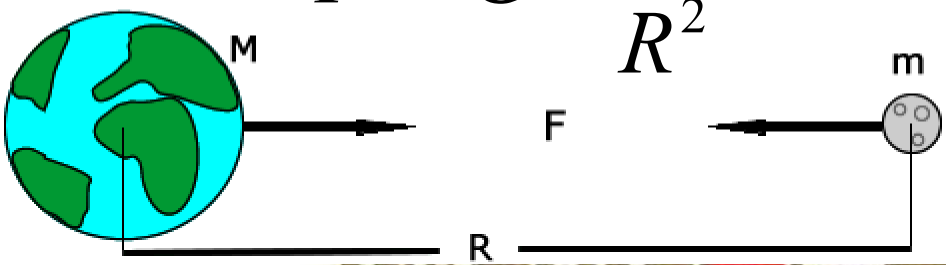
Example of Air's Resistive Force

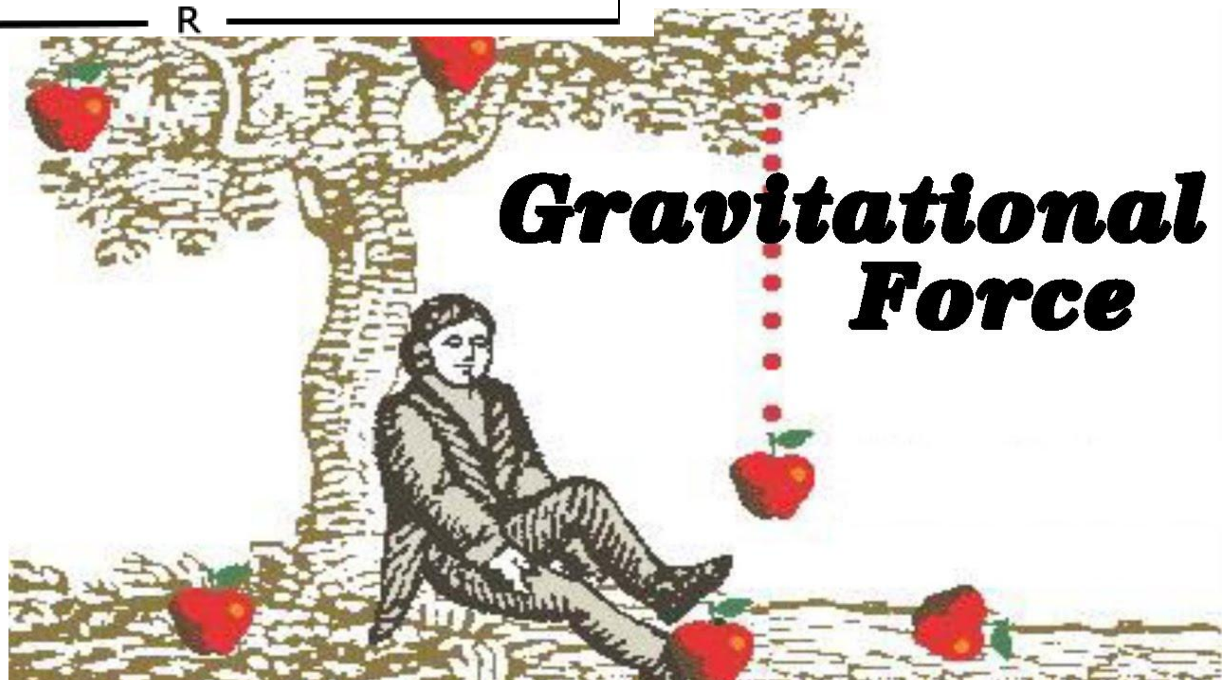


Example of Gravitational Field Force

$$F = G\frac{M \bullet m}{R^2}$$

$$G = 6.67 \times 10^{-11} Nm^2 / kg^2$$

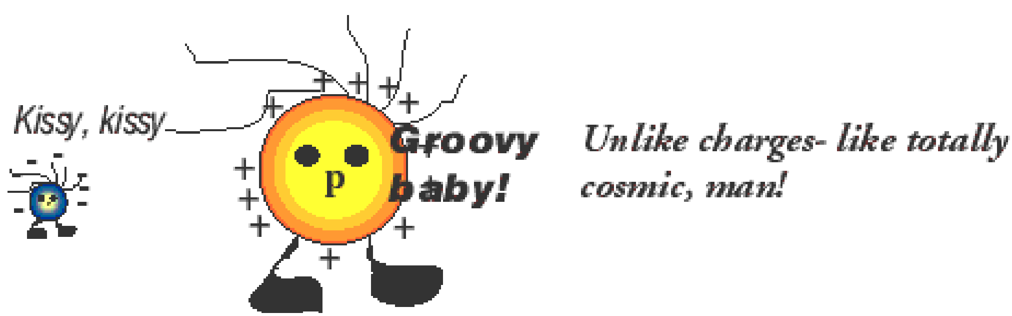




Example of Electric Field Force

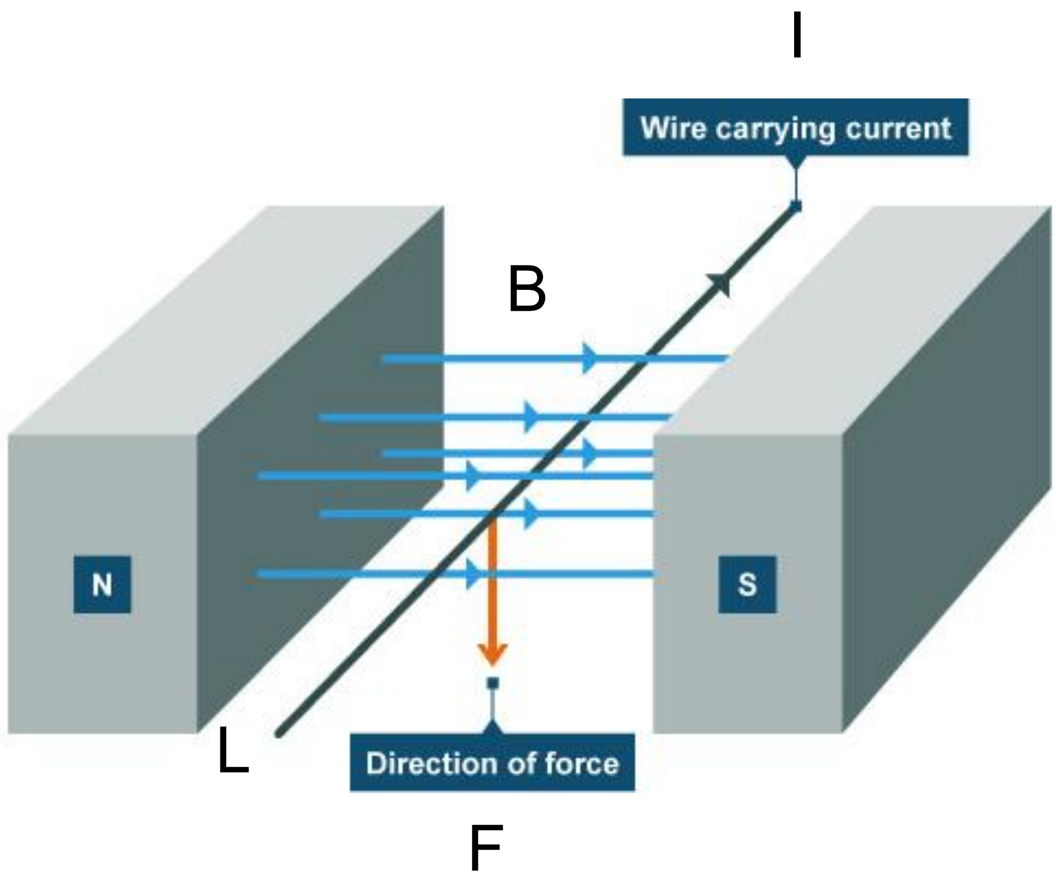
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi\varepsilon_0} = 9.0\times10^9\, Nm^2 / C^2$$



Example of Magnetic Field Force

$$F = L \bullet \vec{I} \times \vec{B}$$



Example of Induced Centrifugal Force

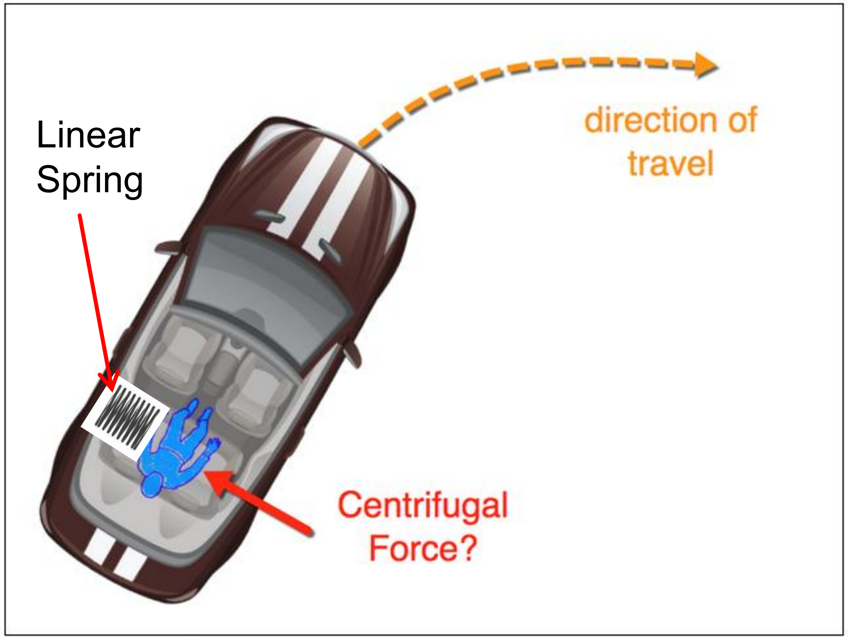
- A car is moving along a circular path with a radius of 45.0 meters. A person of mass of 60.0 kg is sitting inside the moving car. If the car's circular velocity is 60.0 km/h, what is the centrifugal force induced on the person?

- Answer:

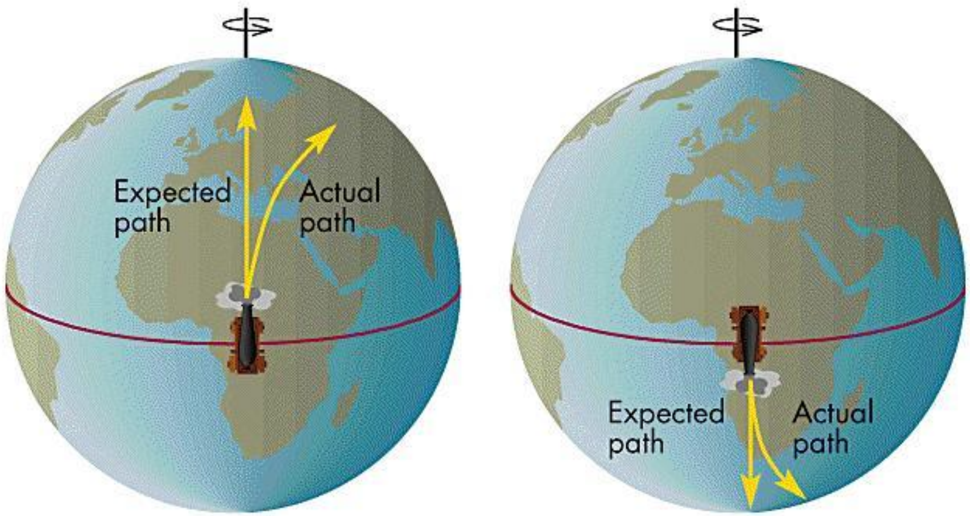
$$v = \frac{60 \times 10^3}{60 \times 60} = 16.667 m/s$$

$$a_c = \frac{v^2}{R} = \frac{16.667^2}{45.0} = 6.173 m^2/s$$

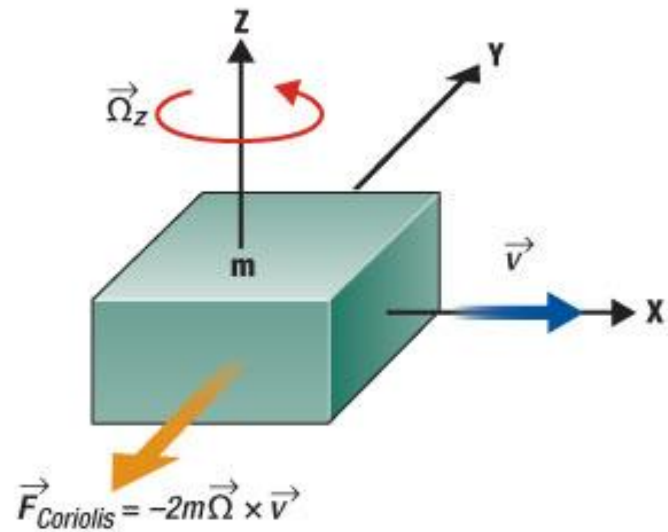
$$F_c = m a_c = 60.0 \times 6.173 = 370.38 N$$

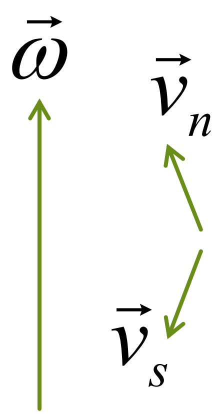


Example of Induced Coriolis Forces



A Projectile fired northward **B** Projectile fired southward



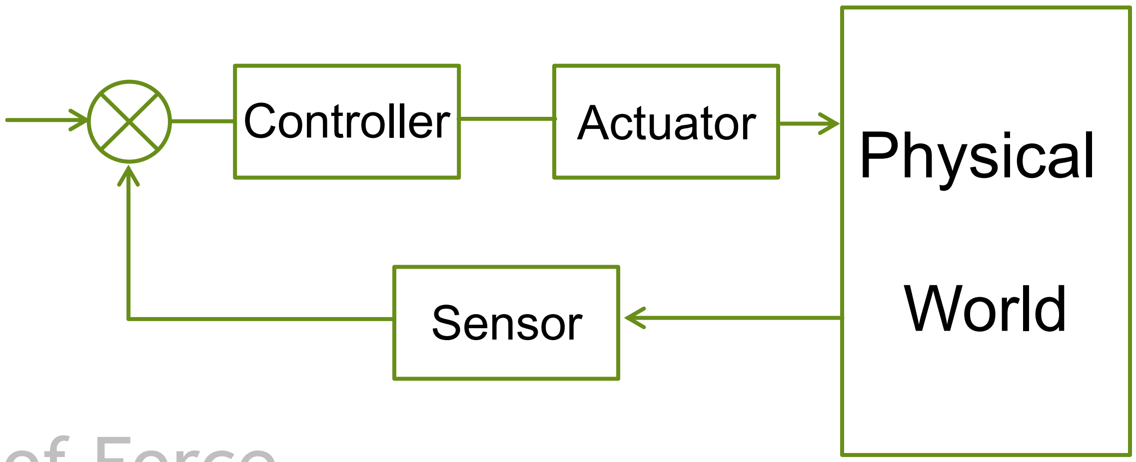


Velocity toward north

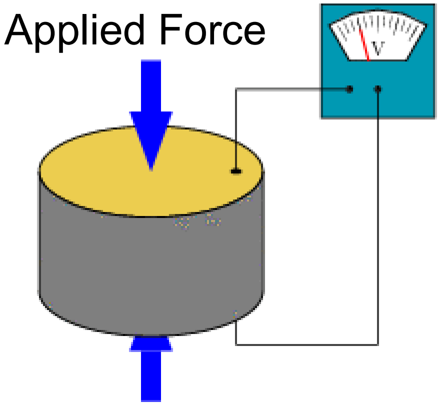
Velocity toward south

$$\vec{F} = -2m\vec{\Omega} \times \vec{v}$$

Outline



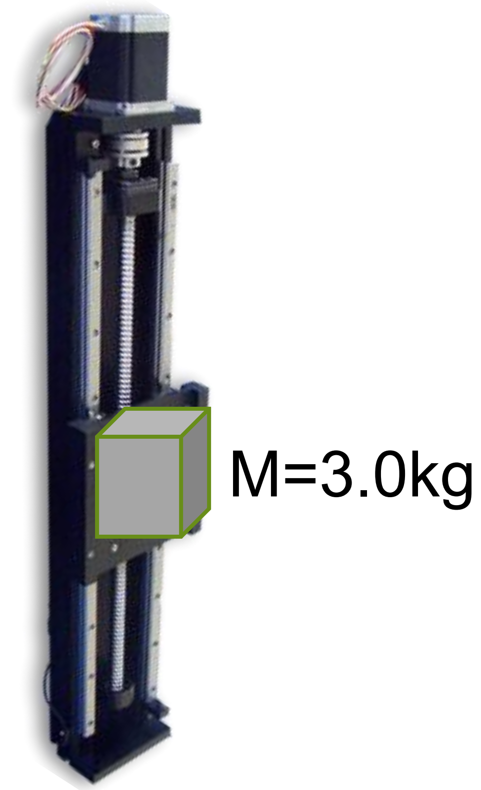
- Understanding of Force
- Computation of Force
- Measurement of Force



Piezo-electrical Materials

Computation of Link’s Acting Force

- A prismatic link joint carries a payload of 3.0 kg in a vertical direction. The kinetic frictional force of the joint is 0.5 N. If the upward acceleration has to be 2.0 m/s^2, what should be the acting force on the prismatic link?



- Answer:

$$F = F_{acting} - F_k - mg = F_{acting} - 0.5 - 3.0 \times 9.8$$

$$F_{acting} - 0.5 - 3.0 \times 9.8 = 3.0 \times 2.0$$

$$F_{acting} = 0.5 + 3.0 \times 9.8 + 3.0 \times 2.0 = 35.9N$$

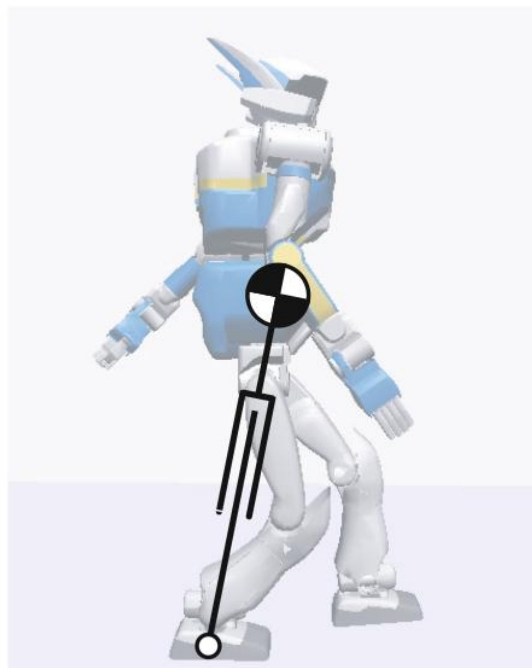
Computation of Leg's Acting Force

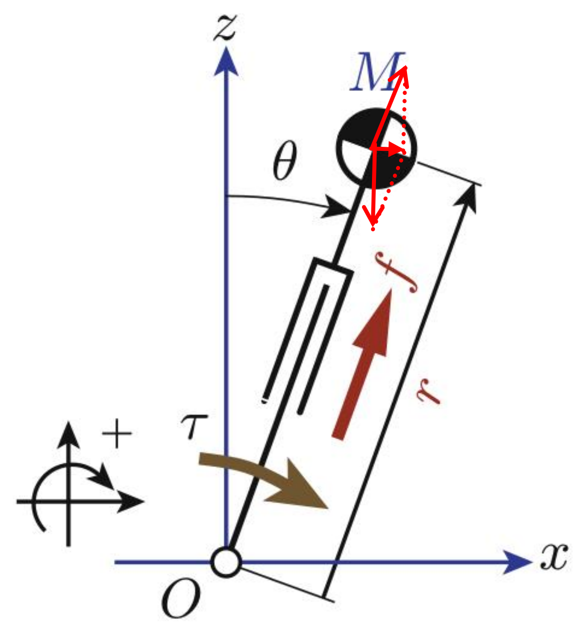
- What should be the leg's acting force so that the net force is in the horizontal direction?

- Answer:

$$f \times \cos(\theta) = Mg$$

$$f = \frac{Mg}{\cos(\theta)}$$

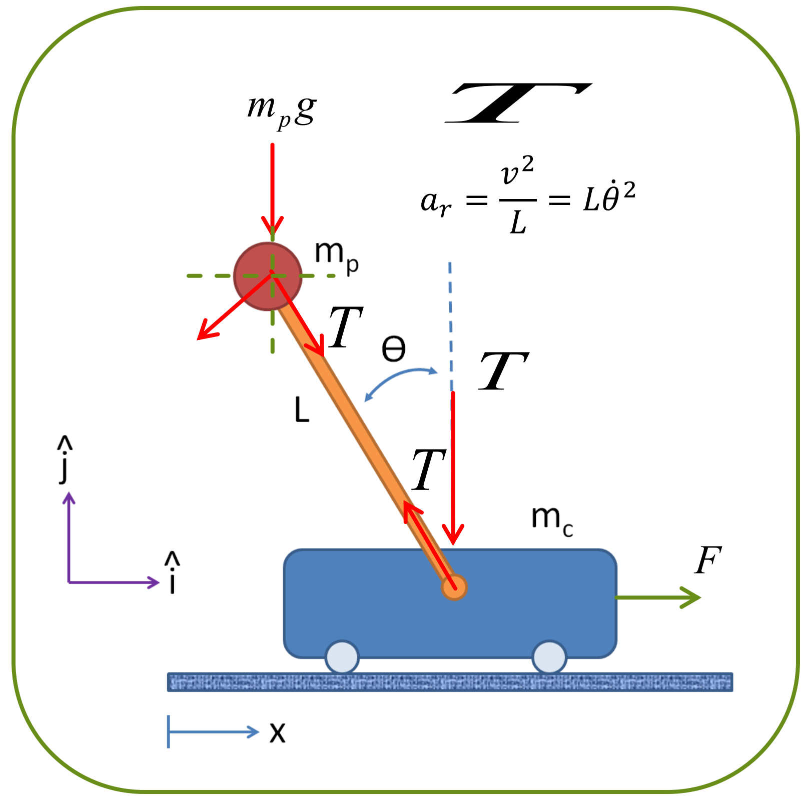




Computation of Driving Force and Tension Force

An inverted pendulum is attached to a cart through a rigid bar.

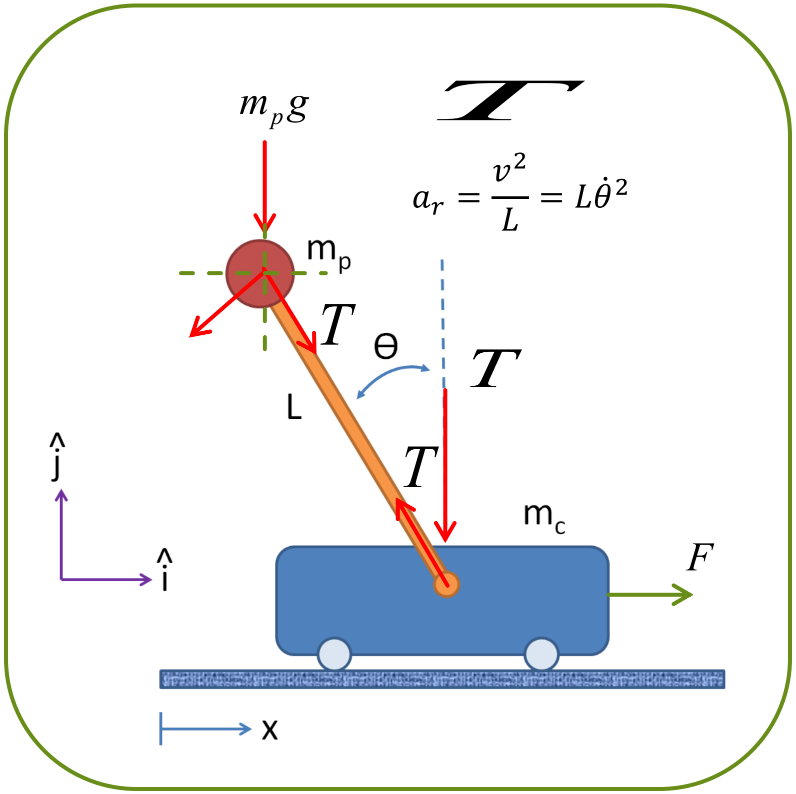
- A) What is the driving force to produce a desired horizontal acceleration?
- B) What is the tension force inside the bar when a desired horizontal acceleration occurs?



$m_c g$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

A) Answer:



$$m_c g$$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

- From the free-body diagram of mass c:

$$F - T\sin(\theta) = m_c \ddot{x}_c$$

- From the free-body diagram of mass p:

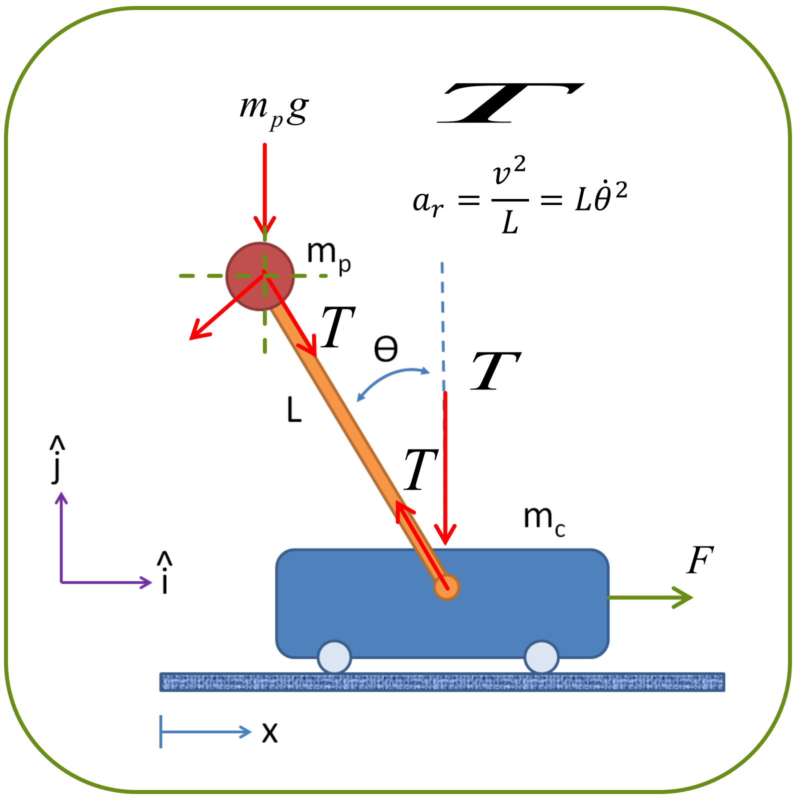
$$T\sin(\theta) = m_p \ddot{x}_p = m_p \ddot{x}_c + m_p \ddot{x}_{p/c}$$

+

The computed force is:

$$F = (m_c + m_p)\ddot{x}_c + m_p \ddot{x}_{p/c}$$

B) Answer:



$m_c g$

$$\ddot{y}_{p/c} = -L\sin(\theta)\ddot{\theta} - L\cos(\theta)\dot{\theta}^2$$

- From the free-body diagram of mass p:

$$T\sin(\theta) = m_p\ddot{x}_p = m_p\ddot{x}_c + m_p\ddot{x}_{p/c}$$

The computed tension force is:

$$T = \frac{1}{\sin(\theta)}(m_p\ddot{x}_c + m_p\ddot{x}_{p/c})$$

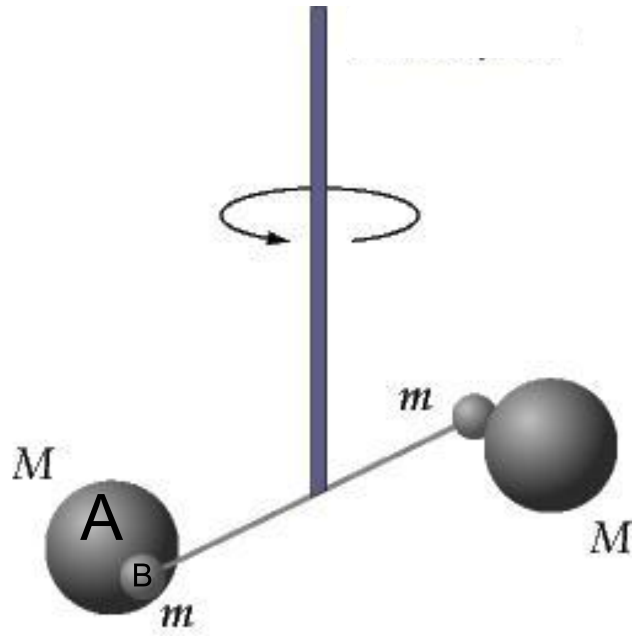
Computation of Gravitational Force

- In the figure below, mass A is 20.0 kg and mass B is 0.1 kg. If the distance between mass A and mass B is 2.0 cm, what is their gravitation force?

- Answer:

$$F = G\frac{M \bullet m}{R^2} = 6.67 \times 10^{-11}\frac{20 \times 0.1}{0.02^2}$$

$$F = 3.335 \times 10^{-7} N$$



Computation of Electrical Force

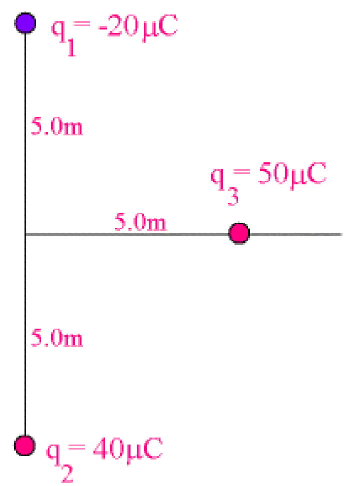
- In the figure below, what is the amplitude of electrical force F_{23}?

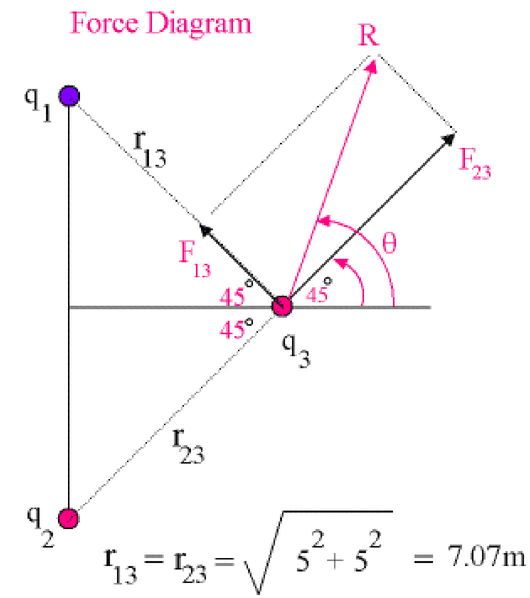
- Answer:

$$F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}^2}$$

$$F_{23} = 9.0\times10^{9}\,\frac{40\times10^{-6}\times50\times10^{-6}}{7.07^2}$$

$$F_{23} = 0.36N$$



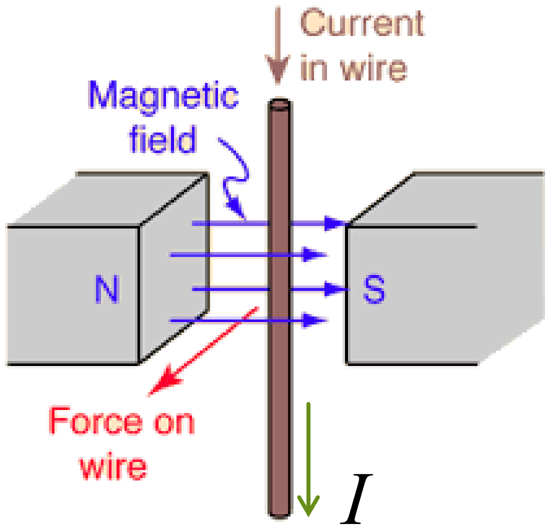


Computation of Magnetic Force

- A wire of length L is placed into a magnetic field of density B. If the current passing through it is I, what is the magnetic force acting on the wire?

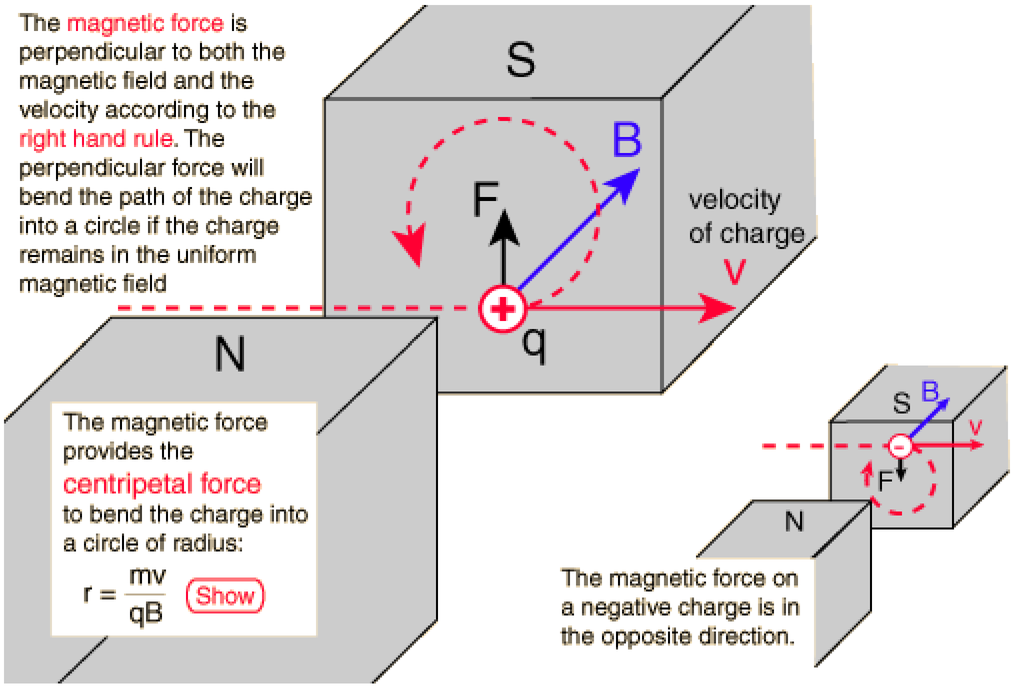
- Answer:

$$\vec{F} = L \bullet \vec{I} \times \vec{B}$$



Computation of Magnetic Force

- A charge of -10 Coulomb enters a magnetic field at the speed of 1.0 km/s. Due to the magnetic force, the charge moves along a circular path of the radius of 2.0 m. If the mass of the charge is 0.01g, what is the magnetic force? And, what is the density of the magnetic field?
- Answer:

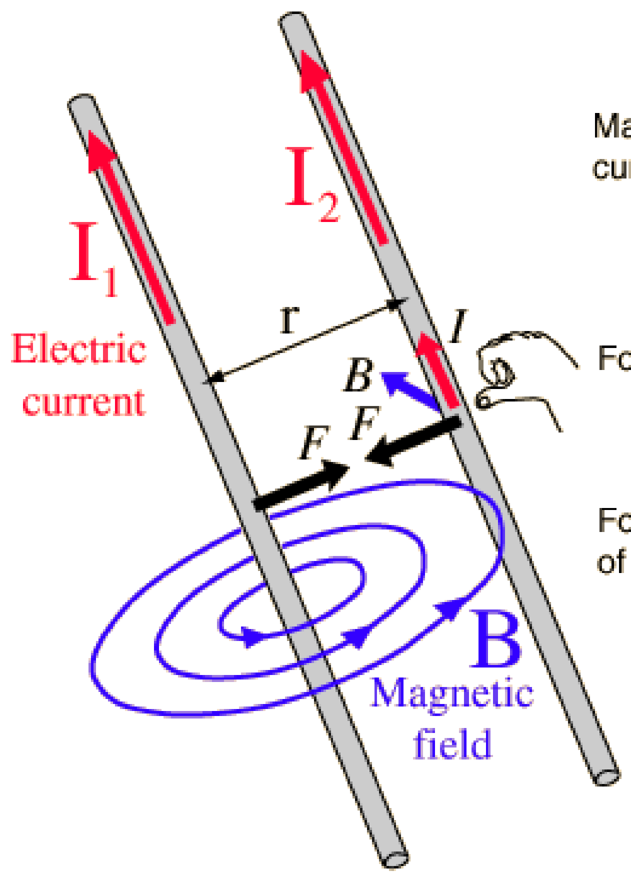


$$F_m = F_c = m\frac{v^2}{r} = 10^{-5}kg\frac{1000^2}{2.0} = 5N$$

$$B = \frac{F_m}{qv} = \frac{5}{10\times 1000} = 5\times 10^{-4}T$$

Computation of Magnetic Force

- Two wires are placed in parallel as shown in the figure below. What is the force received by wire 2?



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

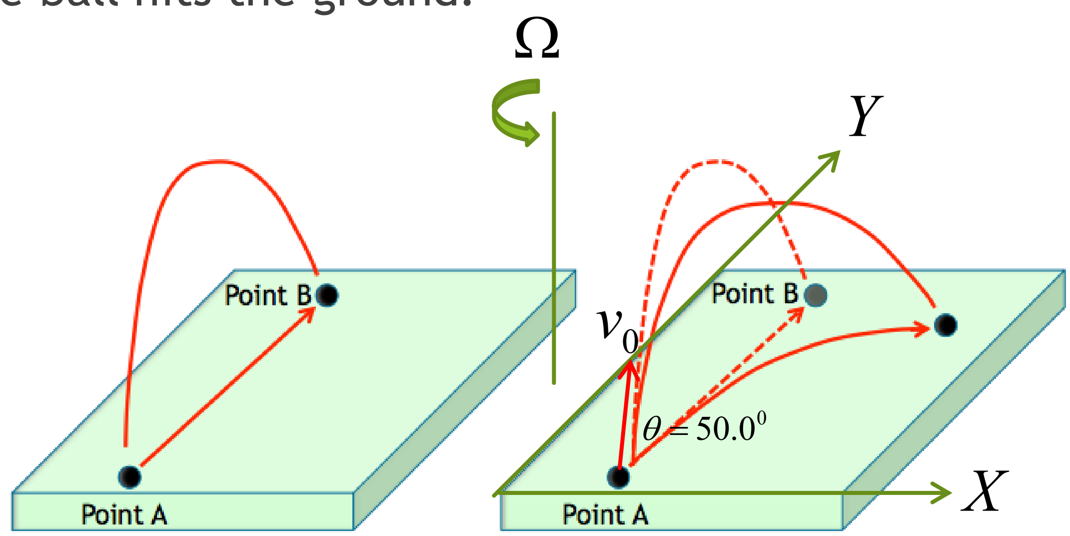
$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Computation of Coriolis Force

- The Earth makes one round of revolution within 24 hours. In a terrain near the north pole, a ball is launched from point A as shown in the figure. The initial speed of the ball is 10.0m/s, and its mass is 0.1 kg. If the ball's take-off angle is 50.0 degrees, what is the Coriolis force acting on the ball? What is the displacement along X axis when the ball hits the ground?
- Answer:



$$F = 2mv_y\Omega$$

$$F = 2 \bullet 0.1 \bullet 10\cos(50^0)\frac{2\pi}{24 \bullet 60 \bullet 60}$$

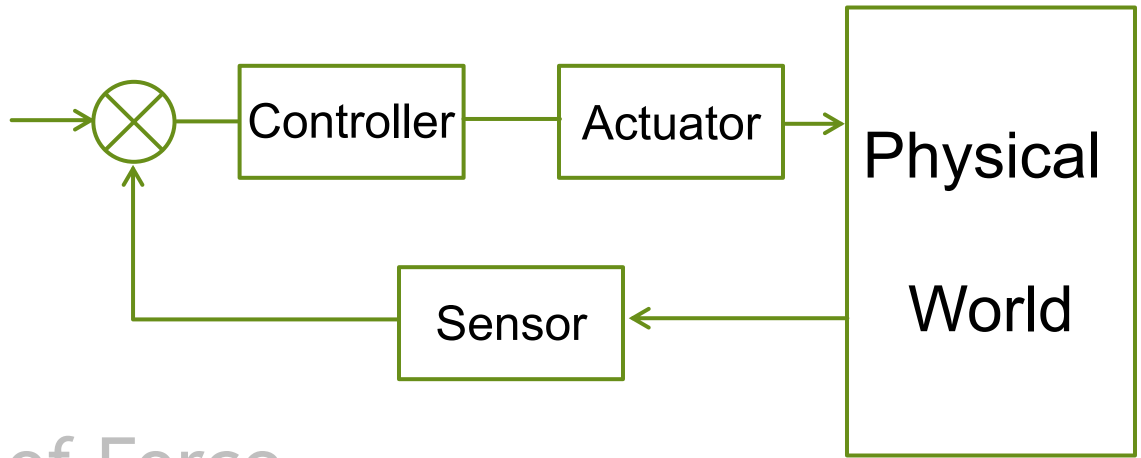
$$F = 9.35 \times 10^{-5} N$$

$$a = \frac{F}{m} = 9.35 \times 10^{-4} m/s^2$$

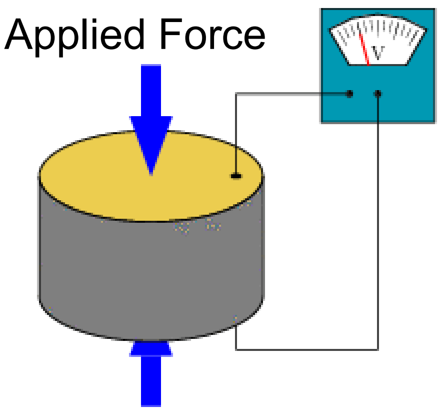
$$t = \frac{2v_0 \sin(\theta)}{g} = \frac{2 \bullet 10.0 \bullet \sin(50)}{9.8} = 1.56s$$

$$d_x = \frac{1}{2}at^2 = \frac{1}{2}9.35 \times 10^{-4} \bullet 1.56^2 = 1.1\ mm$$

Outline



- Understanding of Force
- Computation of Force
- Measurement of Force



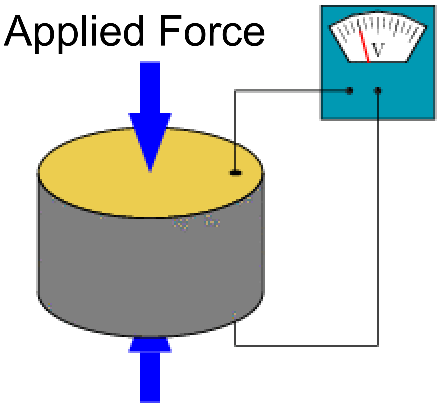
Piezo-electrical Materials

Applications in Human-Robot Interaction

Applications in Human-Computer Interaction

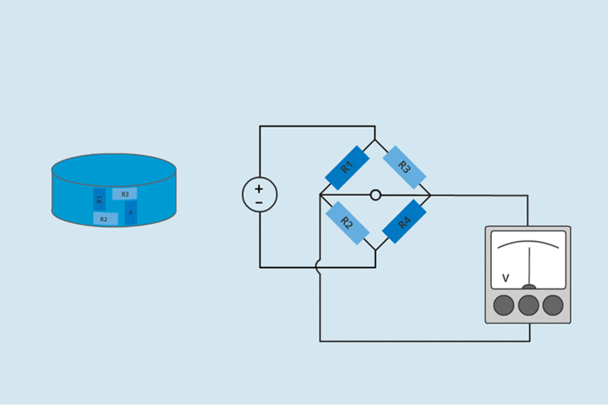
Principles of Measurement

- Principle 1: Force acting on special materials could produce voltages.



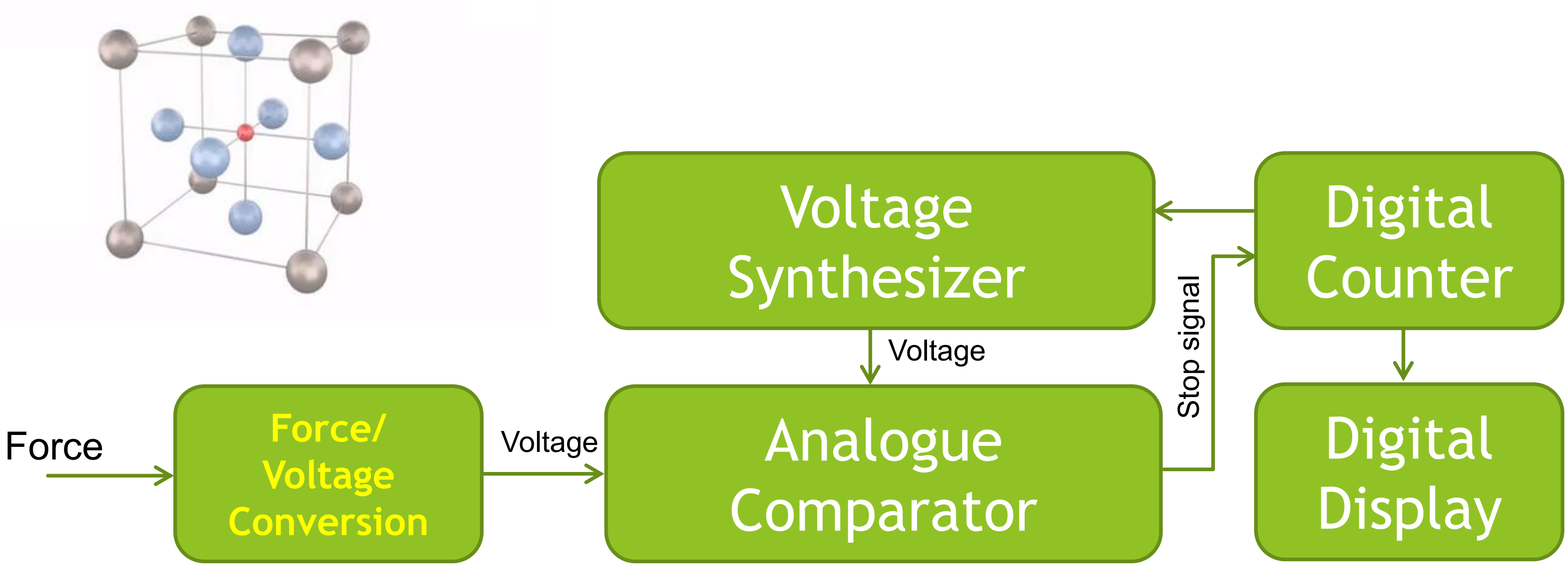
Piezo-electrical Materials

- Principle 2: Force can cause a mass to have deformation or displacement. Deformation can cause the change of electric properties of certain materials:
  - Change of resistance
  - Change of capacitance



How to apply principle 1 to design digital measurement and sensing systems for force?

- Force is converted to voltage by using piezoelectric device, and voltage is measured by digital voltmeter (e.g. microcontrollers).

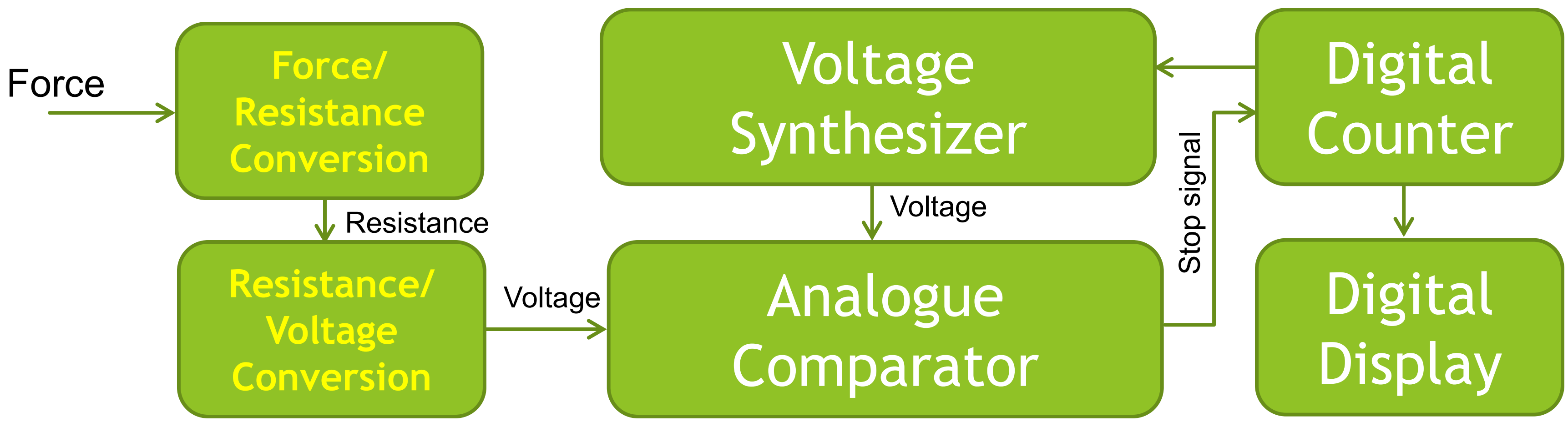


All microcontrollers are programmable digital sensors of voltage!

Example of Doing Force to Voltage Conversion

How to apply principle 2 to design digital measurement and sensing systems for force?

- Force is converted to resistance which is then converted to voltage. Finally, the voltage is measured by digital voltmeter (e.g. microcontrollers).

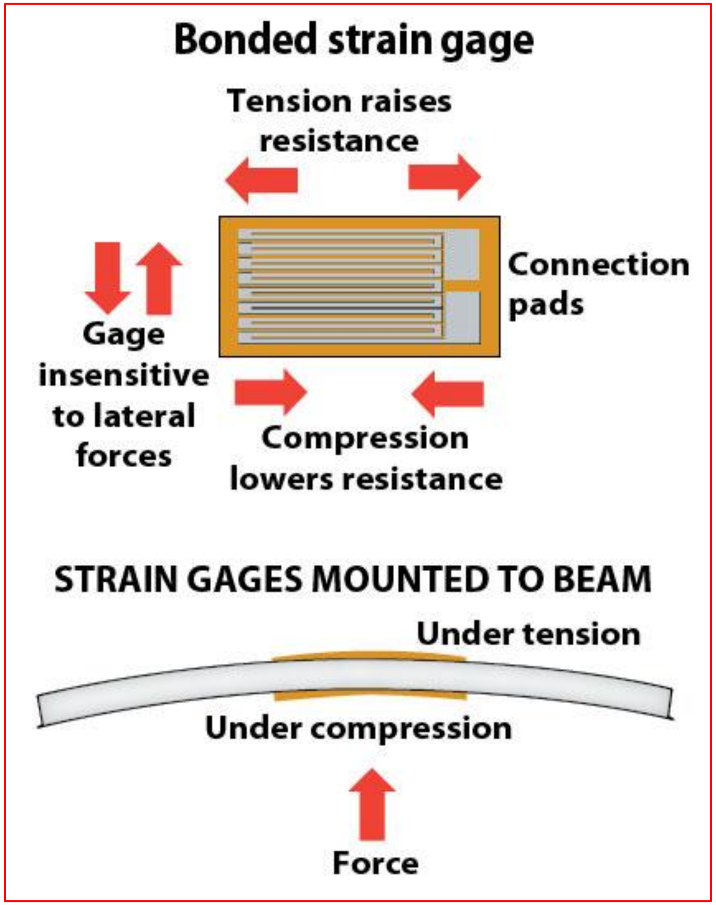


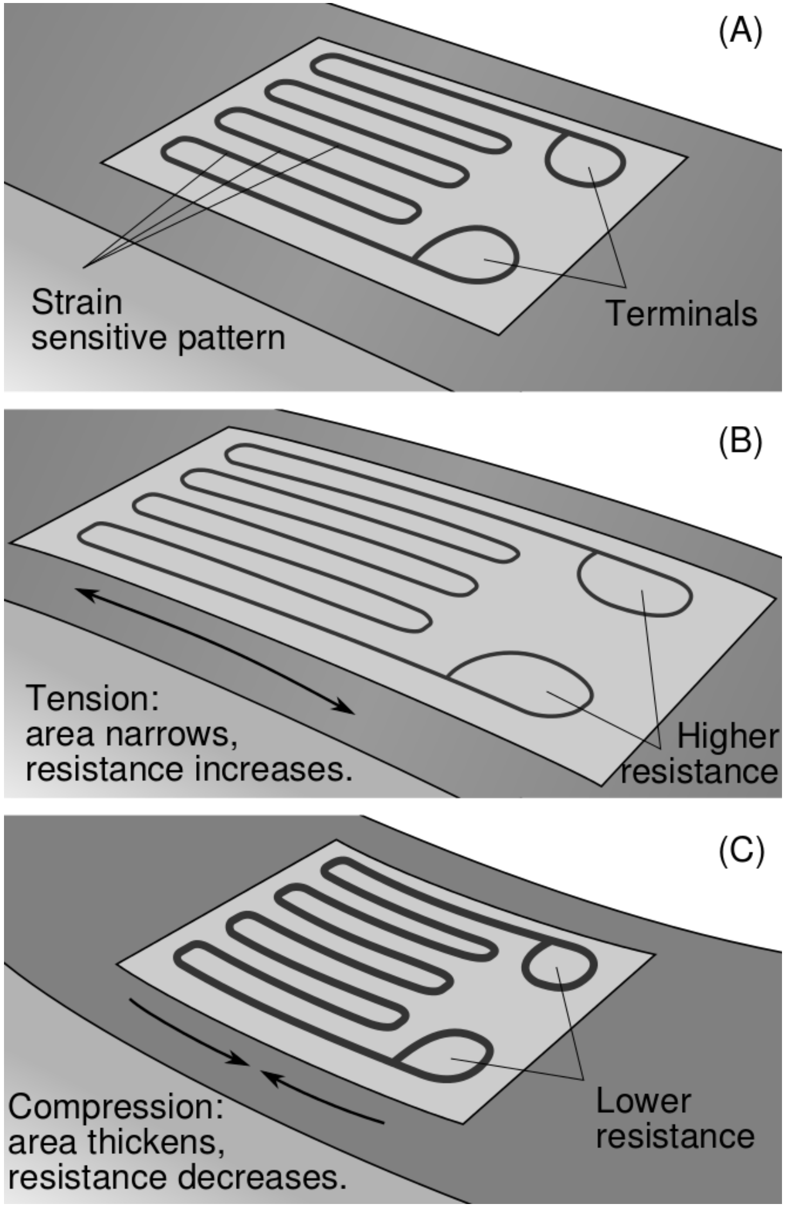
All microcontrollers are programmable digital sensors of voltage!

Force to Resistance Conversion

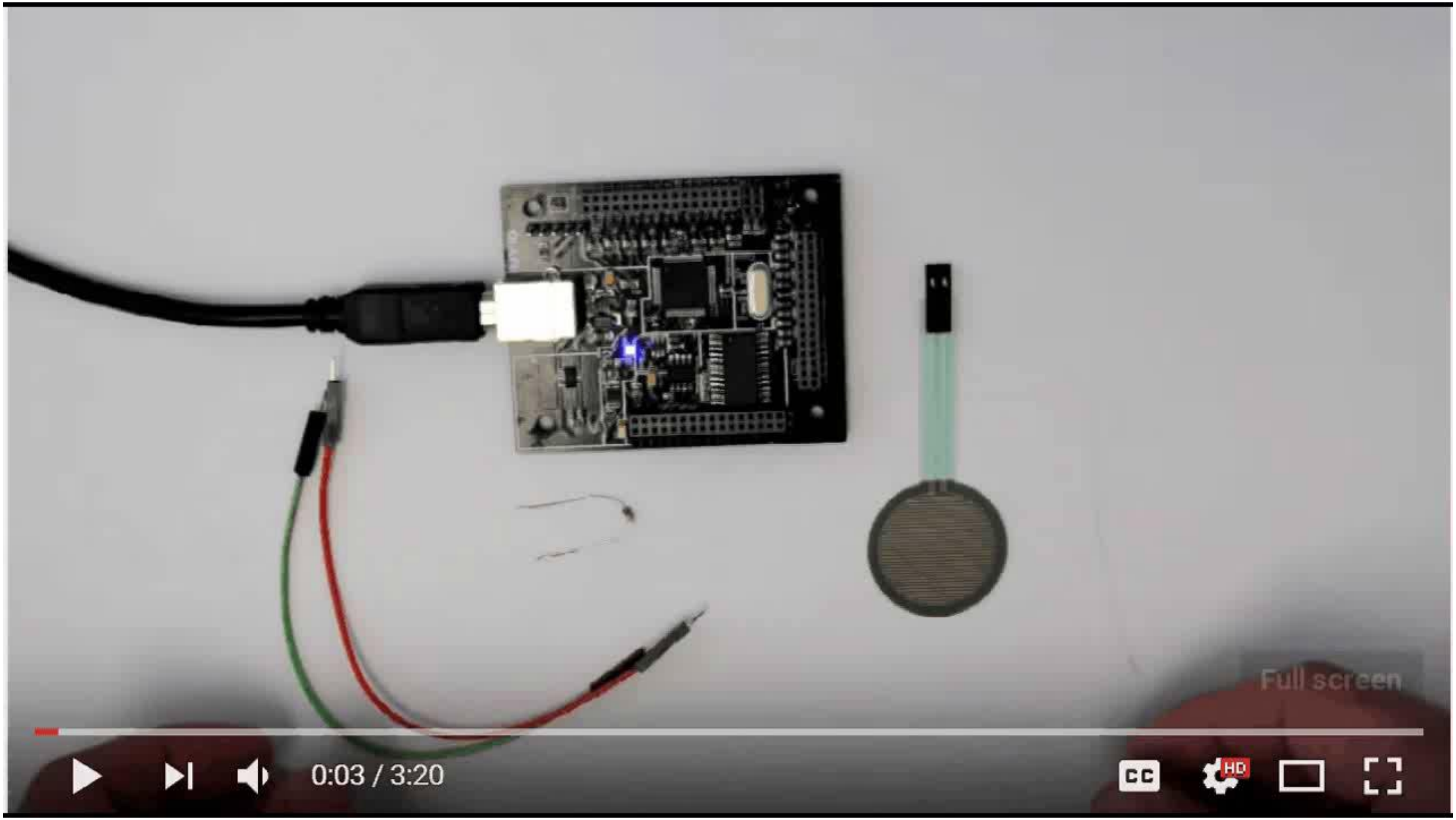
- Details of Hardware:

Force Sensing Resistor





Example of Force Sensing Resistor

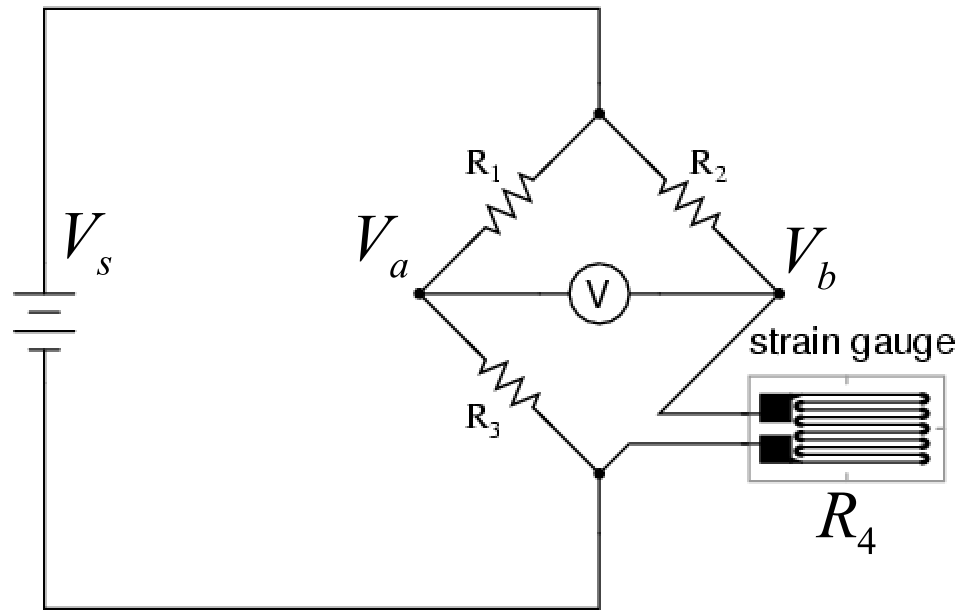


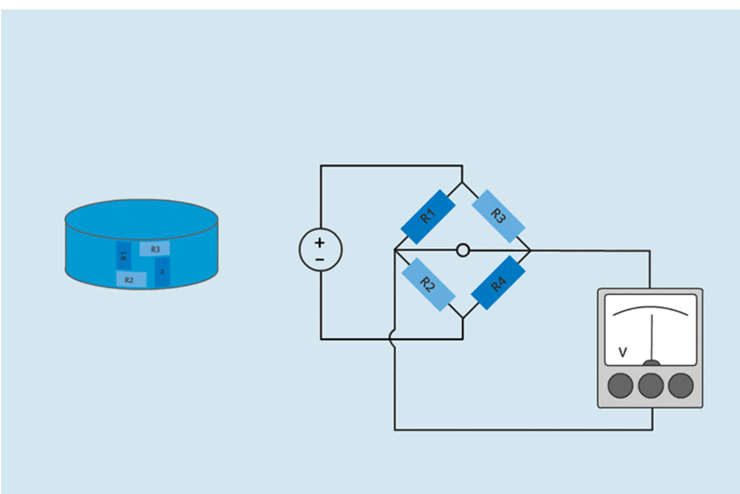
Resistance to Voltage Conversion

- Details of Circuit and Equations:

$$V_a = \frac{R_3}{R_1 + R_3} V_s$$

$$V_b = \frac{R_4}{R_2 + R_4} V_s$$





(to continue …)

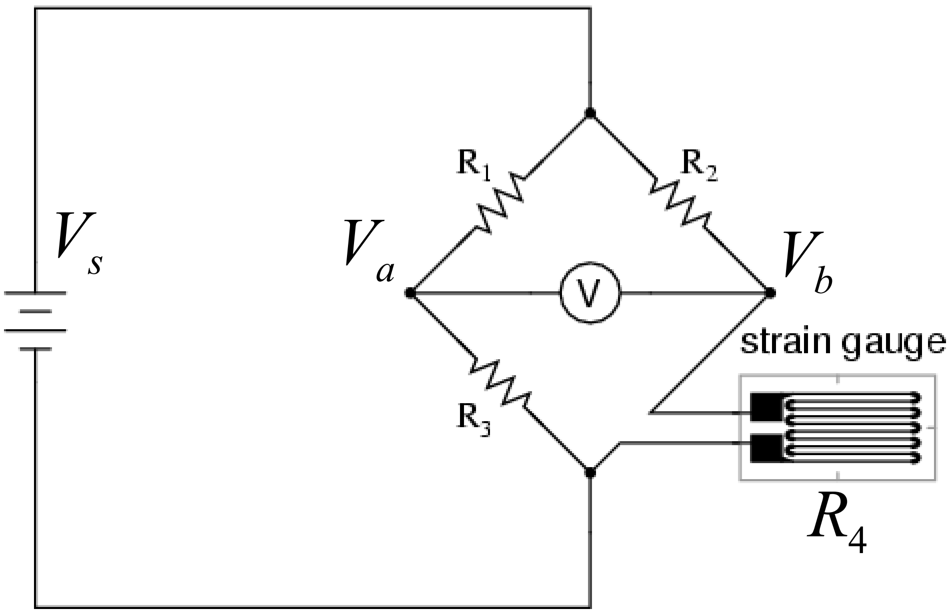
Equation of Converting Resistance to Voltage

$$V = V_a - V_b = \left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right)V_s$$

$$\frac{R_4}{R_2 + R_4} = \frac{R_3}{R_1 + R_3} - \frac{V}{V_s}$$

$$X = \frac{R_3}{R_1 + R_3} - \frac{V}{V_s}$$

$$X = \frac{R_4}{R_2 + R_4} \Rightarrow R_4 = \frac{R_2 X}{1 - X}$$

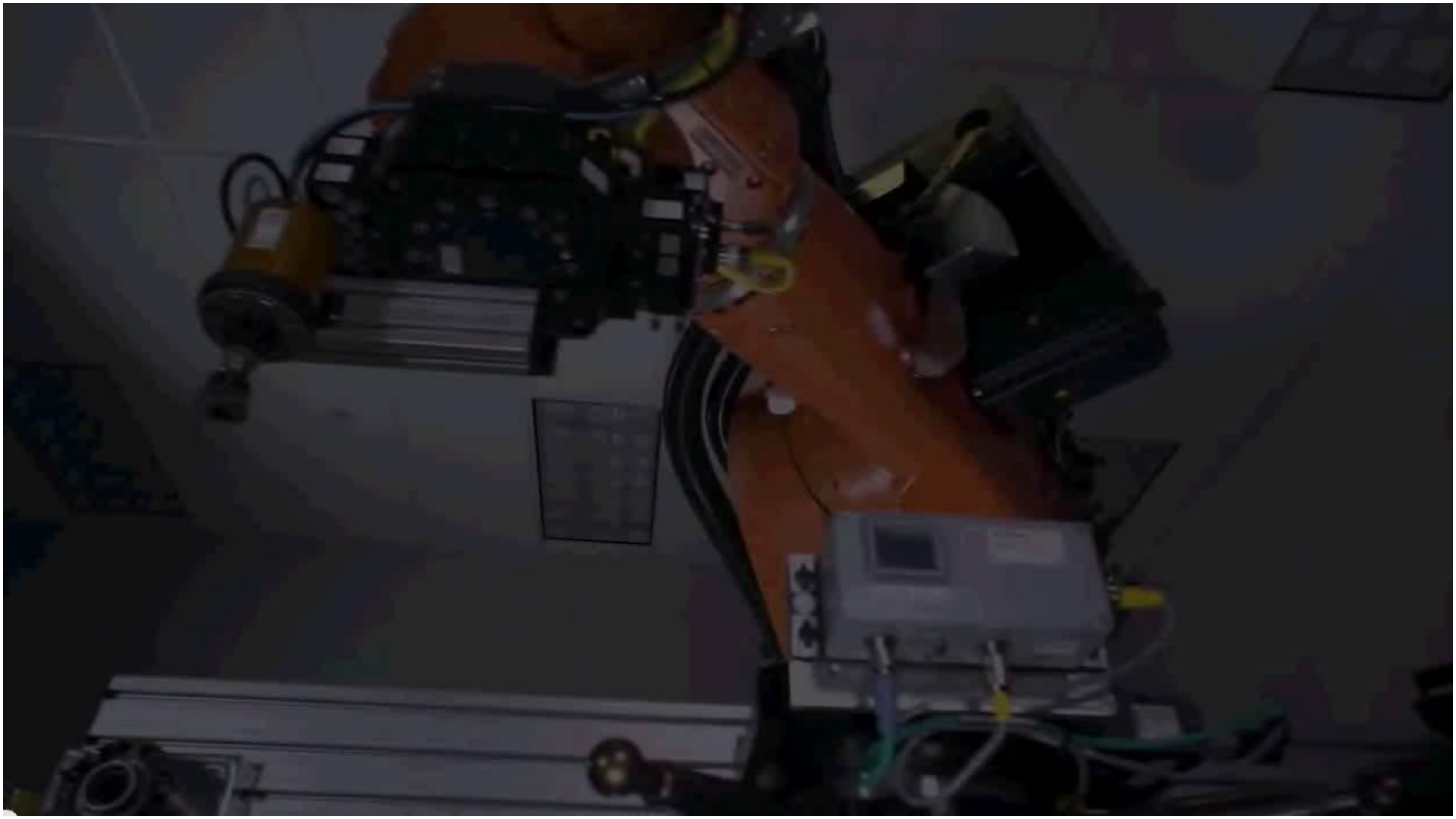


$$\Delta X = -\frac{1}{V_s}\Delta V$$

$$\Delta R_4 = \left\{\frac{R_2}{1 - X} + \frac{R_2 X}{(1 - X)^2}\right\}\Delta X$$

$$F_{measured} = K \times \Delta R_4$$

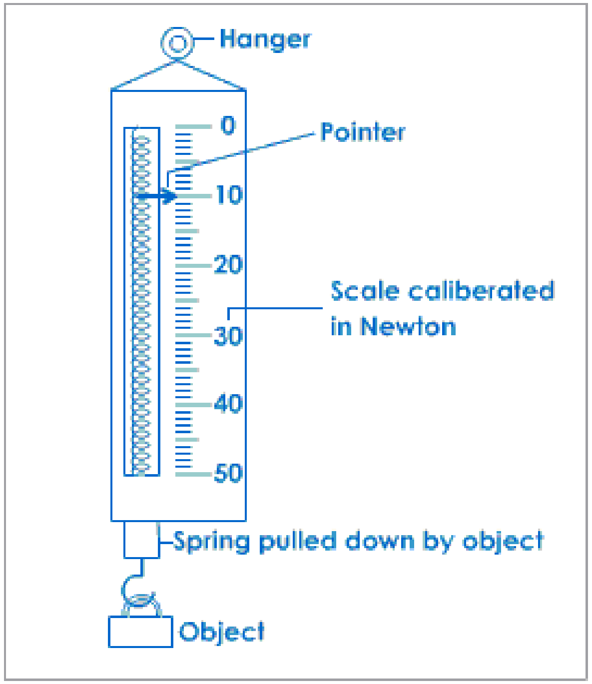
Example of Integrated Force/Torque Sensor

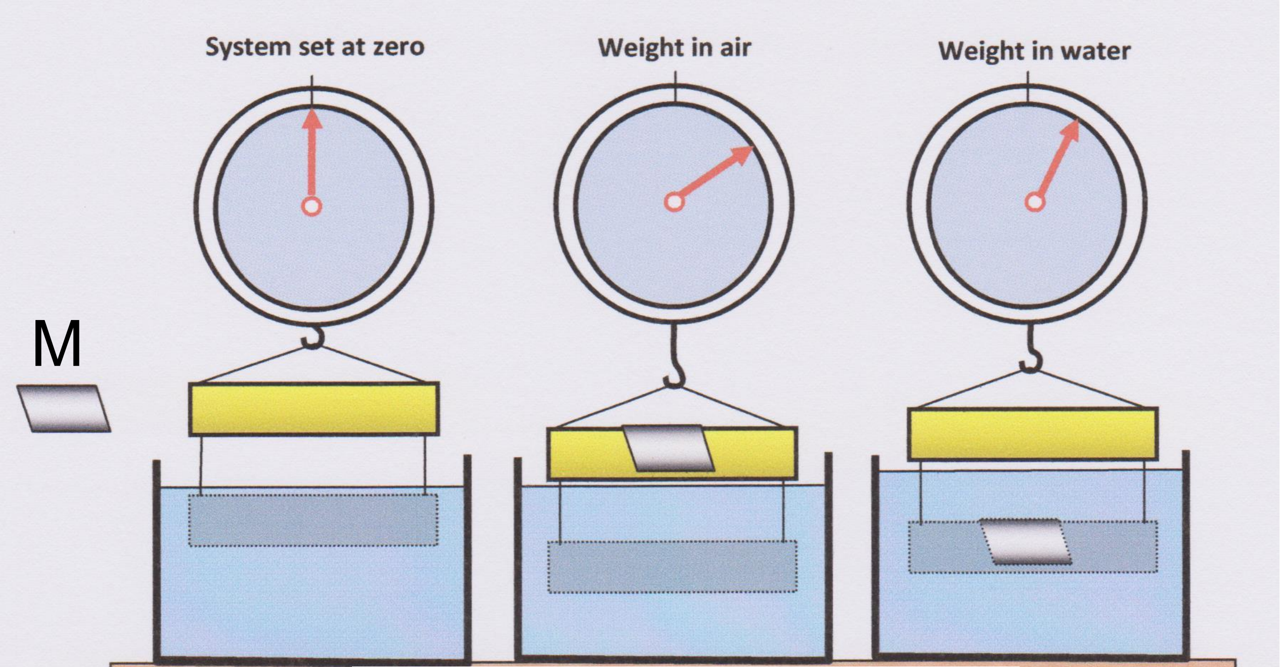


Other Example of Using Force-Displacement Conversion for the Design of Simple Device so as to Manually Measure Force …

- Applied force is proportional to the displacement of spring.

$$F = k \bullet d \qquad \Delta F = k \bullet \Delta d$$

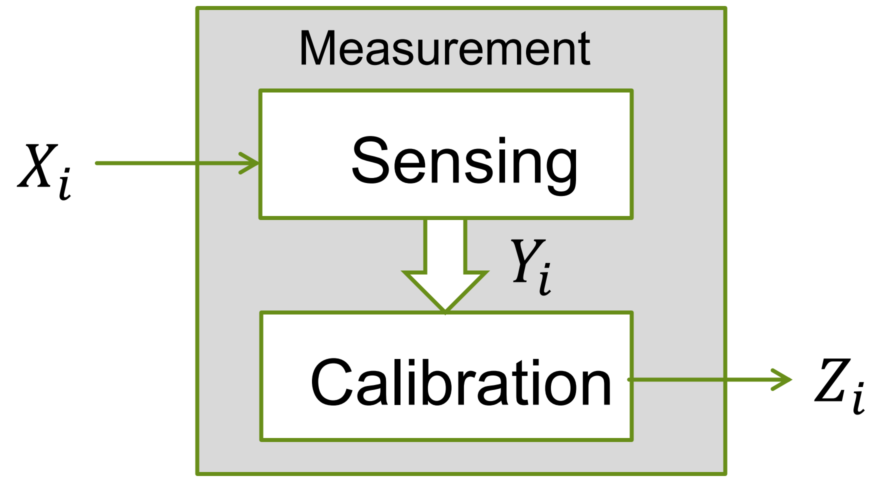


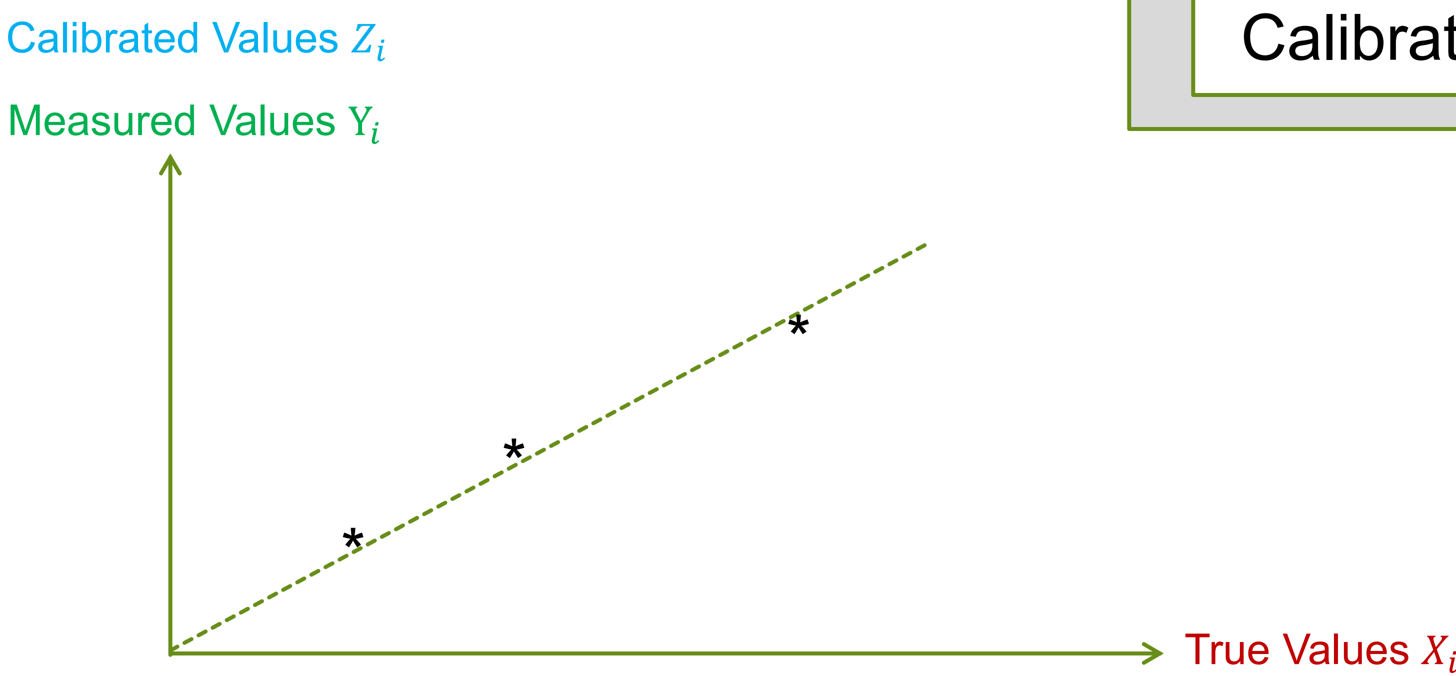


Example of Measuring Buoyant Force

Remember to Do Calibration

- Curve fitting for calibration:
  - Yi is produced by Xi
  - Zi is computed from Yi
  - Zi must be equal to Xi

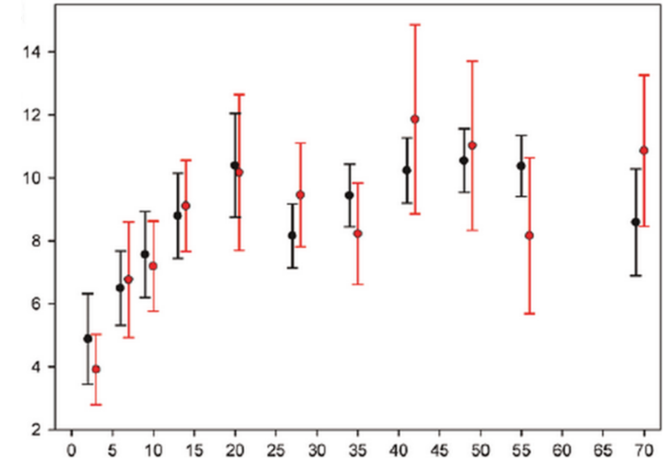


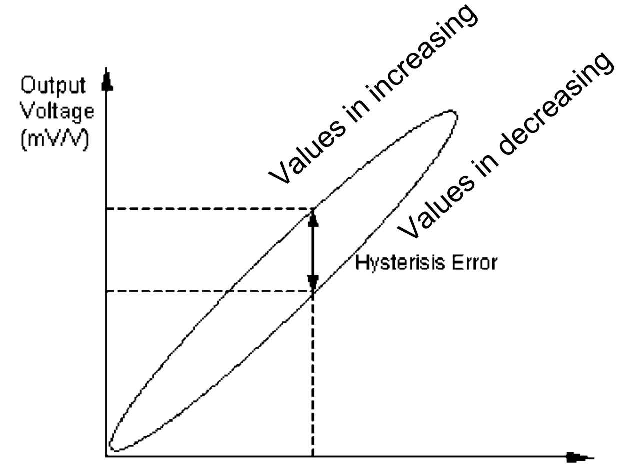


Remember to Do Error Analysis

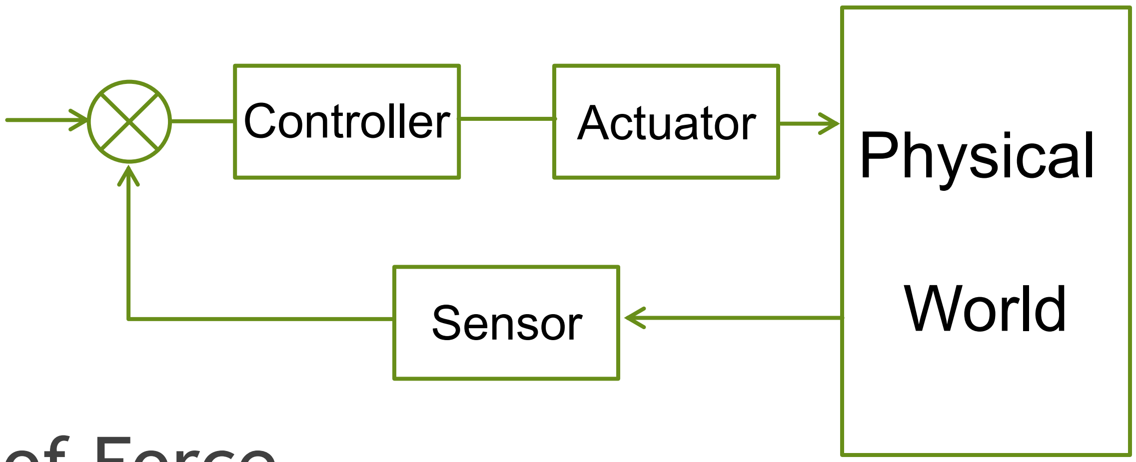
- Systematic error = mean value – true value
- Repeatability error = value with maximum error – mean value
- Accuracy = value with minimum error – mean value
- Hysteresis error = |measured value in increasing – measured value in decreasing|

For each true value, we can do error analysis

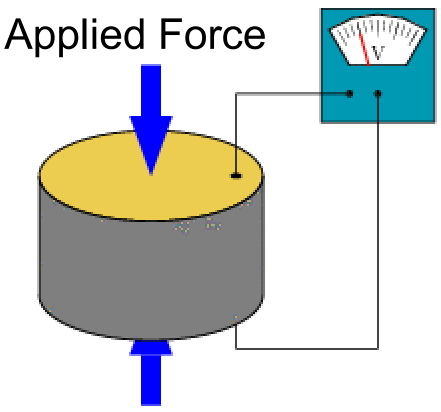




Summary



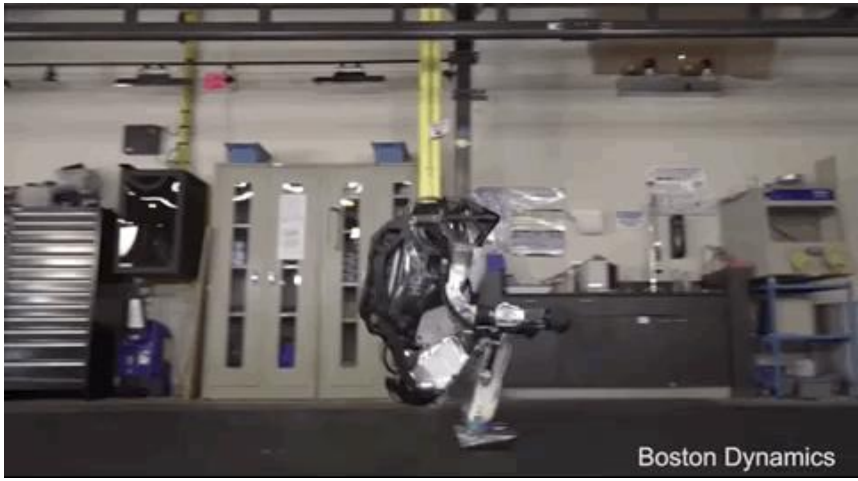
- Understanding of Force
- Computation of Force
- Measurement of Force



Piezo-electrical Materials

Outline of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



NANYANG TECHNOLOGICAL UNIVERSITY

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3 Lecture 5

MA4822

Measurement of Torque

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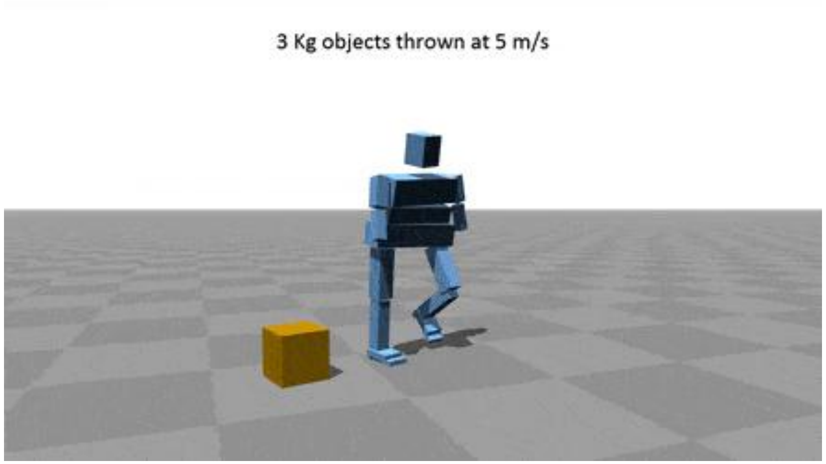
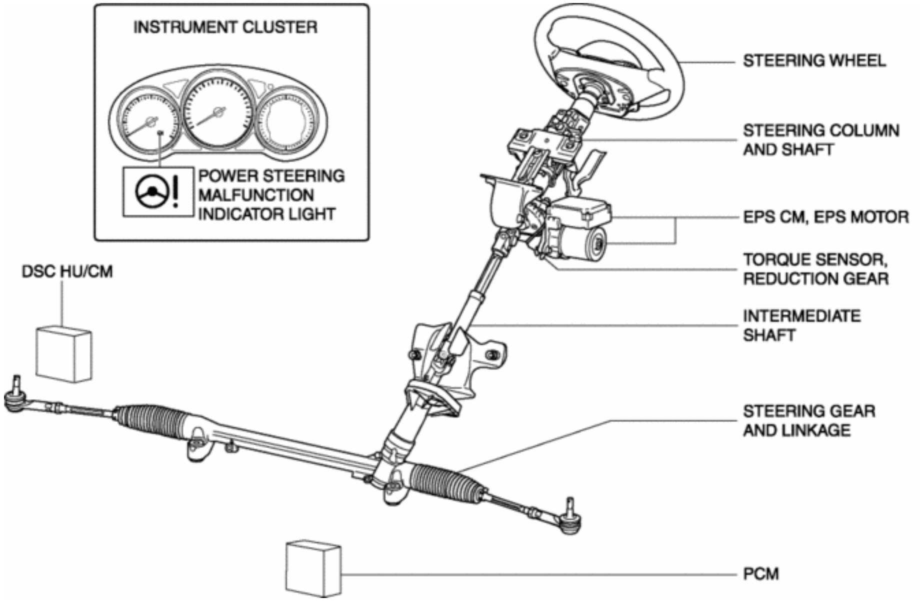
Outline

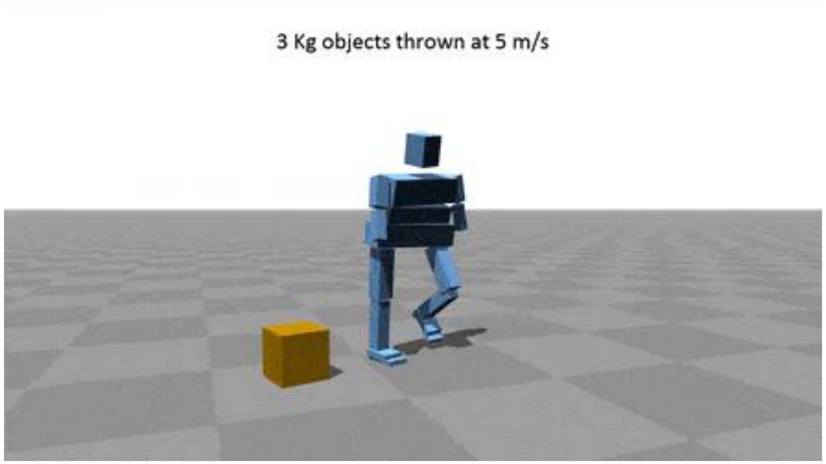


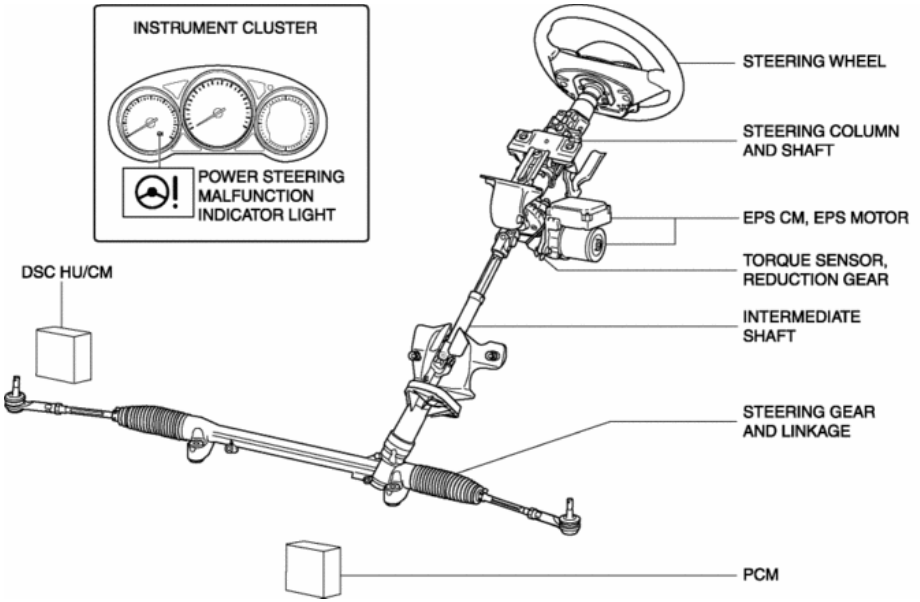
- Understanding of Torque
- Computation of Torque
- Measurement of Torque



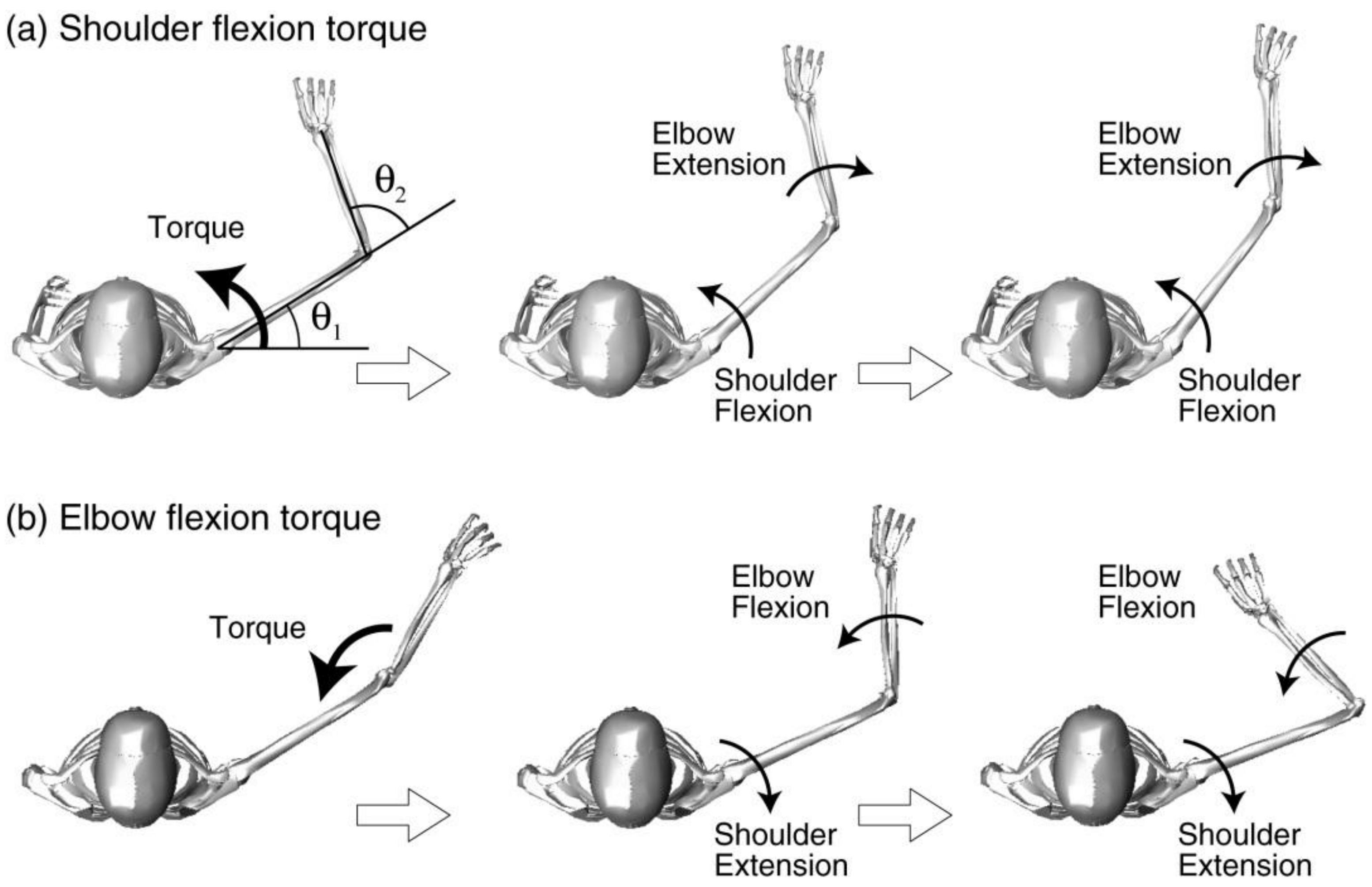
Outline

- Understanding of Torque
- Computation of Torque
- Measurement of Torque



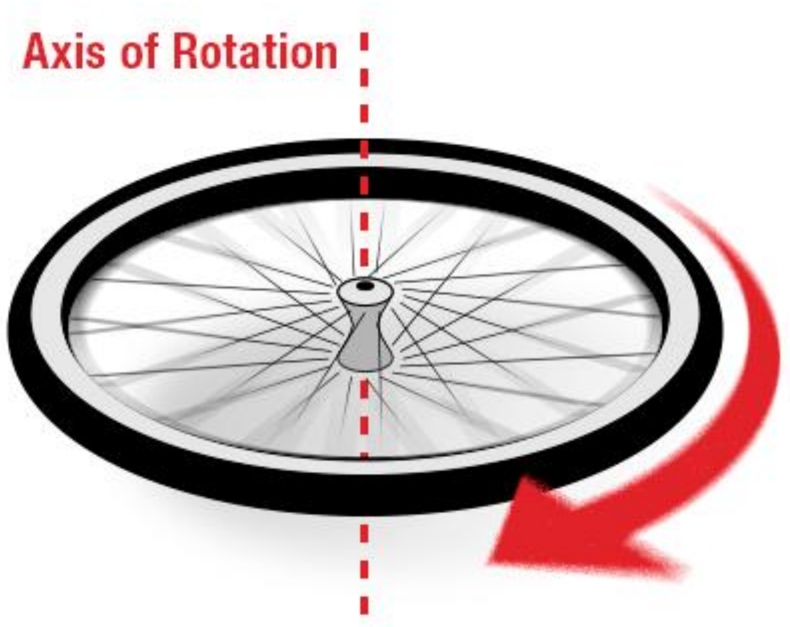


Example Showing Dominance of Angular Motion



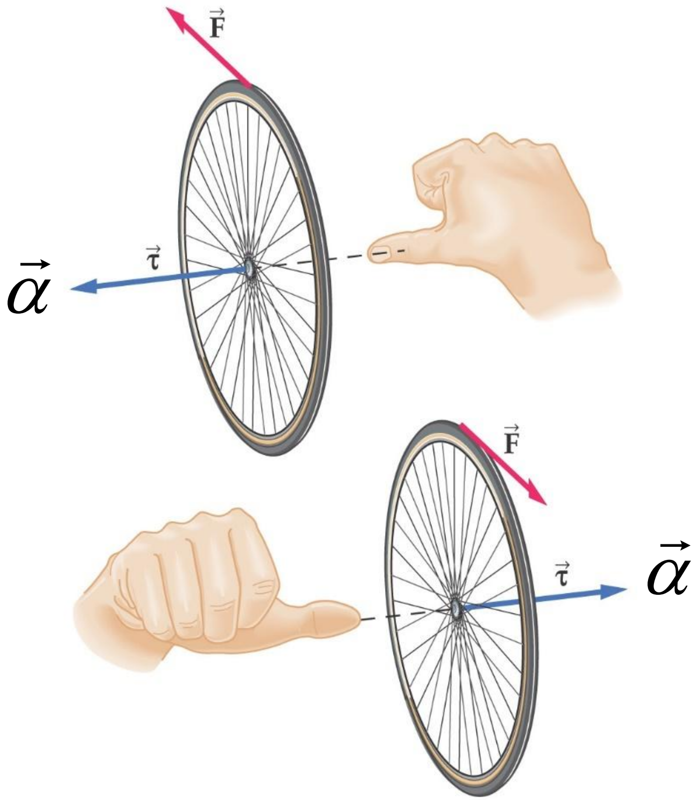
Definition of Torque

- Torque refers to the physical quantity which changes the state of angular motion of an object about an axis.



Understanding Torque (1)

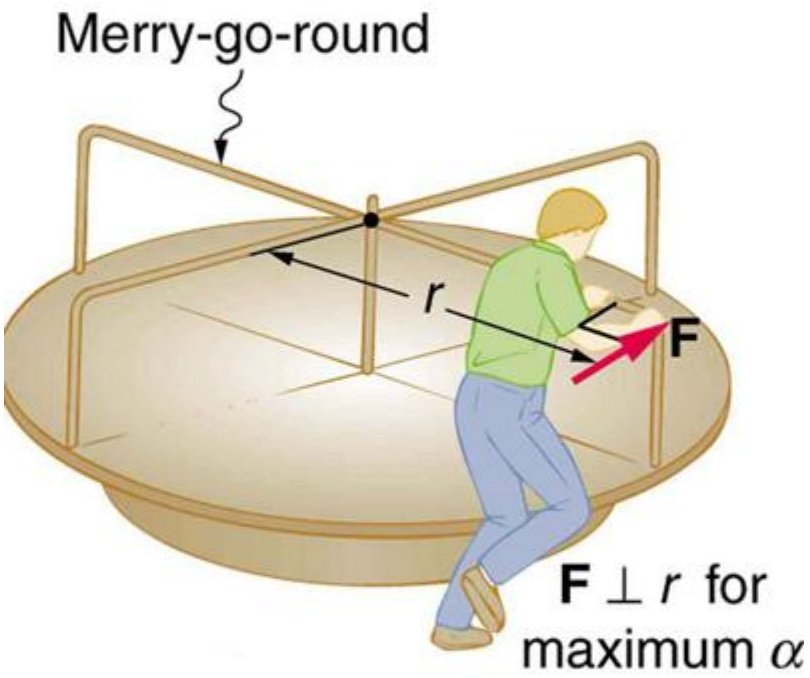
- 1. Torque causes a rotating object to accelerate or decelerate.



$$\tau = I\alpha$$

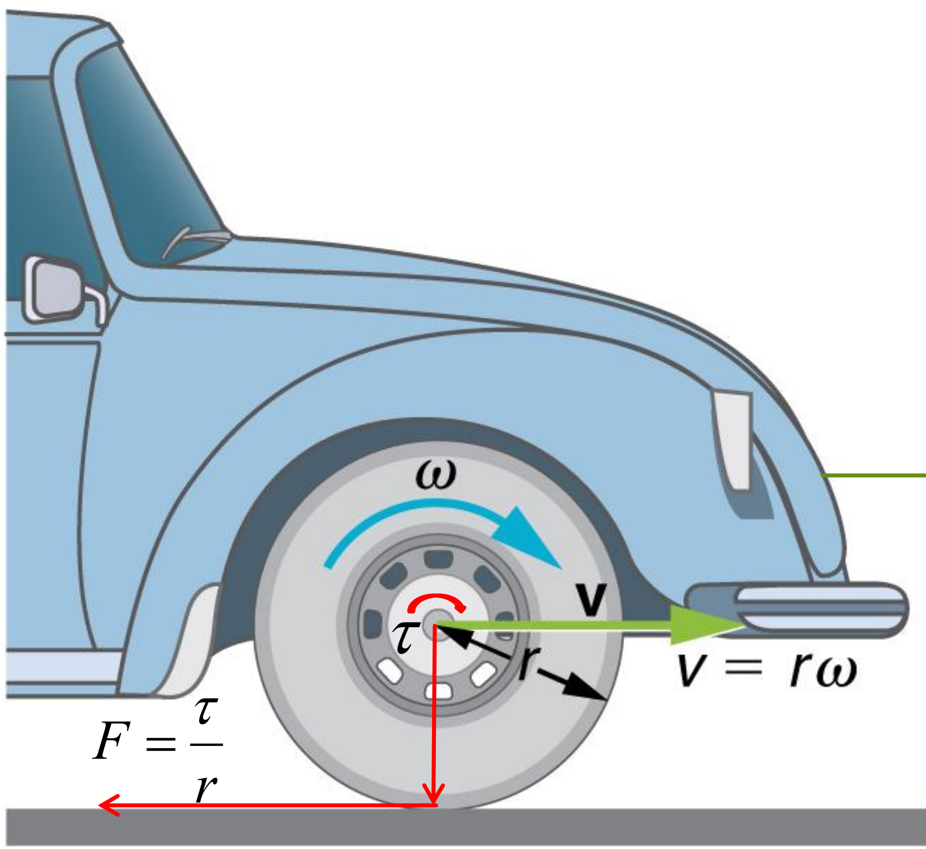
Understanding Torque (2)

- 2. An acting torque can be produced by an acting force.



Understanding Torque (3)

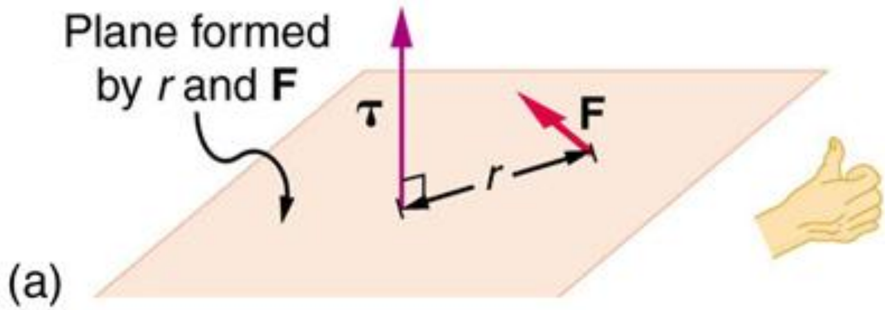
- 3. An acting torque can be transformed into an acting force.



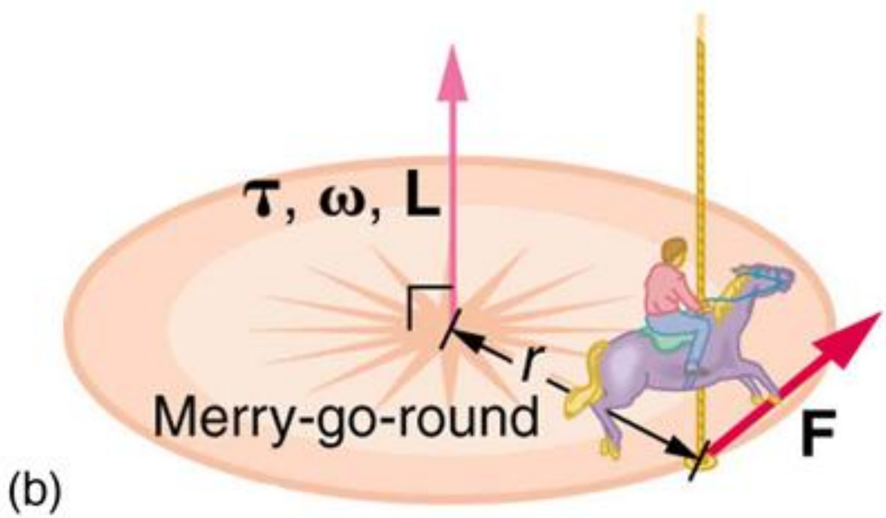
Reaction force on the car

Acting force on the ground

Example of Conversion Between Force and Torque



$$\vec{\tau} = \vec{r} \times \vec{F}$$

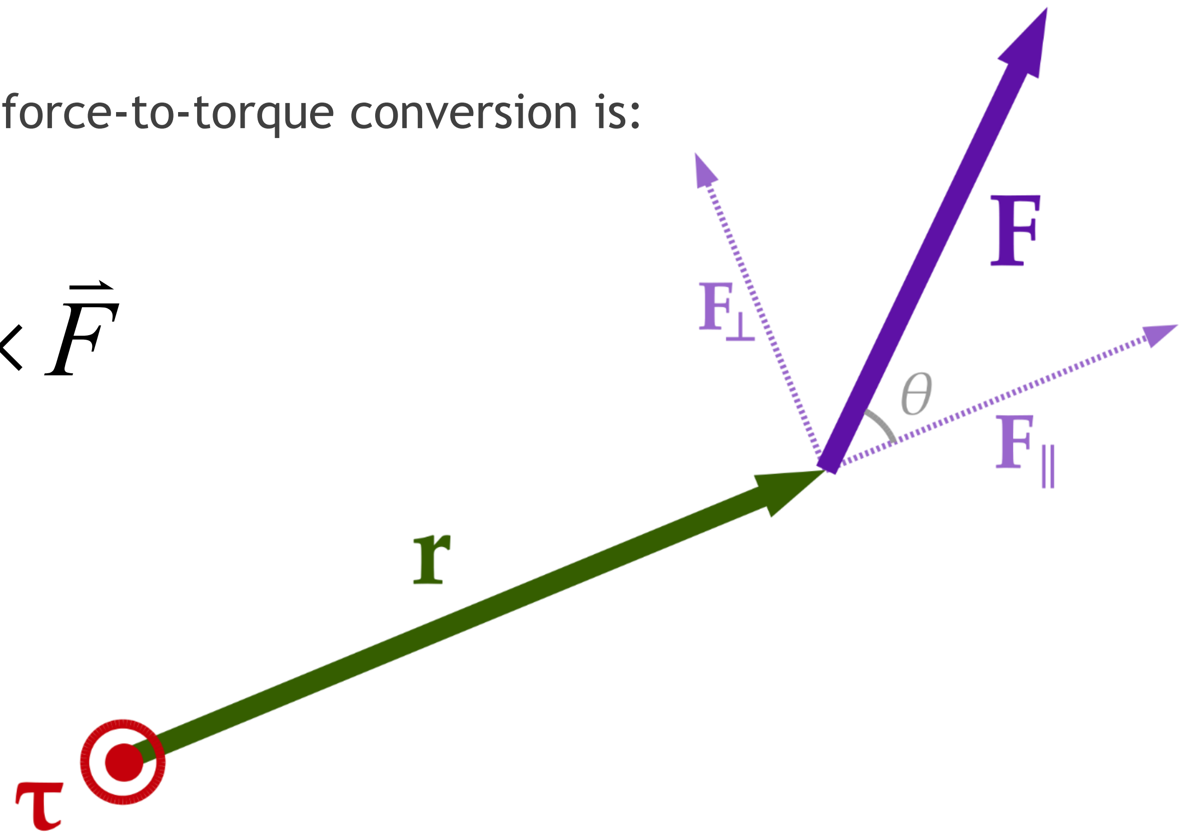


$$\vec{F} = \vec{\tau} \times \vec{r}$$

Understanding Torque (4)

- 4. Equation of force-to-torque conversion is:

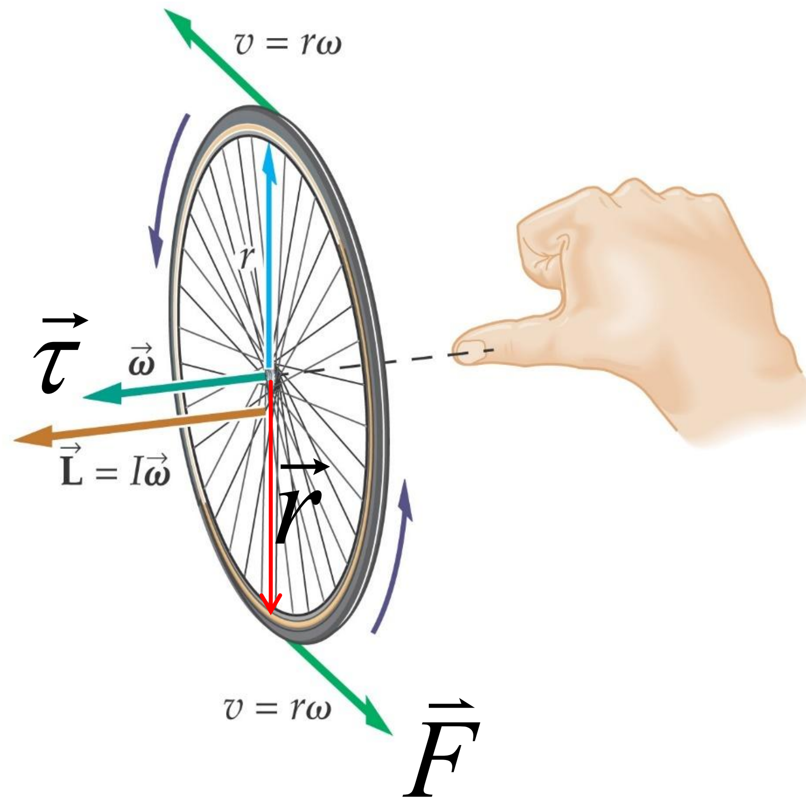
$$\vec{\tau} = \vec{r} \times \vec{F}$$



Understanding Torque (5)

- Equation of torque-to-force is:

$$\vec{F} = \vec{\tau} \times \vec{r}$$



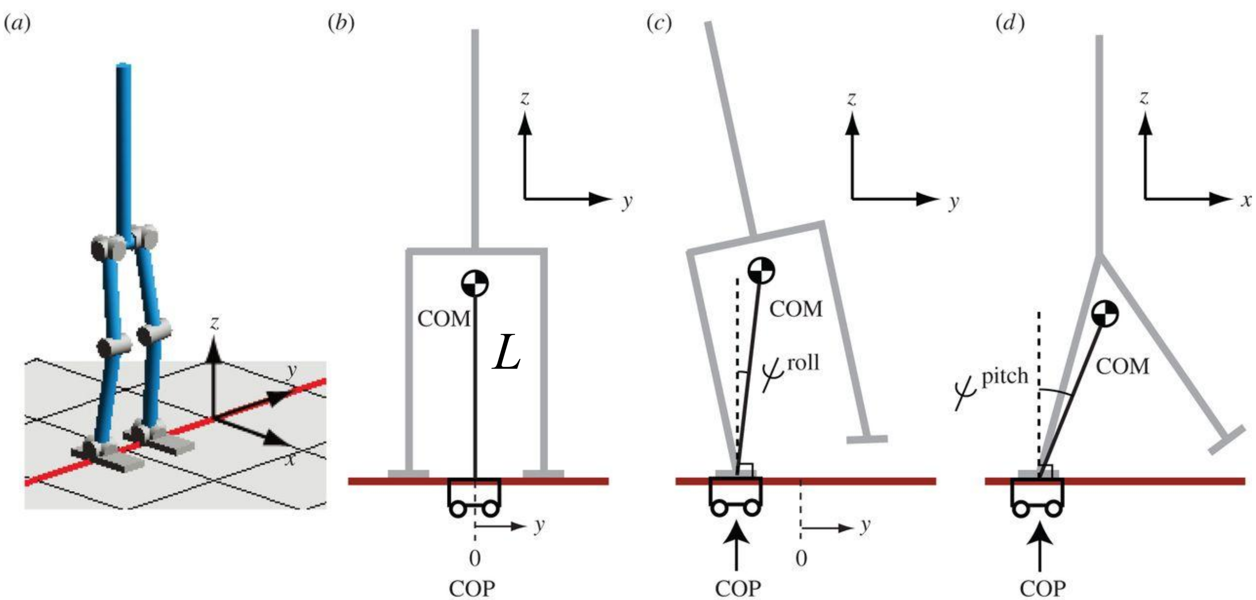
Example of Determining Torques

Torque about X axis:

$$\tau_x = M_{body} gL \sin(\varphi_{roll})$$

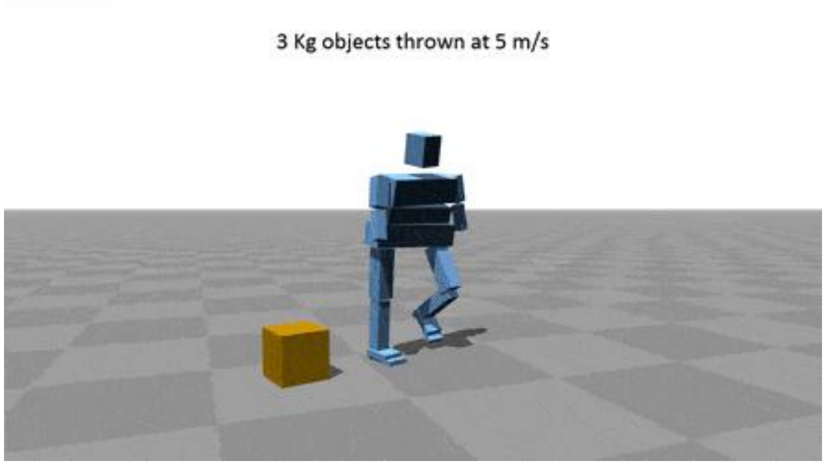
Torque about Y axis:

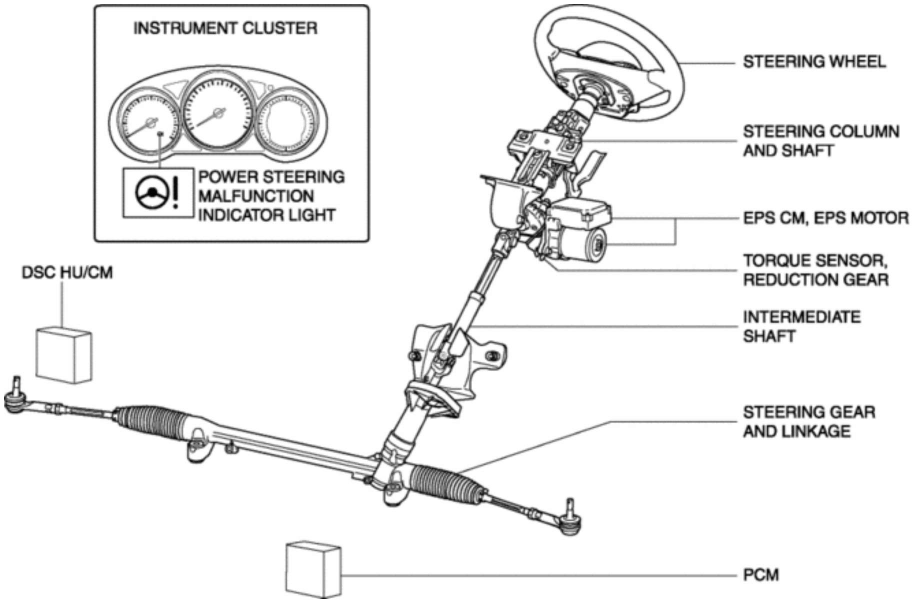
$$\tau_y = M_{body} gLsin(\varphi_{pitch})$$



Outline

- Understanding of Torque
- Computation of Torque
- Measurement of Torque

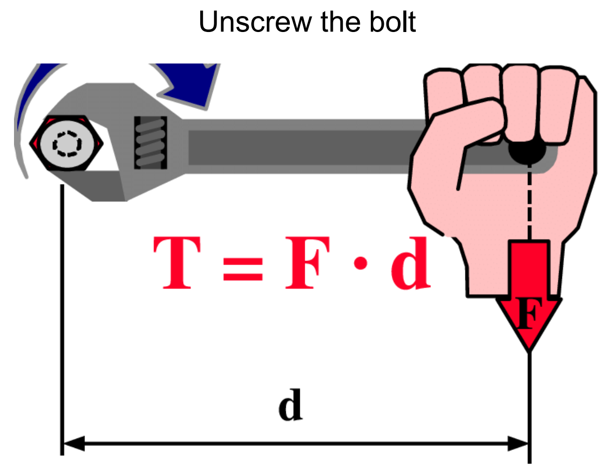




Formula of Computation

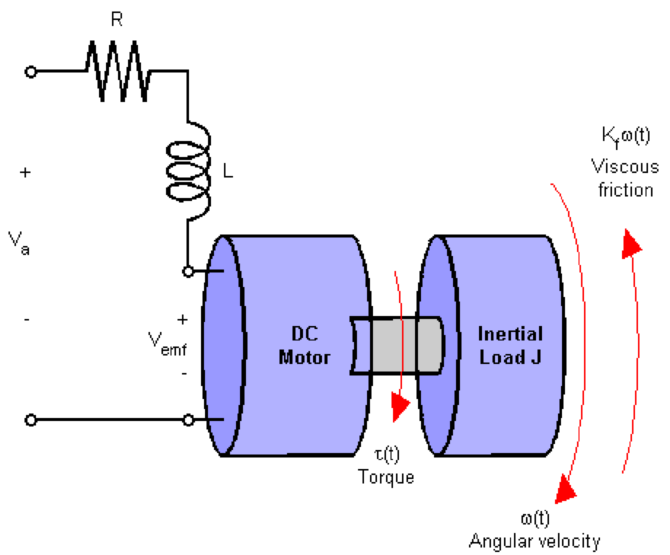
- If force is known, then:

$$\vec{\tau} = \vec{r} \times \vec{F}$$



- If angular acceleration is known, then:

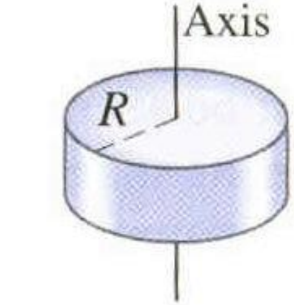
$$\vec{\tau} = I\vec{\alpha}$$



How to determine the moments of inertia?

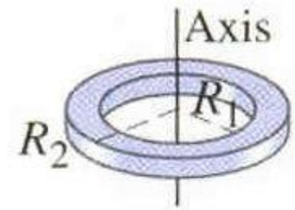
Formula of Moment of Inertia

(c) **Solid cylinder,** radius R — Through center

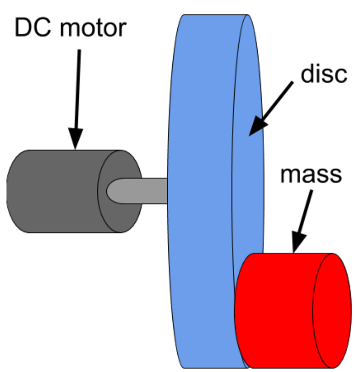


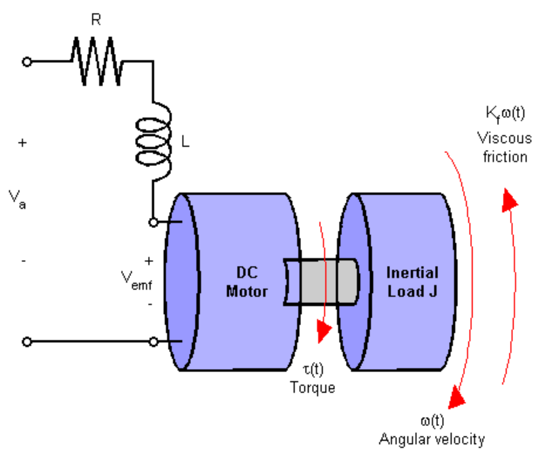
$$\frac{1}{2}MR^2$$

(d) **Hollow cylinder,** inner radius R_1 outer radius R_2 — Through center

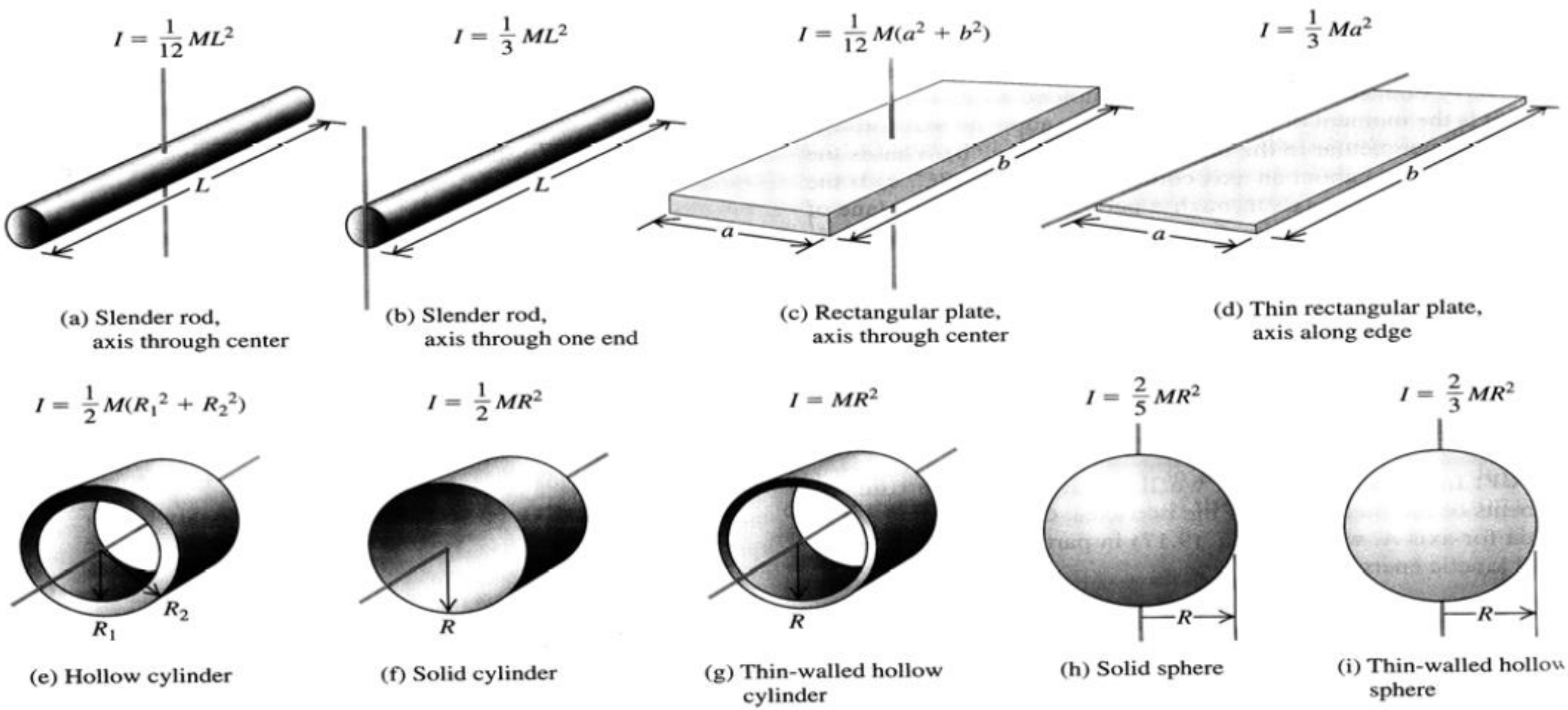


$$\frac{1}{2}M(R_1^2 + R_2^2)$$

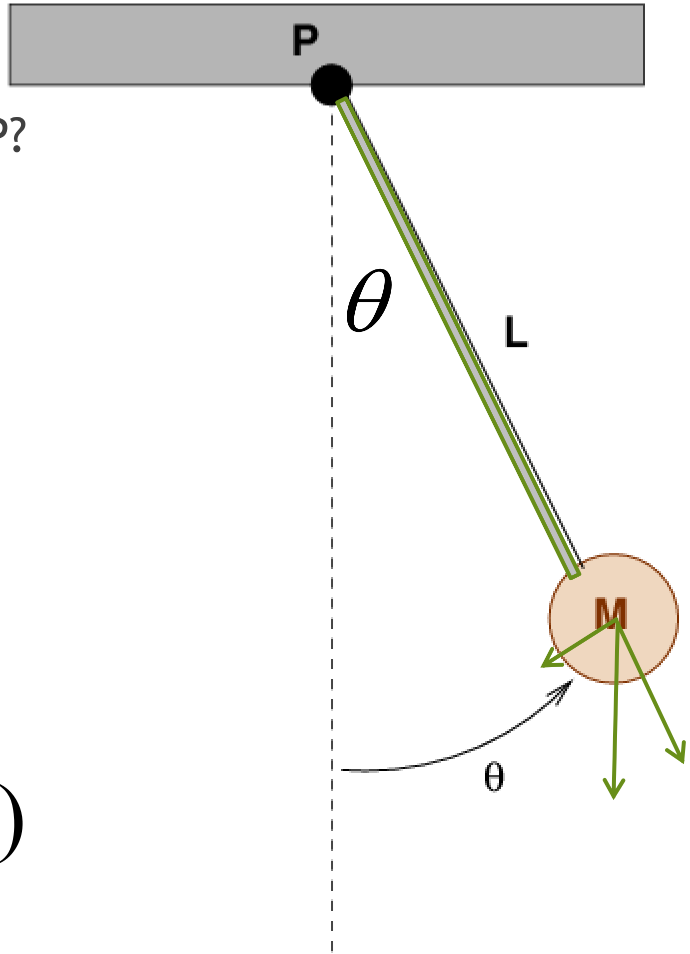




More Formula of Moment of Inertia



Example 1 of Computing Torque



- What is the torque acting at joint P?
- Answer:

$$F = mg\sin(\theta)$$

$$\tau = L \times F = Lmg\sin(\theta)$$

Example 2 of Computing Torque

- As shown in the figure, joint C is a passive joint while joint O is an actuated joint. What should be the torque actuated on joint O in order to maintain such static configuration?

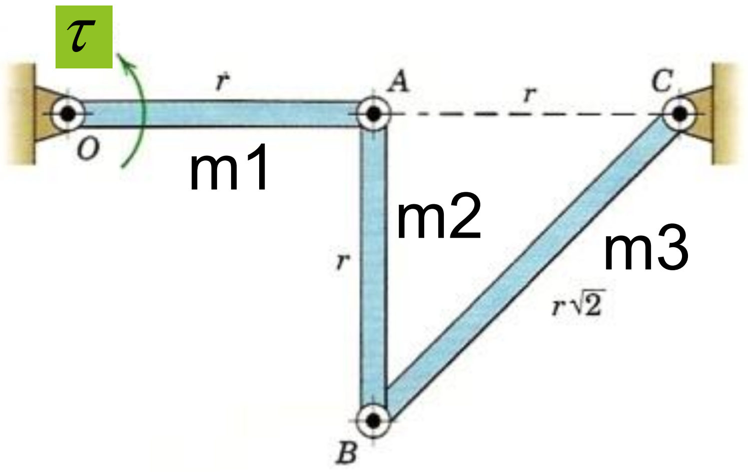
- Answer:

$$\tau_1 = m_1 g \frac{r}{2}$$

$$\tau_2 = m_2 g \bullet r$$

$$\tau_3 = \frac{1}{2} m_3 g \bullet r$$

$$\tau = \tau_1 + \tau_2 + \tau_3$$



Example 3 of Computing Torque

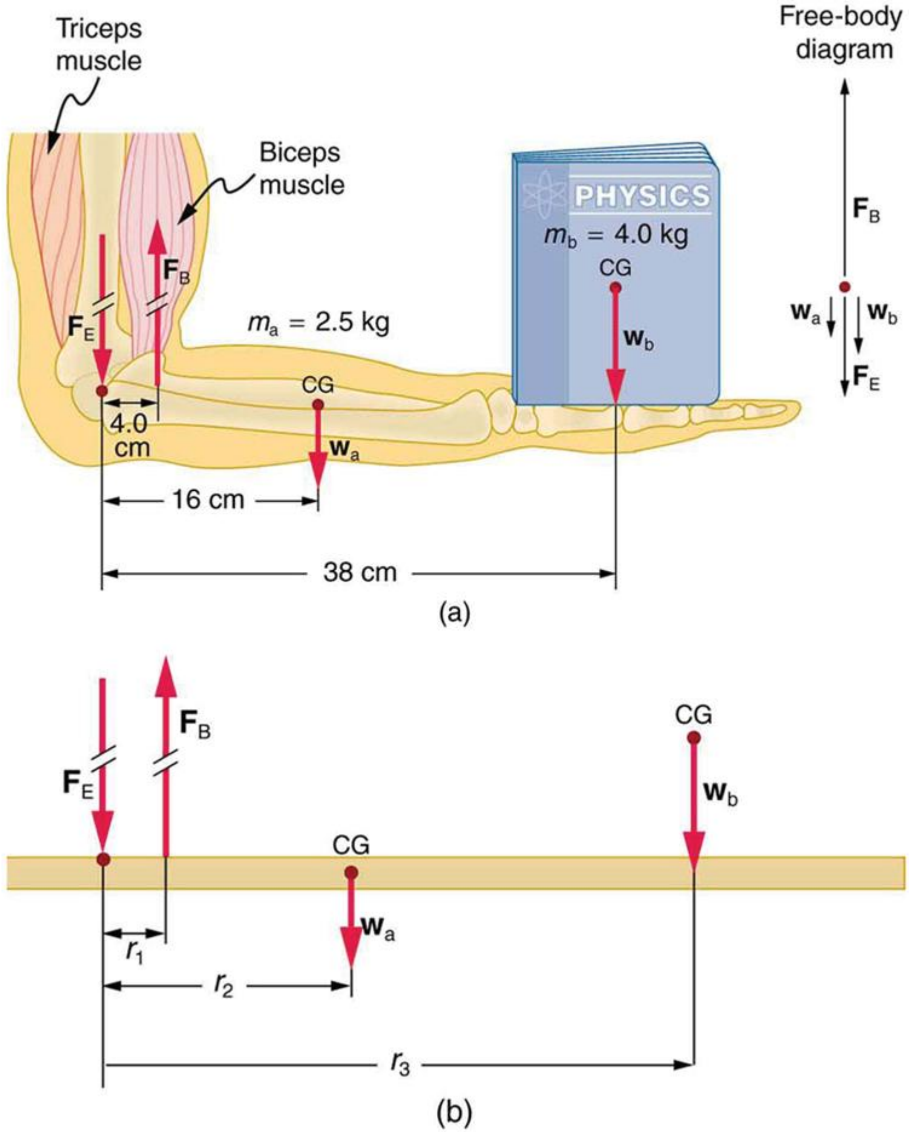
- As shown in the figure, what is the torque at elbow joint? What is the force from biceps muscle?

- Answer:

$$\tau = w_b \times 0.38 + w_a \times 0.16 - F_B \times 0.04 = 0$$

$$F_B = \frac{4.0 \times 9.81 \times 0.38 + 2.5 \times 9.81 \times 0.16}{0.04}$$

$$F_B = 470.88N$$



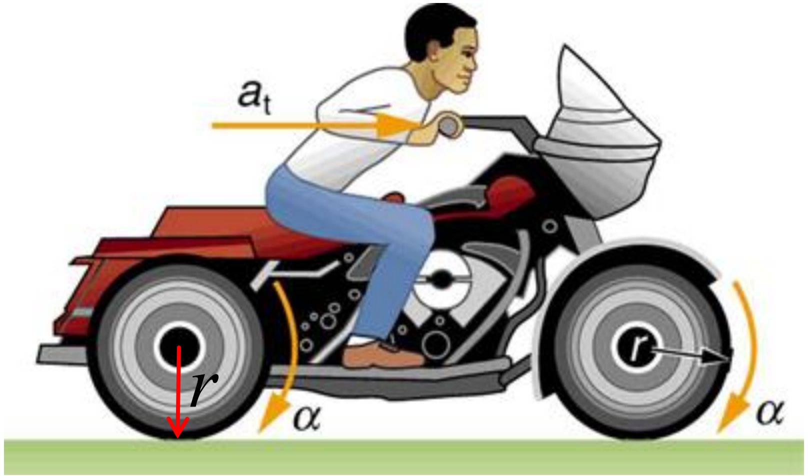
Example 4 of Computing Torque

- As shown in the figure below, the total mass of the motorcycle is 200.0 kg. What should be the torque acting at the rear wheel in order to achieve the linear acceleration of 10.0 m/s² if the radius of the wheel is 50.0 cm?

- Answer:

$$F = m \times a = 200.0 \times 10.0 = 2000N$$

$$\tau = r \times F = 2000 \times 0.5 = 1000\text{N.m}$$



Example 5 of Computing Torque

- q1 is link 1's angular position with respect to Z axis. q2 is link 2's angular position with respect to link 1. q3 is link 3's angular position with respect to link 2. If the robot stands at rest in the posture of squat, what is the torque from joint 3?

- Answer:

Orientation of Link 3 with respect to Z axis:

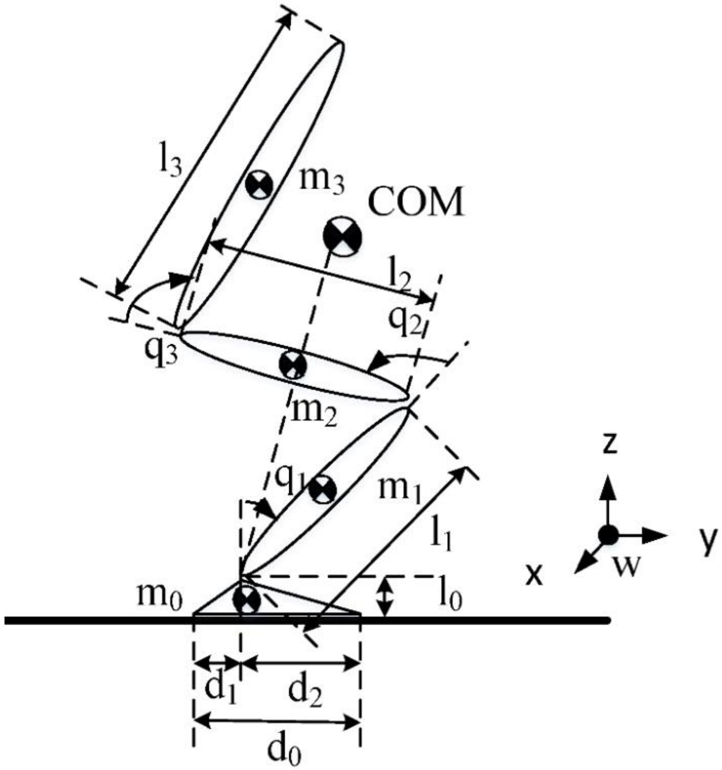
$$\theta_3 = q_1 + q_2 + q_3$$

Moment Arm of Link 3 to Joint 3's Axis :

$$a_3 = \frac{l_3}{2}\sin(\theta_3)$$

Torque from Joint 3:

$$\tau_3 = \frac{l_3}{2}\sin(\theta_3) \times m_3 g$$



Posture of Squat

Example 6 of Computing Torque

- q1 is link 1's angular position with respect to Z axis. q2 is link 2's angular position with respect to link 1. q3 is link 3's angular position with respect to link 2. If the robot stands at rest in the posture of squat, what is the torque from joint 2?

- Answer: $\theta_2 = q_1 + q_2$

$$\theta_3 = q_1 + q_2 + q_3$$

  - Moment Arm of Mass 3 to Joint 2's Axis:

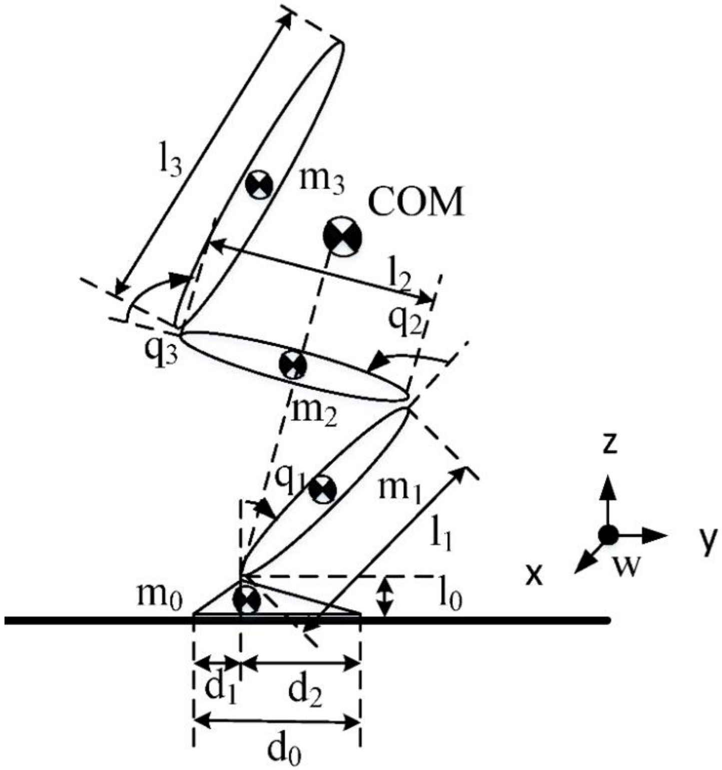
$$a_3 = l_2 \sin(\theta_2) - \frac{l_3}{2}\sin(\theta_3)$$

  - Moment Arm of Mass 2 to Joint 2's Axis:

$$a_2 = \frac{l_2}{2}\sin(\theta_2)$$

  - Torque from Joint 2:

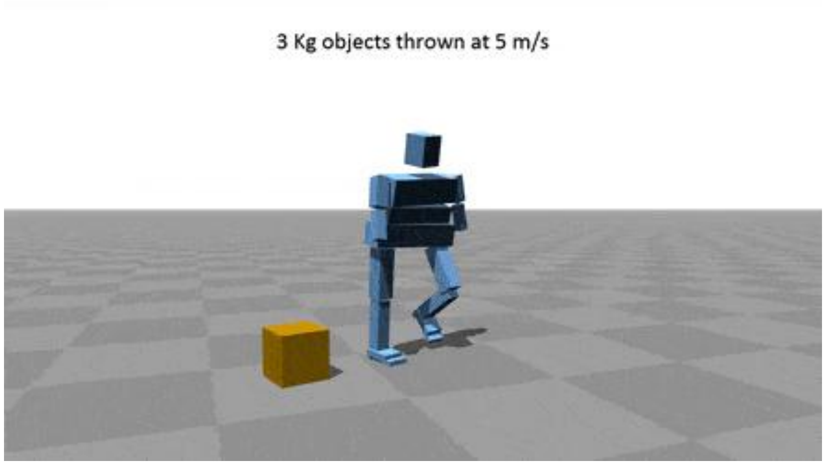
$$\tau_2 = a_2 \times m_2 g + a_3 \times m_3 g$$

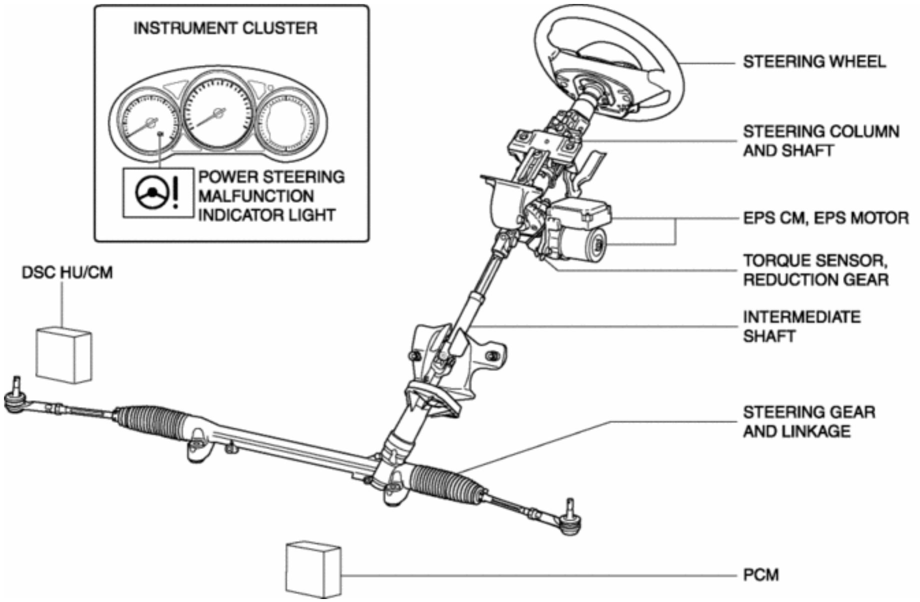


Posture of Squat

Outline

- Understanding of Torque
- Computation of Torque
- Measurement of Torque





Applications in Automobiles …

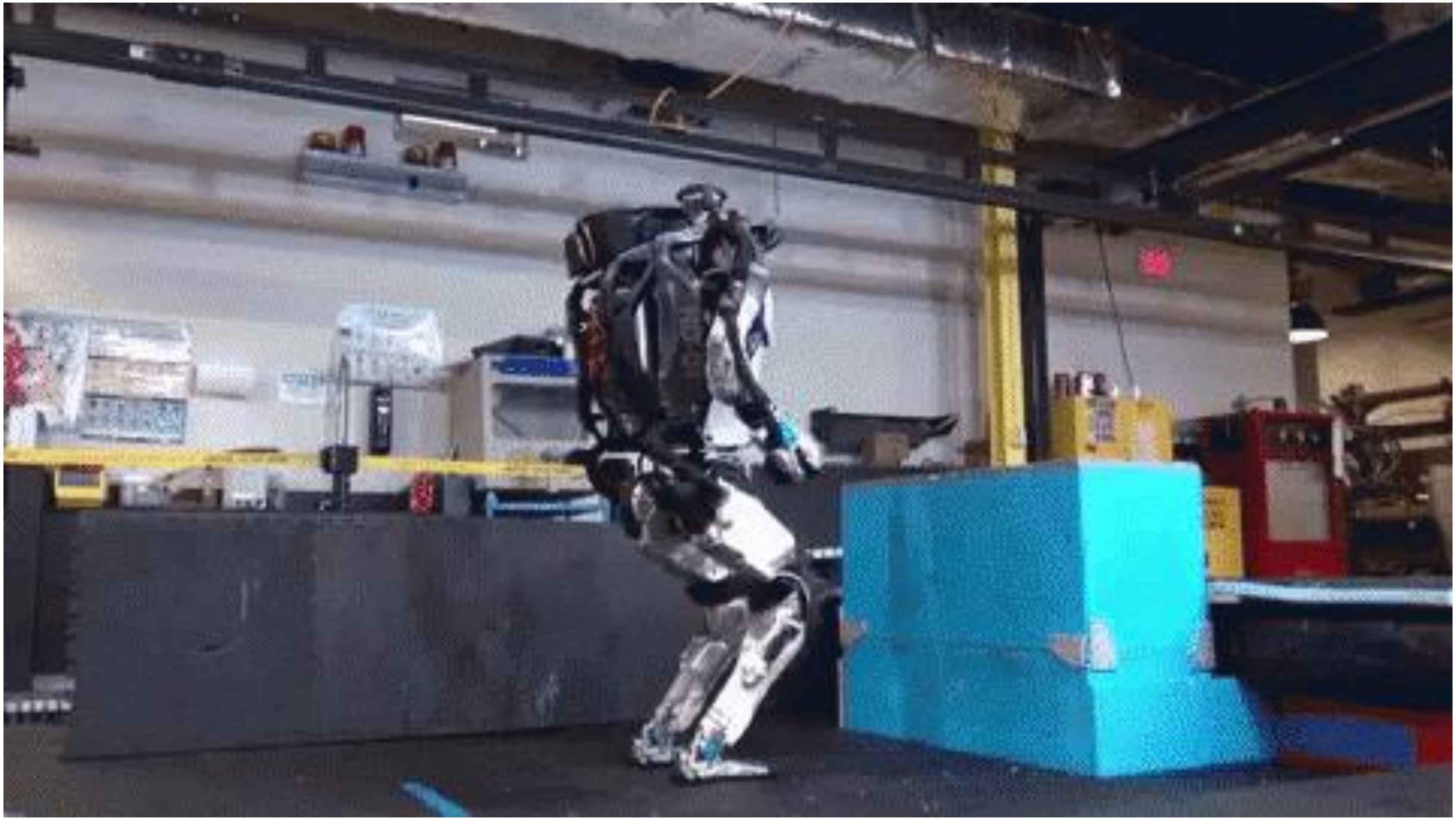
Steer by Wire with Torque Feedback

Applications in Automobiles …

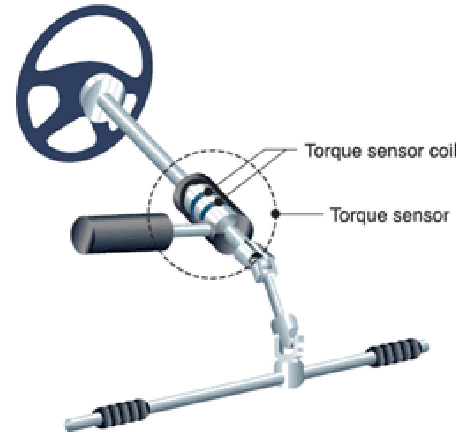
Transmission with Torque Feedback Control

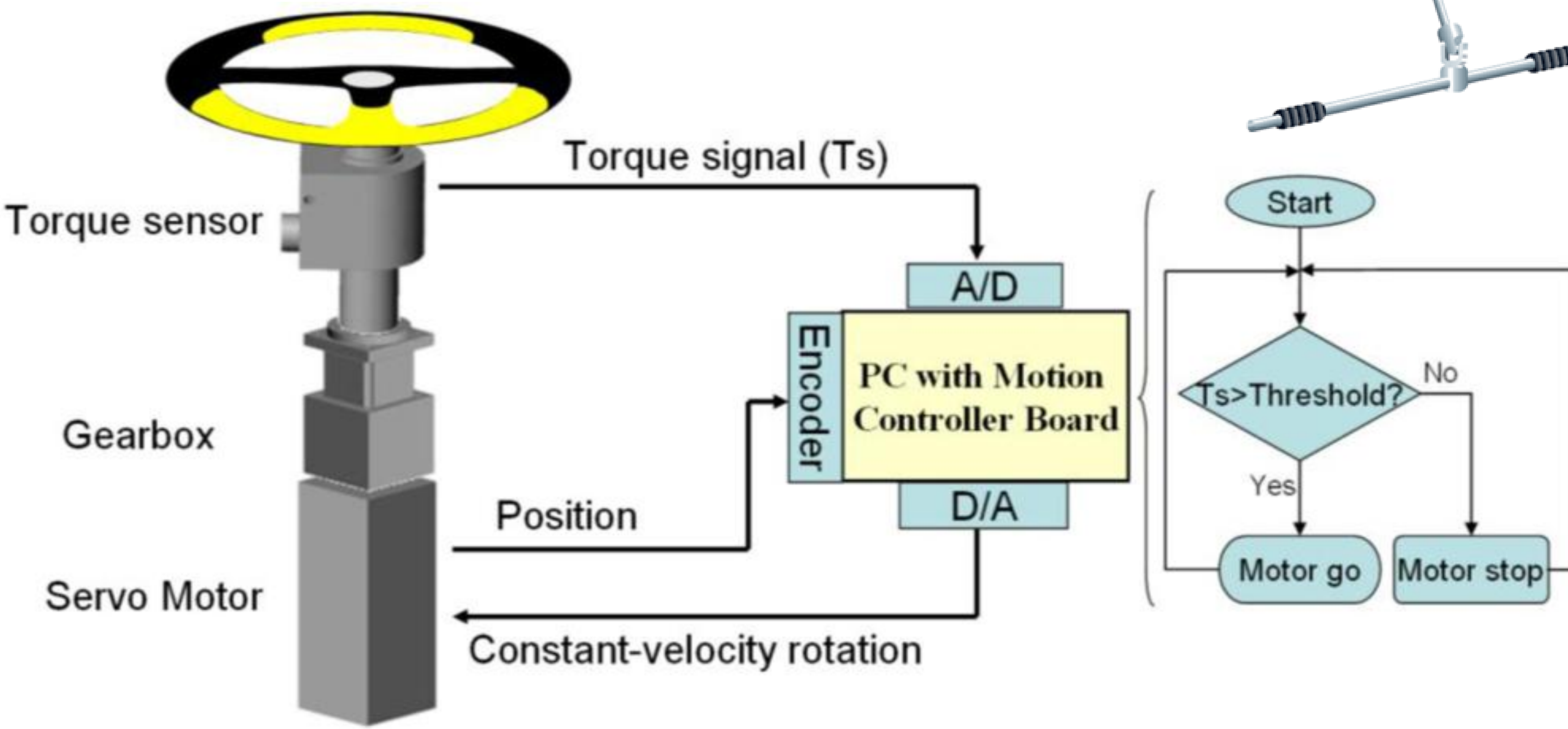
Applications in Robotics …

Applications in Robotics …



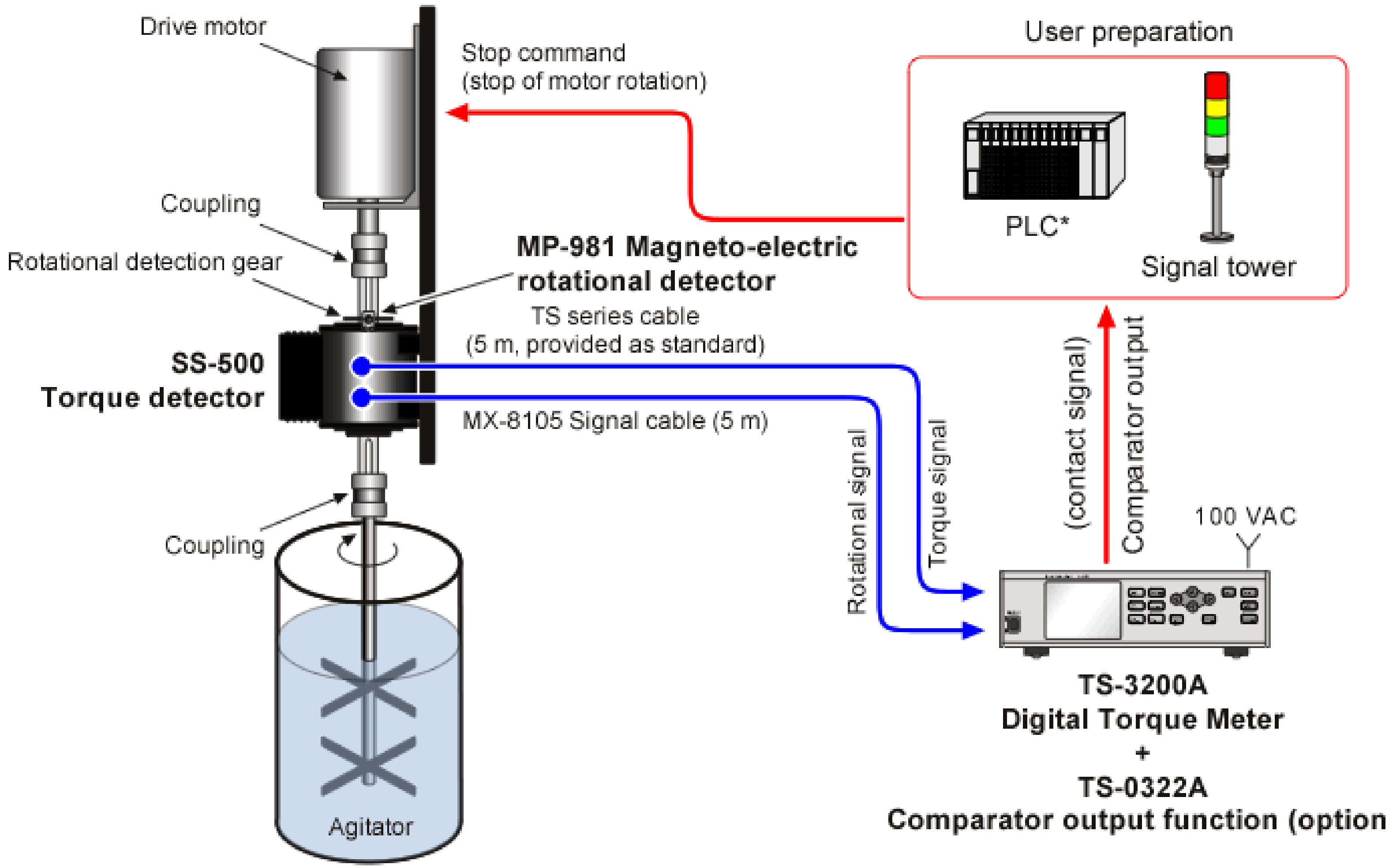
Torque Measurement in Automobile



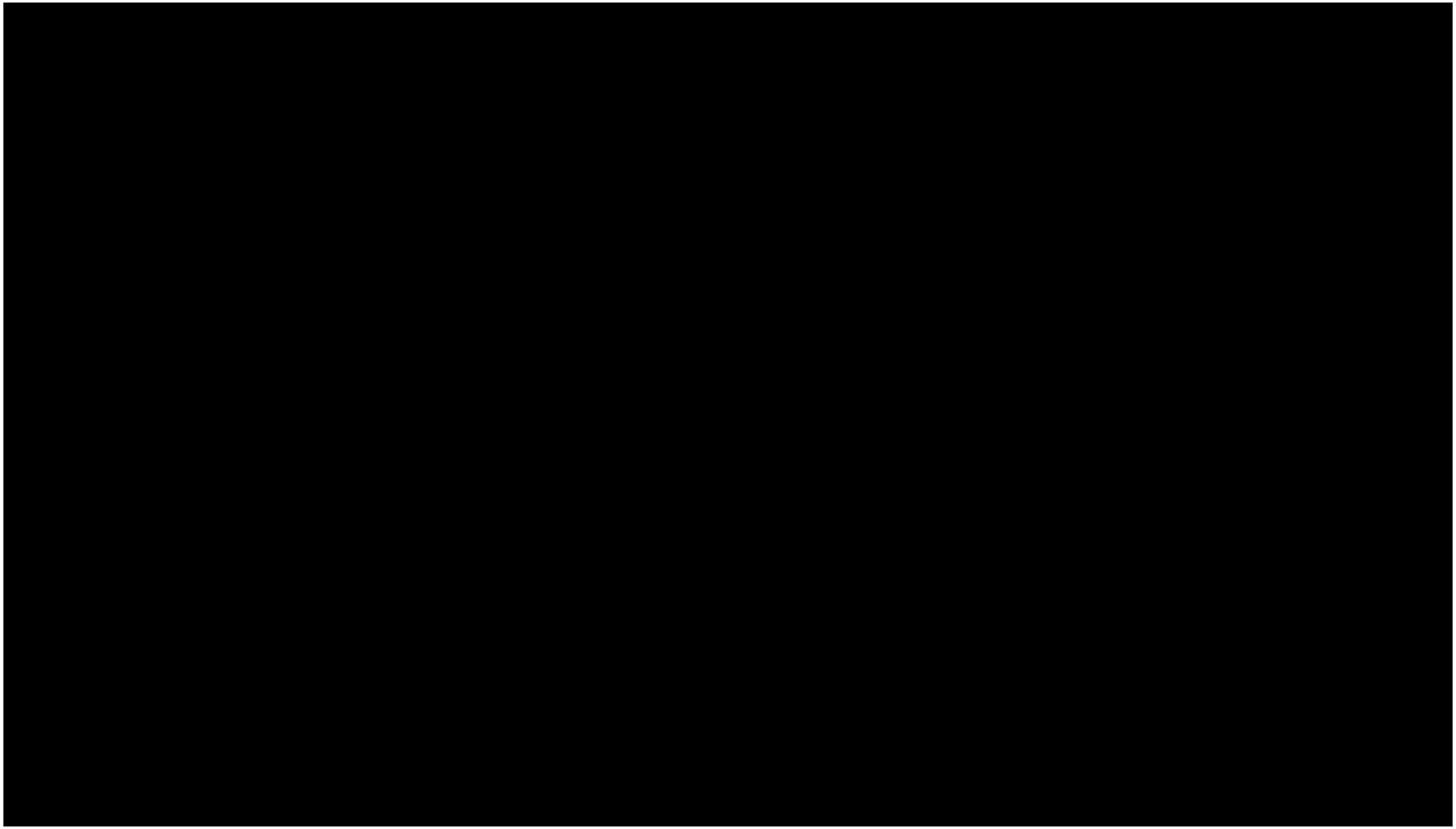


(a) Steering subsystem and servo motor control (b) Algorithm of controller

Torque Measurement in Industry



Torque Measurement in Robotics



Example of Torque Sensors

Force/Torque Sensors

Principles of Measurement

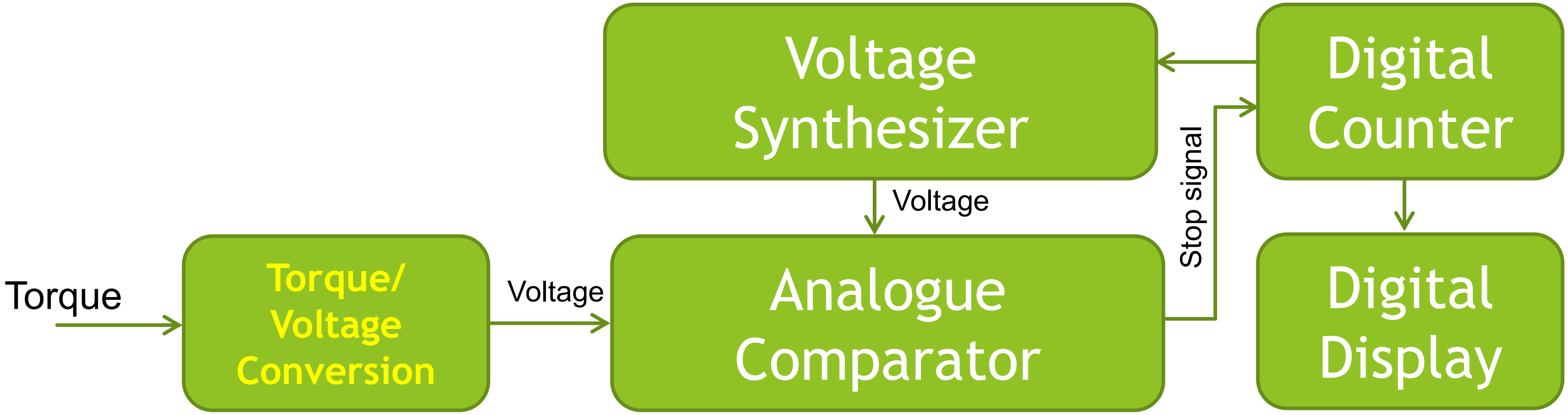
- Principle 1: Torque could cause a rigid body to make angular displacement, which could be transformed into the variations of voltages. Through the measurement of voltages, torque could be measured.

- Principle 2: Torque could cause a rigid body to deform, which could be transformed into the variations of resistance. Through the measurement of resistance, torque could be measured.

- Principle 3: Torque could cause a rigid body to make angular displacement, which could be transformed into the variations of capacitance. Through the measurement of capacitance, torque could be measured.

How to apply principle 1 to design digital measurement and sensing systems for torque?

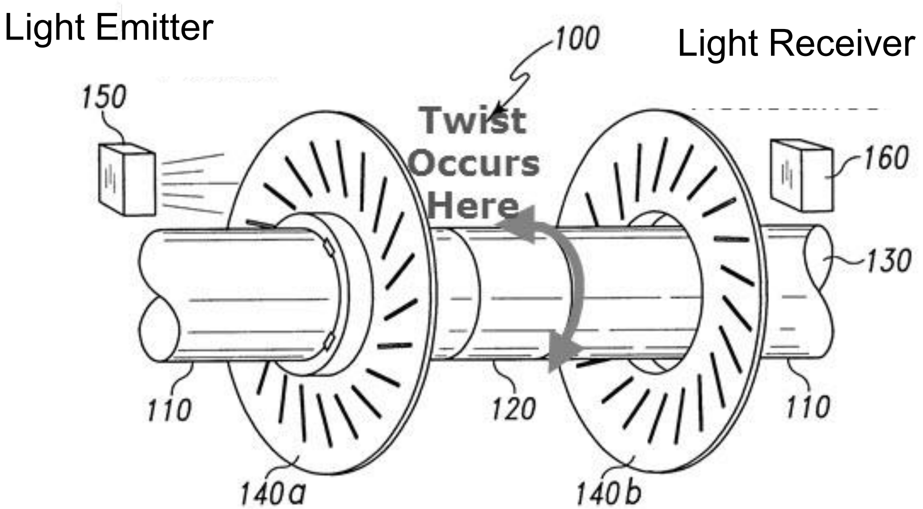
- Torque is converted to voltage which is measured by digital voltmeter (e.g. microcontrollers).

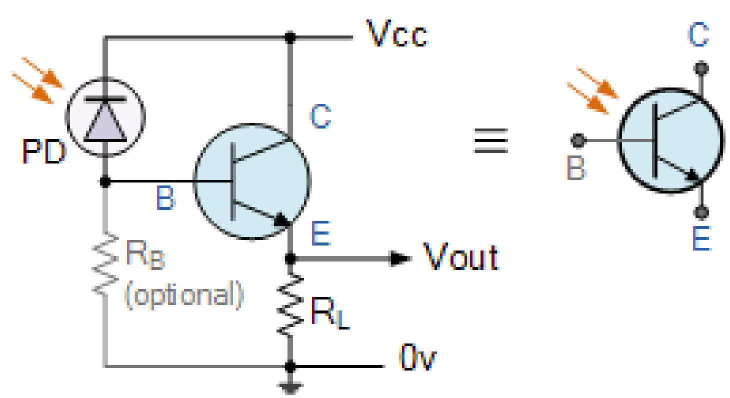


All microcontrollers are programmable digital sensors of voltage!

How to convert torque to voltage?

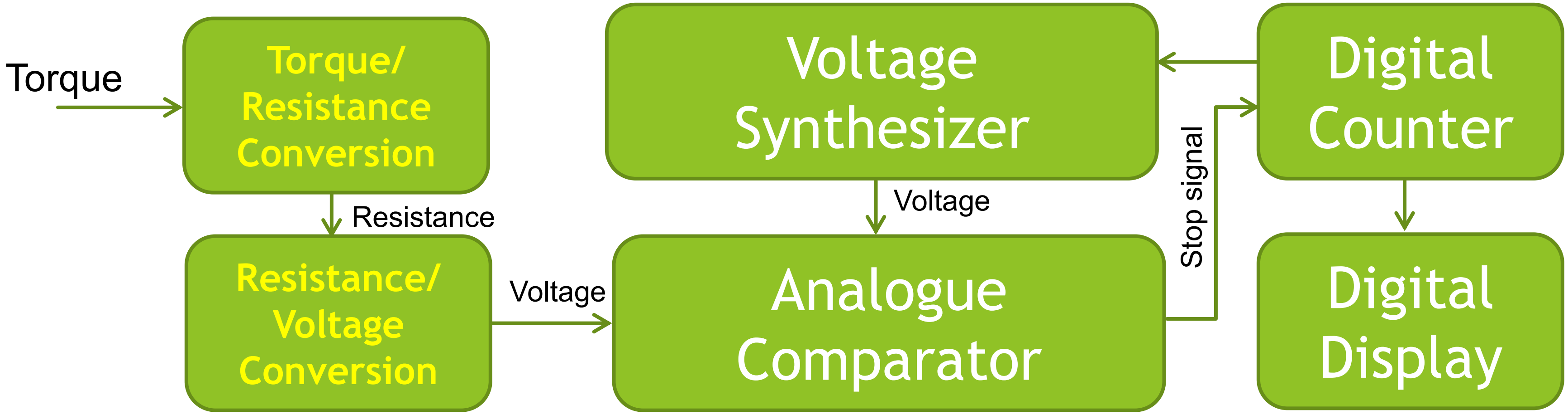
- A shaft can be treated as an torsional spring.
- An applied torque to the shaft will make it to undergo an angular displacement.
- Such displacement can be sensed by a pair of light emitter and light receiver, which converts light intensities into output of DC voltages.





How to apply principle 2 to design digital measurement and sensing systems for torque?

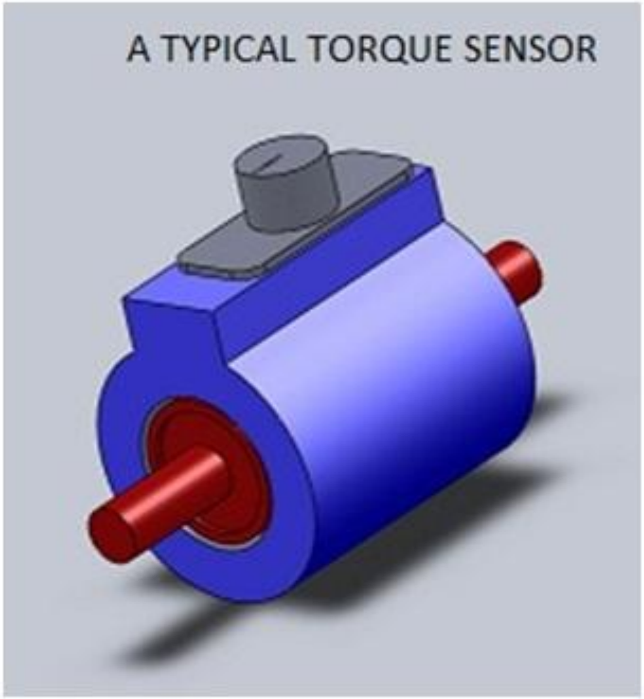
- Torque is converted to resistance which is then converted to voltage. Finally, the voltage is measured by digital voltmeter (e.g. microcontrollers).

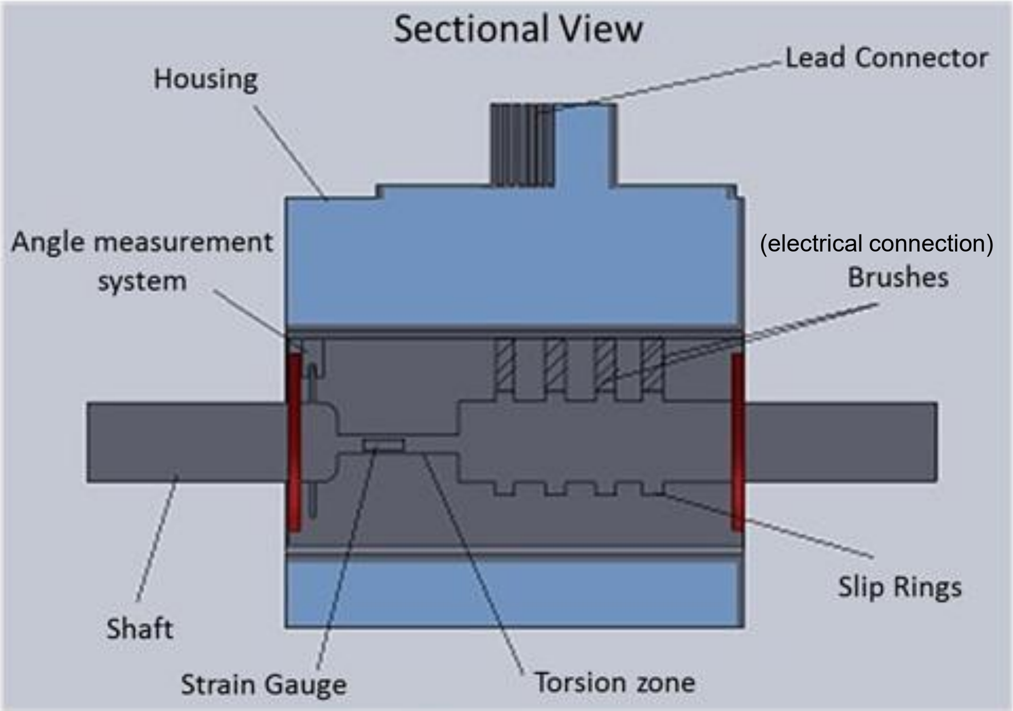


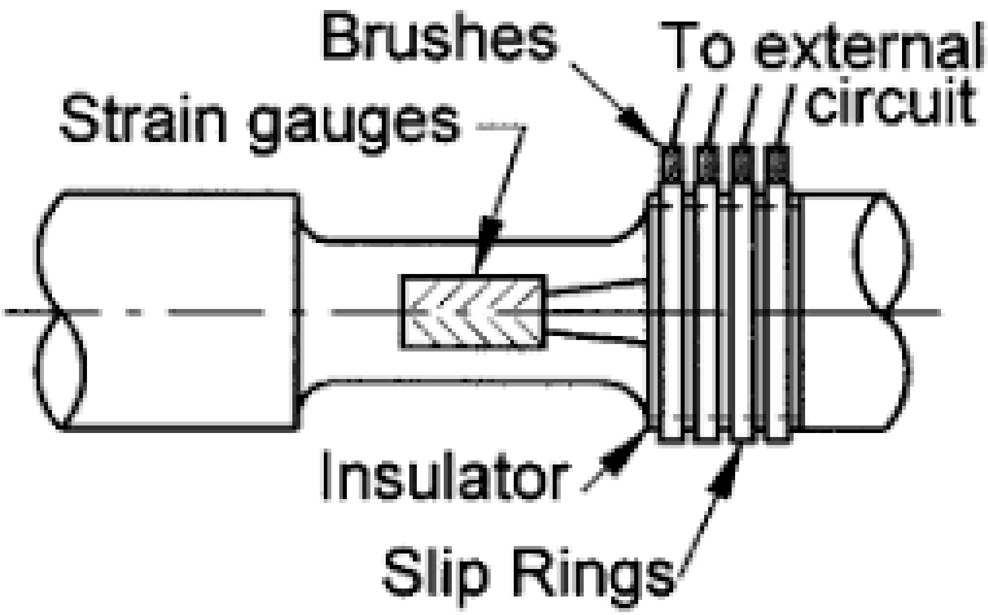
All microcontrollers are programmable digital sensors of voltage!

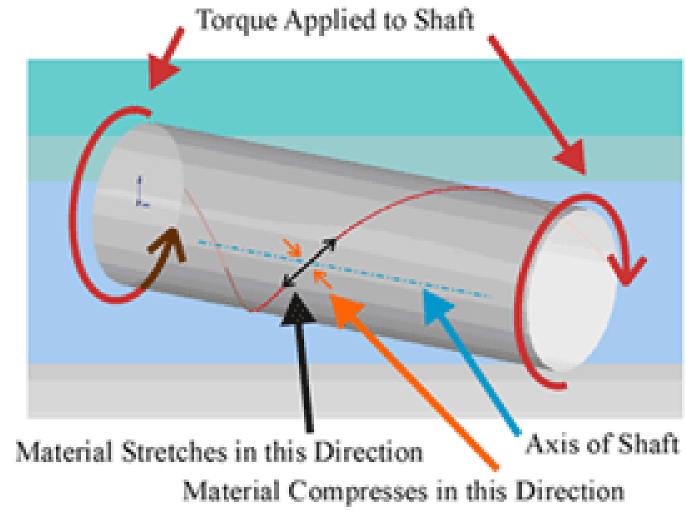
How to convert torque to resistance?

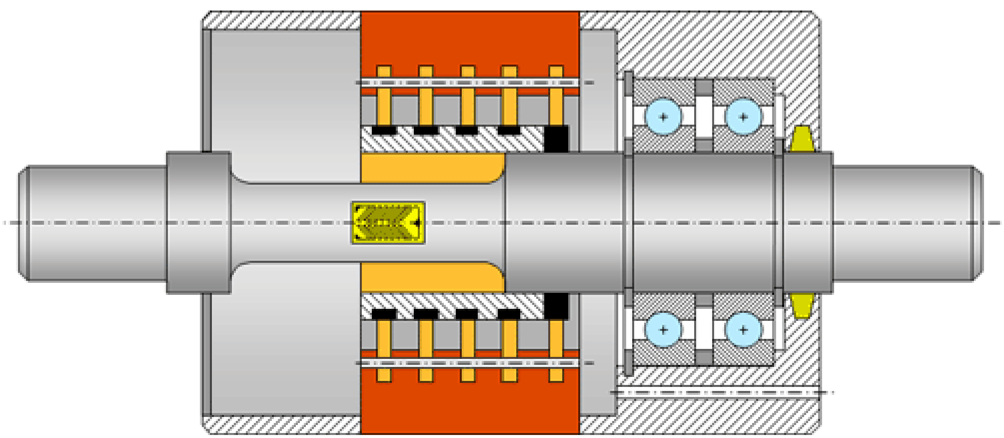
- A strain gauge is placed at the torsion zone of a rotating shaft.
- Brushes and slip rings supply current to strain gauges.
- An applied torque will cause the strain gauge to deform.
- The variation of torque will cause the change of the strain gauge's resistance.

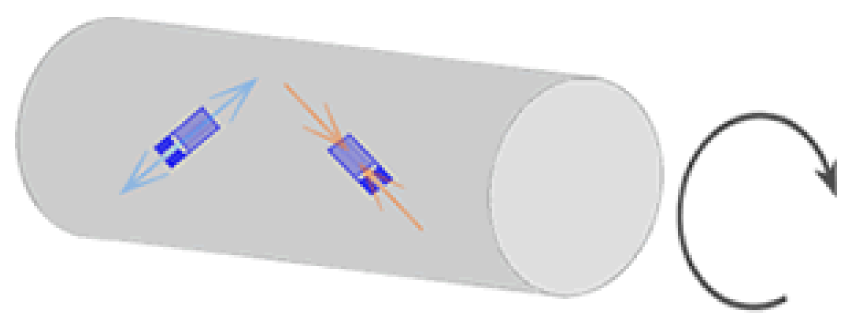






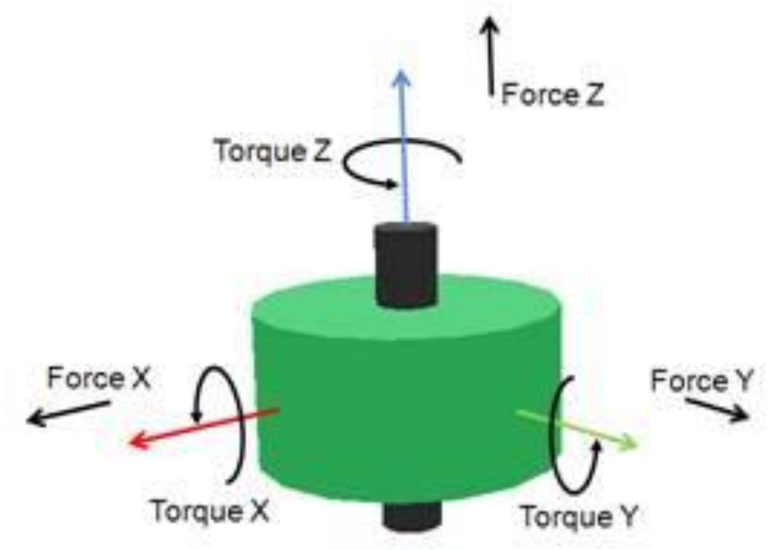


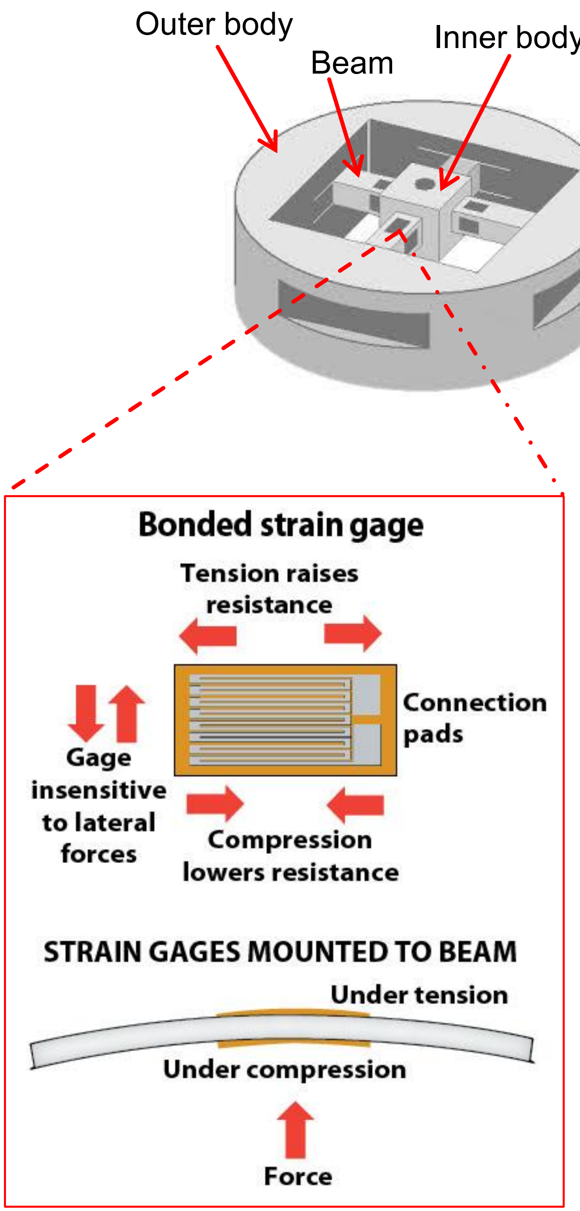




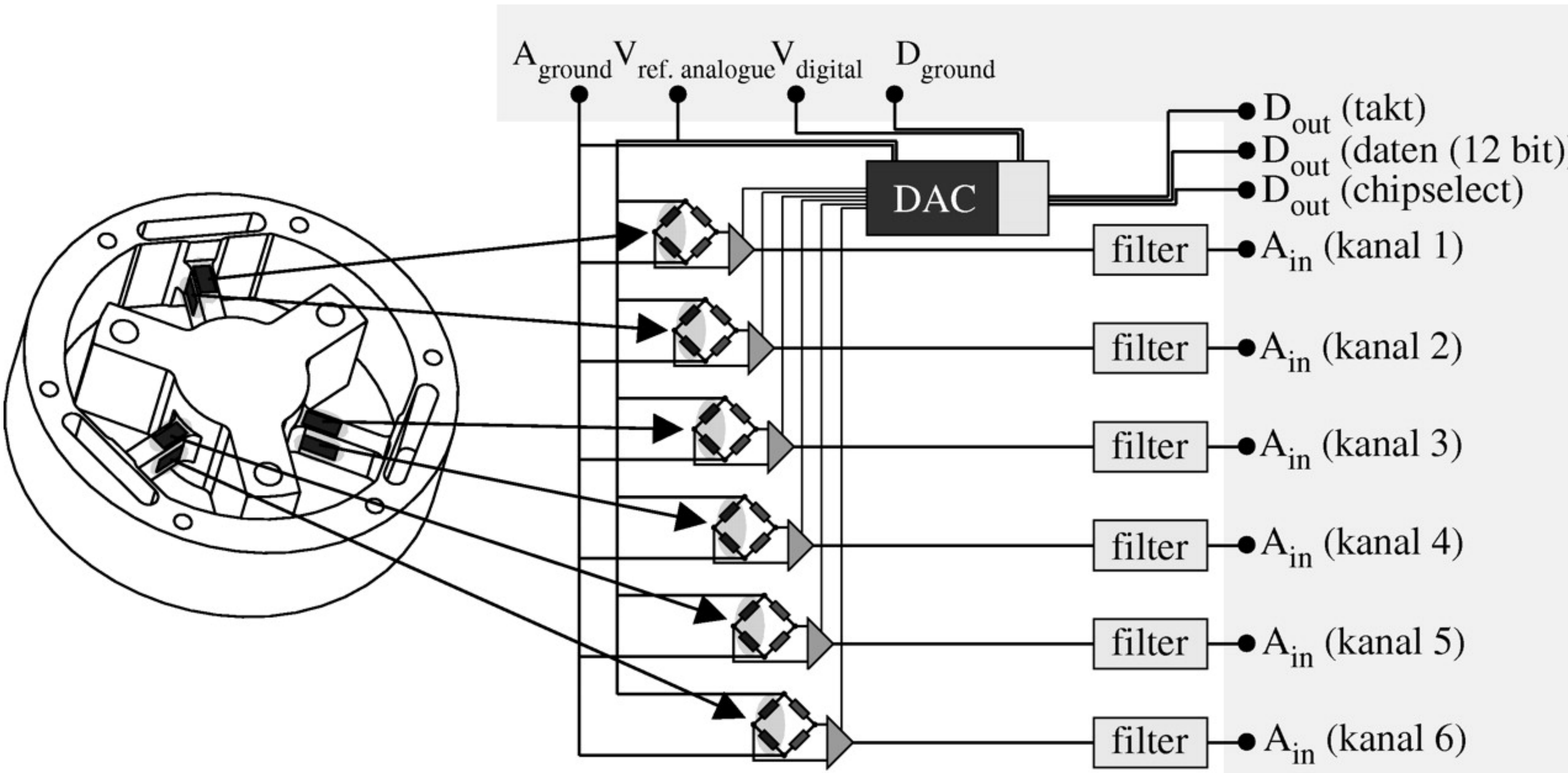
Strain Gauges Bonded to a Torque Shaft

How to achieve the measurement of force and torque within a single sensor?





Solution 1

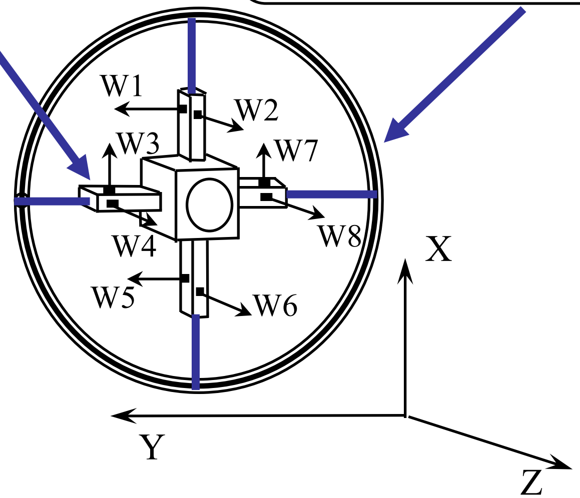


A ground
V ref. analogue
V digital
D ground
DAC
D out (takt)
D out (daten (12 bit))
D out (chipselect)
filter
A in (kanal 1)
filter
A in (kanal 2)
filter
A in (kanal 3)
filter
A in (kanal 4)
filter
A in (kanal 5)
filter
A in (kanal 6)

Solution 2:

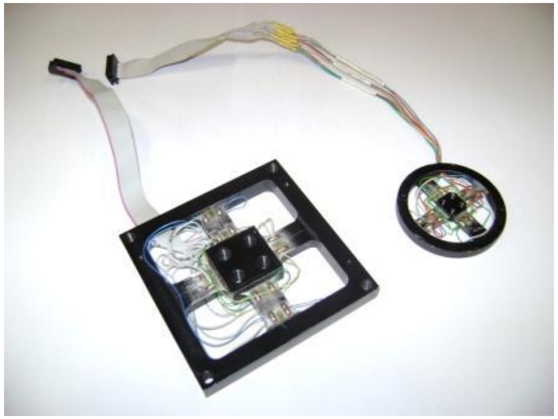
1. Each beam has two pairs of strain gage sensor which are placed at two adjacent facets.

2. Four beams have eight pairs of strain gage sensors. So, eight values of resistances can be measured.

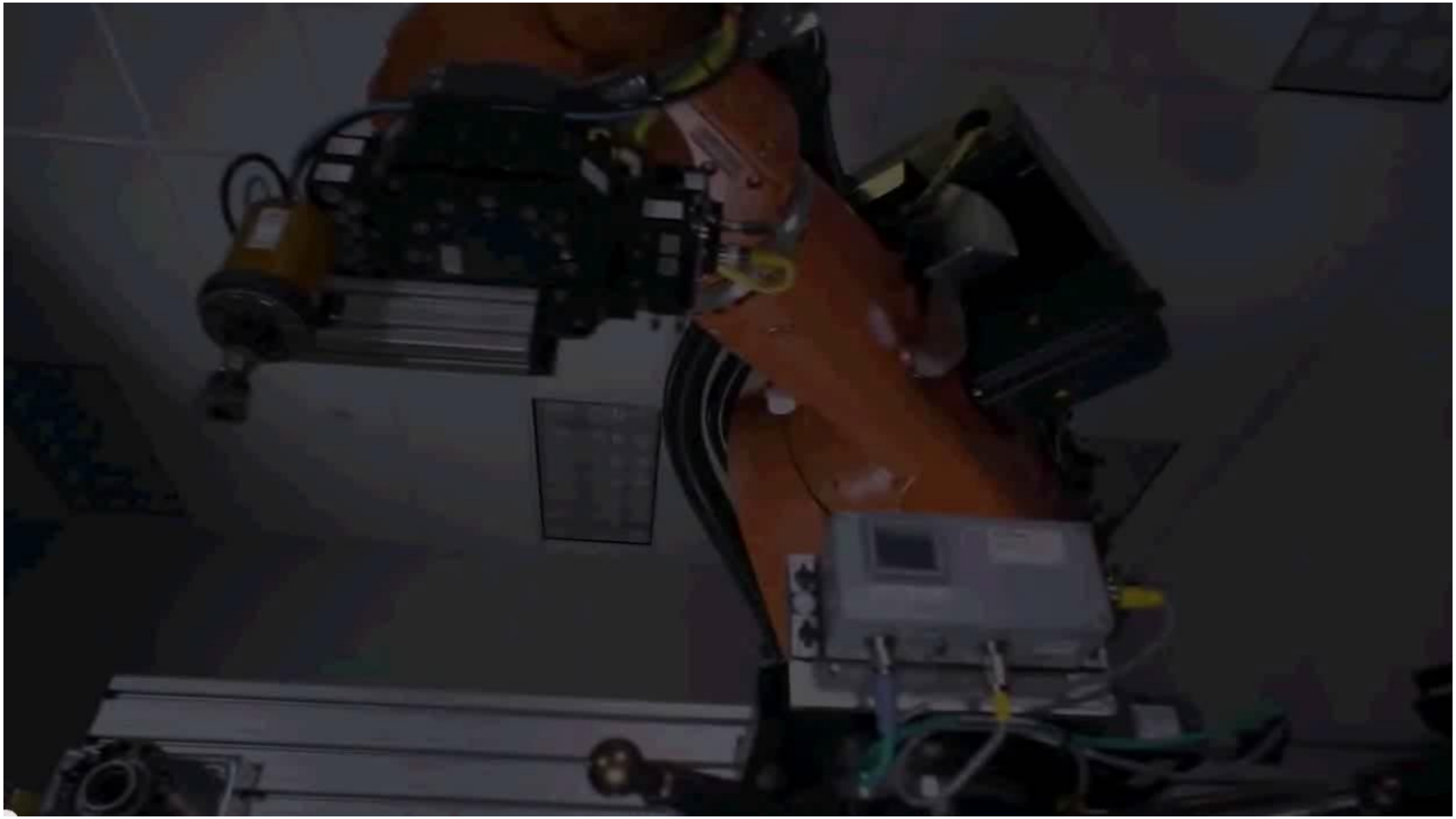


3. The eight values of measurements can be mapped to the output of three forces and three torques by a matrix:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{pmatrix} 0 & 0 & K_{13} & 0 & 0 & 0 & K_{17} & 0 \\ K_{21} & 0 & 0 & 0 & K_{25} & 0 & 0 & 0 \\ 0 & K_{32} & 0 & K_{34} & 0 & K_{36} & 0 & K_{38} \\ 0 & 0 & 0 & K_{44} & 0 & 0 & 0 & K_{48} \\ 0 & K_{52} & 0 & 0 & 0 & K_{56} & 0 & 0 \\ K_{61} & 0 & K_{63} & 0 & K_{65} & 0 & K_{67} & 0 \end{pmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix}$$

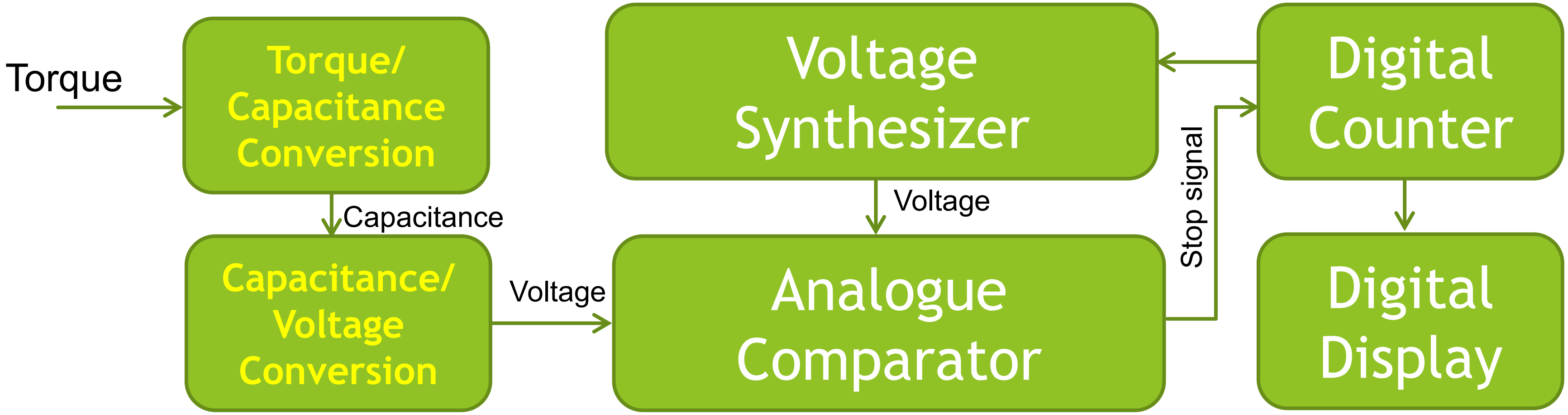


Example of Implementation



How to apply principle 3 to design digital measurement and sensing systems for torque?

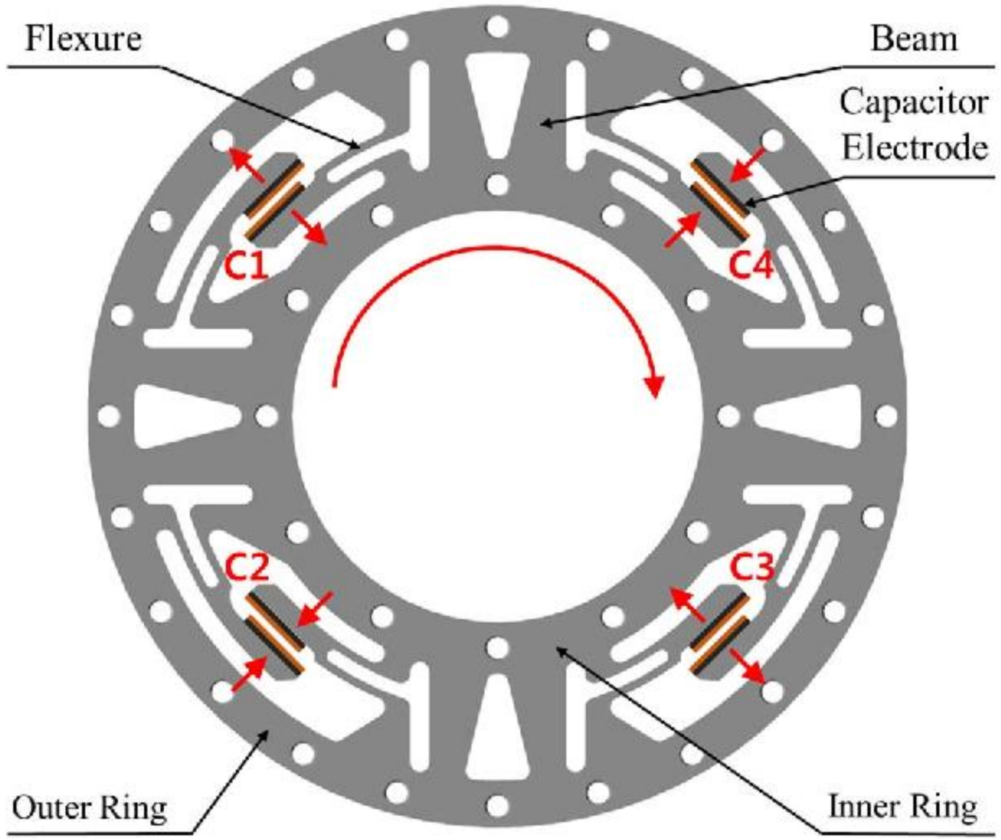
- Torque is converted to capacitance which is then converted to voltage. Finally, the voltage is measured by digital voltmeter (e.g. microcontrollers).



All microcontrollers are programmable digital sensors of voltage!

How to convert torque to capacitance?

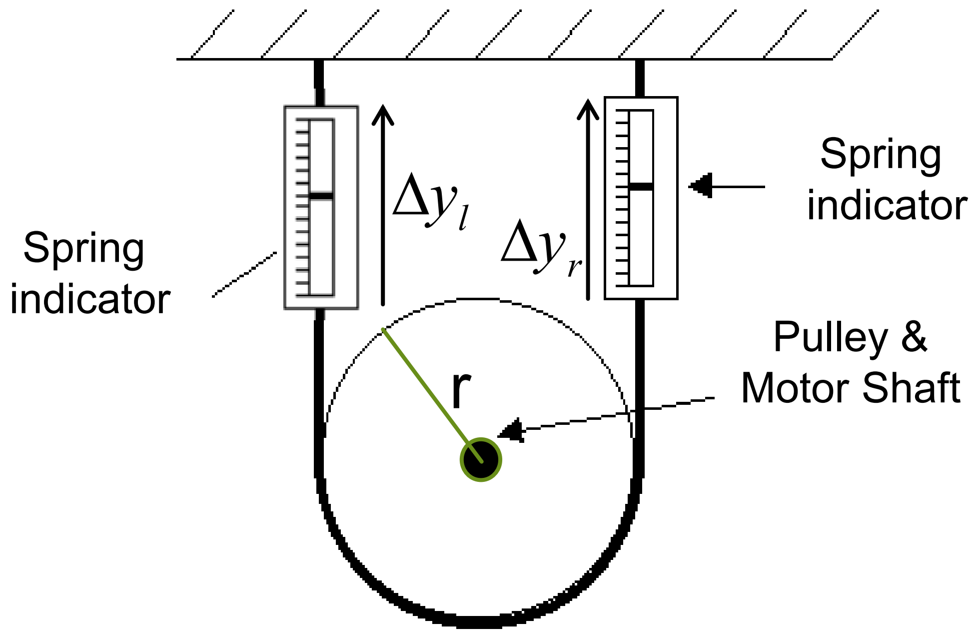
- An applied torque will produce angular displacements.
- Angular displacement will change the effective area between two parallel plate in a capacitor. Such change will vary the capacitance which could be measured.



- Clockwise rotation increases capacitances of C1 and C3
- Counter-clockwise rotation increases capacitances of C2 and C4

Other methods …

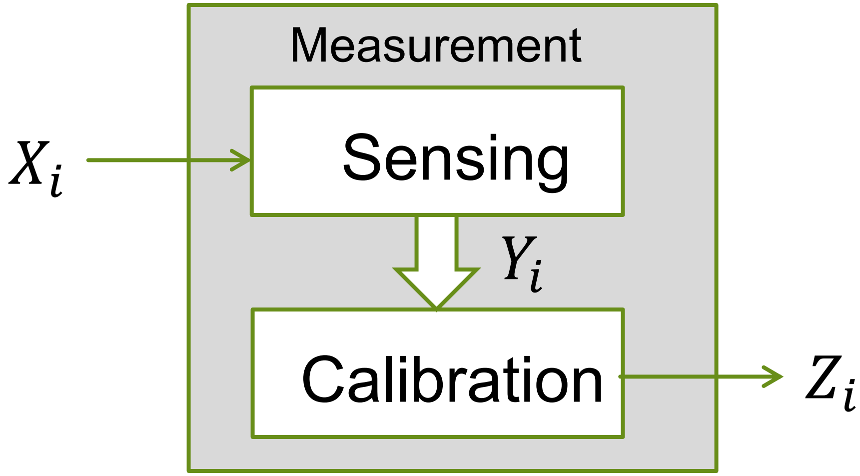
$$\tau = k\Delta y_l \bullet r - k\Delta y_r \bullet r$$ (τ *is* net torque at shaft)

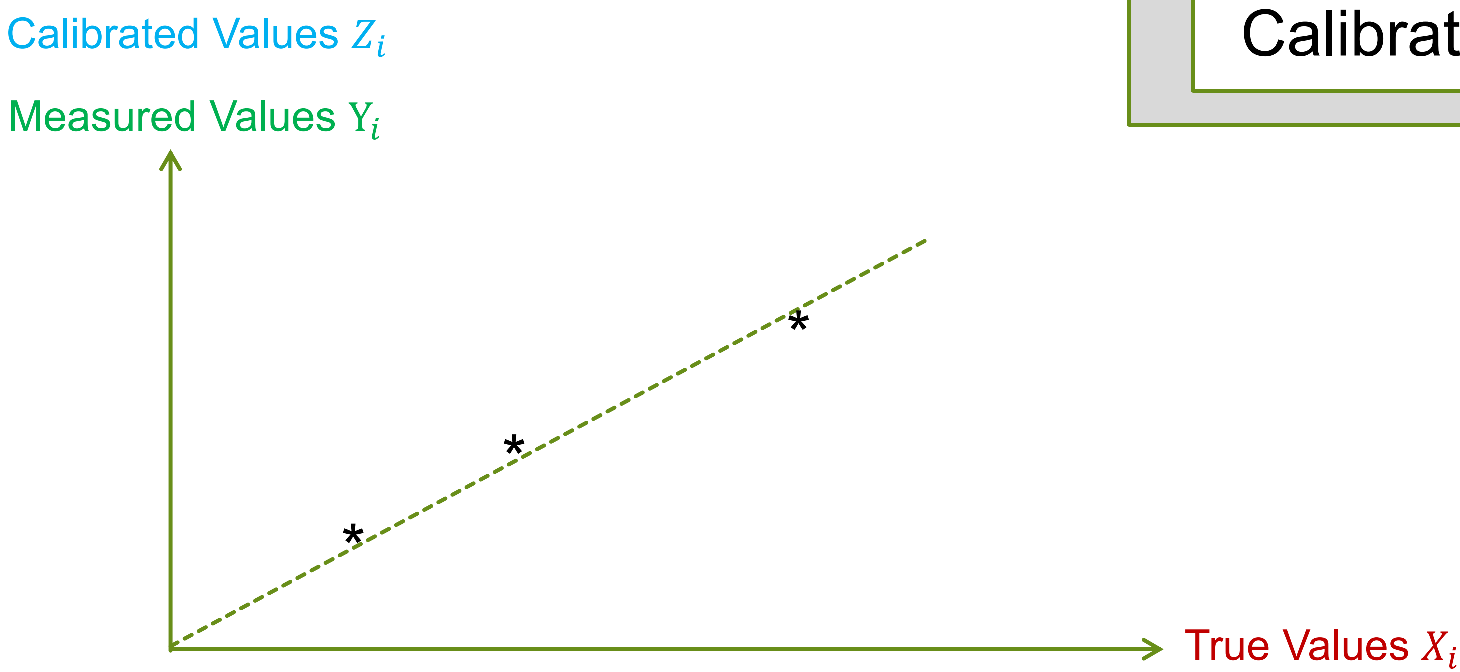


Pulley

Remember to Do Calibration

- Curve fitting for calibration:
  - Yi is produced by Xi
  - Zi is computed from Yi
  - Zi must be equal to Xi

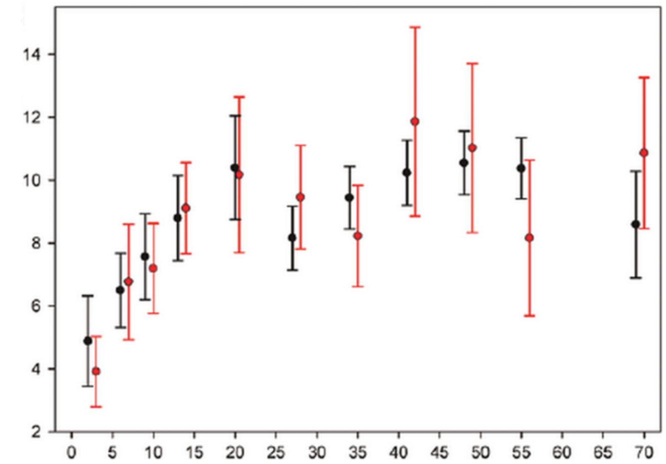


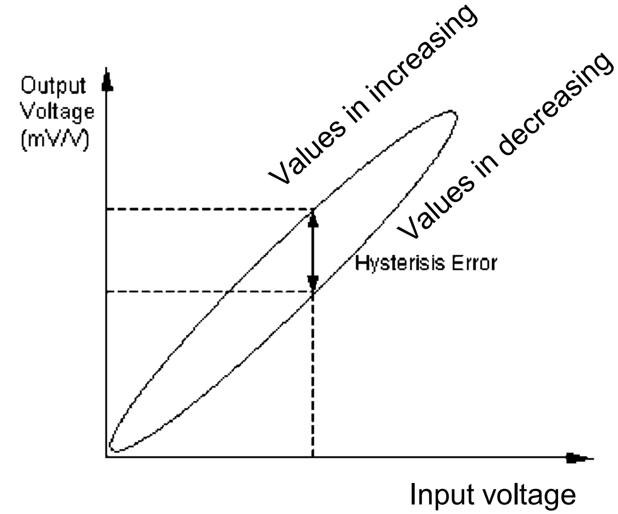


Remember to Do Error Analysis

- Systematic error = mean value – true value
- Repeatability error = value with maximum error – mean value
- Accuracy = value with minimum error – mean value
- Hysteresis error = |measured value in increasing – measured value in decreasing|

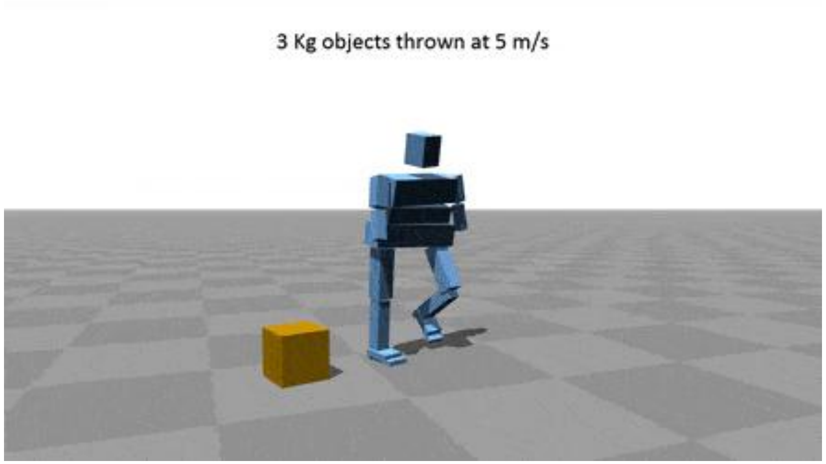
For each true value, we can do error analysis

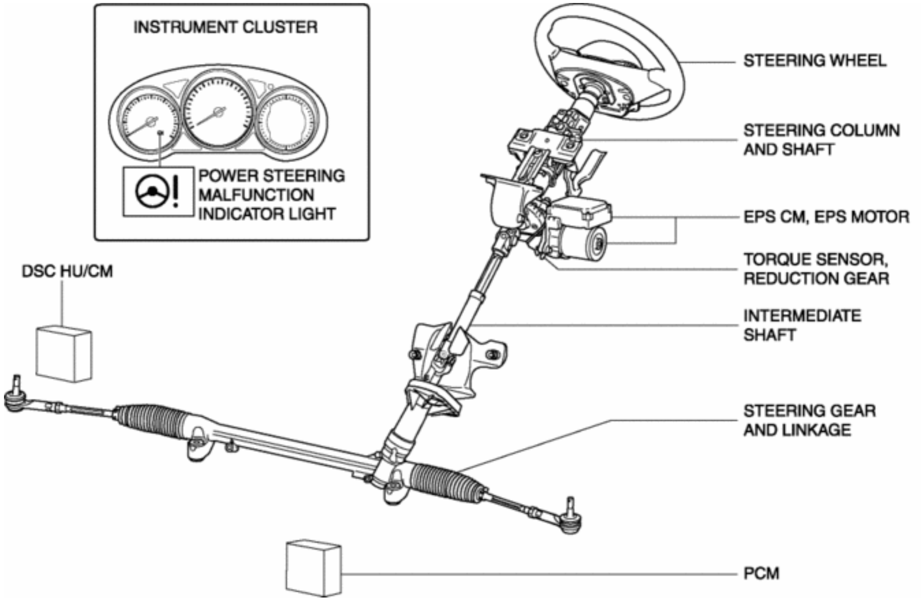




Summary

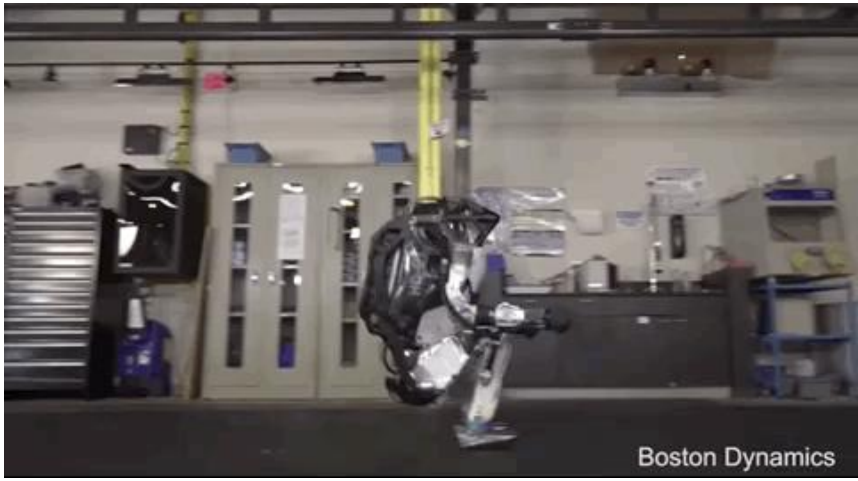
- Understanding of Torque
- Computation of Torque
- Measurement of Torque





Summary of Module 3

- Lecture 1:
  - Measurement of Position
- Lecture 2:
  - Measurement of Velocity
- Lecture 3:
  - Measurement of Acceleration
- Lecture 4:
  - Measurement of Force
- Lecture 5:
  - Measurement of Torque



"Ask not what your country can do for you – ask what you can do for your country," - John F. Kennedy

"Do not think that you are needy – think that you are needed in the world", - Manis Friedman

"Study will make you knowledgeable, resourceful, and hence more needed", - Xie Ming

Thank You for Listening!